

## $s$ - $d$ -Fermi-liquid model for high $T_c$ superconductivity involving suppression tunnelling by decoherence

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We consider a model system of two interacting Fermi-liquids, one of which is light and the other much heavier. In the normal state the lighter component provides a quantum mechanical bath coupled 'ohmically' to the heavier component in the sense of Caldeira and Leggett, suppressing thereby the band (tunnelling) matrix elements of the heavier component. Thus we lose the energy of delocalization. On the other hand, a superconducting ordering stiffens the bath spectral function at low energies and so restores the tunnelling. The resulting regain of the delocalization energy bootstraps so as to stabilize the superconducting order that caused it. It is conceivable that the motions parallel to the easy  $ab$ -plane and along the hard  $c$ -axis may also effectively correspond to the light and the heavy Fermi-liquids, respectively.

We propose a purely electronic mechanism of superconductivity for a system of two interacting Fermi-liquids, of which one is light and the other much heavier, corresponding, respectively, to two overlapping partially filled bands, one wide and the other narrow. This is based on a dynamical feature of such a two-component system that follows generally from the Born-Oppenheimer adiabatic approximation modified by quantum dissipation, as discussed in detail by Caldeira and Leggett<sup>1</sup>, namely that the higher (faster) subsystem acts as a dissipative 'bath' coupled to the heavier (slower) subsystem, blocking its coherent motion (tunnelling). This results in the loss of delocalization energy of the heavier component (i.e. band narrowing). This blocking is very effective if the coupling is 'ohmic', that is, if the bath spectral function  $J(\omega)$  vanishes linearly with  $\omega$  as  $\omega$  tends to zero. A superconducting ordering of the lighter component would, however, stiffen the spectral function for  $\hbar \omega < 2\Delta$  due to the opening up of the excitation gap  $2\Delta$ . This in turn should restore the tunnelling matrix element of the heavier component to its original magnitude, regaining thereby the energy of delocalization. The latter now bootstraps so as to stabilize the superconducting order energetically. Recently, in the context of the normal state transport, we had successfully invoked this mechanism for the suppression of tunnelling along the (hard)  $c$ -axis due to its 'ohmic' coupling to the motion parallel to the  $ab$  (easy)-plane<sup>2</sup>. Here, we now invoke this mechanism to explain superconductivity itself. We will do this within the framework of a highly truncated but still non-trivial model Hamiltonian that brings out the essential points

of this mechanism clearly. We believe that the mechanism is robust and will survive realistic modifications of the Hamiltonian.

Consider first a light Fermi-liquid described by a partially filled ( $s$ -like) band, but with a pair of tight-binding ( $d$ -like) Wannier orbitals introduced at the Fermi-level at neighbouring sites. The minimum Hamiltonian for our purpose is then

$$H = -t_d(d_1^\dagger d_2 + d_2^\dagger d_1) + \sum_{k\alpha} \epsilon_k C_{k\alpha}^\dagger C_{k\alpha} + (V/N) \sum_{q\alpha} (e^{iq \cdot R_1} d_1^\dagger d_1 + e^{iq \cdot R_2} d_2^\dagger d_2) C_{k+q,\alpha}^\dagger C_{k,\alpha} \equiv H_0 + H_B + H_{0B}, \quad (1)$$

where  $H_0$  describes the two tight-binding orbitals at sites  $R_1$  and  $R_2$ , with bare tunnelling matrix element  $t_d$  between them.  $H_B$  is the light Fermi-liquid 'bath', and  $H_{0B}$  is the dissipative coupling to bath fluctuations  $\rho_{\alpha(q)}$  with  $\rho_{\alpha(q)} \equiv \sum_k C_{k+q,\alpha}^\dagger C_{k,\alpha}$ . Here  $V$  is the local interaction with the bath, and  $\epsilon_k = \hbar v_F(k - k_F)$ .  $N$  is the number of lattice sites. Other symbols have their usual meaning. Our system is assumed to be two dimensional, anticipating layered materials (i.e. the  $\text{CuO}_2$  sheets).

As stated before, this is a highly truncated Hamiltonian. We have taken the  $d$ -like (heavy) Fermions to be spinless and hence no  $s-d$  mixing and no on-site Hubbard  $U$ . (In fact the latter is infinite here.) However, the interaction  $V$  is all important and represents 'interband coulomb' repulsion (i.e. repulsion between the nominally  $\text{Cu}^{2+}:3d$  and the  $\text{O}^{2-}:2p$  bands).

We can readily re-write  $H_{0B}$  as

$$H_{0B} = i(V/N) \sigma_z \sum_{q\alpha} \sin(q \cdot R) \rho_{q\alpha}, \quad (2)$$

where  $\sigma_z$  is the usual Pauli matrix, now for the two states (the two Wannier orbitals), and  $2R$  is the vector separating the two sites 1 and 2. We can now reduce our problem to the usual spin-Boson problem<sup>1</sup> by treating the density fluctuations  $\rho_{q\alpha}$  as bosonic operators with energies  $\hbar V_F q$  in the spirit of the Tomonaga approximation<sup>3</sup>. All we are saying really is that the light Fermionic system can be approximated as a Bosonic electron-hole bath which is well known<sup>1</sup>. Then, the bath spectral function<sup>1</sup> can be written as (for two dimensions).

$$J(\omega) = \pi/2 \sum_q (V^2/N \hbar) (\sin q \cdot R)^2 \delta(\omega - \omega_q) = \eta \omega \quad \text{for } \omega < \omega_c$$

with  $\eta = (\beta/4 \hbar) (Va/V_F)^2$ . (3)

Here  $w_c$  is an upper cut-off for the bath excitations that couple effectively to the heavier subsystem. We have replaced the form factor  $(\sin q \cdot R)^2$  by its average over the two-dimensional Brillouin zone and called it  $\beta$ .

The linear  $w$  dependence of  $J(w)$  for small  $w$  makes the dissipative coupling ohmic. The dimensionless coupling constant is now

$$\alpha = \eta/2\pi \hbar = \beta/8\pi (Va/\hbar V_f)^2 \quad (4)$$

We expect  $\alpha > 1$  for large enough  $V$ . Now, for  $\alpha > 1$  we know that the tunnelling matrix element  $t_d$  renormalizes to zero<sup>1</sup>. Hence we lose the delocalization energy completely in the normal state. Now, consider the possibility of a superconducting order at 0 K. Let us anticipate the superconducting order with the gap parameter  $2\Delta$ . The opening of such a gap (i.e. absence of low-lying excitations) in the excitation spectrum will now stiffen  $J(w)$  making it vanish for  $\hbar w < 2\Delta$ . For  $\hbar w > 2\Delta$ , however, it is essentially unaffected. Thus, the tunnelling matrix element is now re-normalized as<sup>1</sup>:

$$t_d \rightarrow \tilde{t}_d = t_d \exp \left[ - \int_{2\Delta}^{\hbar w_c} \frac{1}{2\pi \hbar} \frac{J(w)}{w^2} dw \right] \\ = t_d (2\Delta/\hbar w_c) \alpha \quad (5)$$

We, therefore, regain the delocalization energy due to the finite tunnelling  $\tilde{t}_d$ . We now assume that the energetic effect of having  $N$  such  $d$ -like Wannier orbitals (i.e. a heavy band) is additive so as to get an extensive contribution. We thus get the energy stabilization  $\Delta E$  per site in the superconducting state (for a two-dimensional system):

$$\Delta E = 2t_d \left( \frac{2\Delta}{\hbar w_c} \right) \alpha \quad (6)$$

This must be equated now with the condensation energy  $1/2 N_F \Delta^2$  normally associated with the superconducting state, in the spirit of bootstrap. This gives two solutions

$$2\Delta = 0, \quad \text{or} \quad 2\Delta = \hbar w_c \left[ \frac{N_F (\hbar w_c)^2}{16 t_d} \right]^{(\frac{1}{\alpha-2})} \quad (7)$$

The second solution is a stable fixed point for  $\alpha > 2$ , implying strong coupling. Thus, to stabilize the superconducting state there is a critical threshold  $\alpha = 2$  that must be exceeded.

It is to be noted here that the above treatment, based on an appeal to the total energy stabilization does not manifestly give the details of the nature of the ordered state, except that the latter has a gap in its excitation spectrum, i.e. absence of low-lying excitations. Such an

energy stabilization provides the thermodynamic stability but as to how the latter entrains the Fermions to produce the ordered state would require a detailed microscopic treatment. It seems natural to extend Hamiltonian to the 'resonance level model' along lines initiated by C. M. Varma and his collaborators<sup>4</sup>.

Finally, we would like to point out that the kinetic stabilization of the superconducting state by the energy gain due to delocalization is somewhat reminiscent of the interlayer pairing discussed by Wheatley, Hsu and Anderson<sup>5</sup>, where the pairing is energetically favoured by the pair delocalization along the  $c$ -axis and consequent energy gain.

*Note added in proof.* The excitation energy gap in question must be a co-moving gap, that is an 'uncompensated' gap, that does not forbid band motion. Superconductivity gap is the only one of this kind known. The author thanks Pramod Gupta for raising this question.

1. Caldeira, A. O and Leggett, A. I., *Ann. Phys.*, 1983, 149, 374-456
2. Kumar, N and Jayannavar, A. M., *Phys. Rev.*, 1992, B45, 5001-5004.
3. See Mahan, G. D., *Many Particle Physics*, Plenum Press, NY, Second edition. 1990.
4. See the recent preprints, Giamarchi, T., Varma, C. M., Ruckenstein, A. E. and Nozières, P., (1992). Also, Perakis, I. E., Varma, C. M. and Ruckenstein, A. E. (1992).
5. Wheatley, J. M., Hsu, T. C. and Anderson, P. W., *Phys. Rev.*, 1988, B37, 5897-5900.

Received 4 March 1993, accepted 6 March 1993

## Large size fractals in ion conducting polymers: A novel experimental observation

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A novel experiment is described in which large size fractals have been seen in ion conducting polymers obtained by doping polyethylene oxide (PEO) with  $\text{NH}_4\text{I}$ . The addition of  $\text{Al}_2\text{O}_3$  in the polymer complex helped in the initiation of cluster nucleation. The measured fractal dimension ( $\sim 1.7$ ) is typical of 'diffusion limited aggregates'.

Ever since the discovery of Mandelbrot about the widespread presence of fractals in nature, physicists have diverted their attention towards it to discuss a range of phenomena occurring in disordered systems for which fractal concepts provide a natural framework. Fractals are generally observed in far-from-equilibrium phenomena whether it is in nature (mountains, snow