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ON THE PROBLEM OF NONTRANSPORT OF ELECTRONS IN DISORDERED SYSTEMS (ANDERSON LOCALIZATION)

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ABSTRACT

The problem of non-transport (Anderson localisation) in cellularly disordered systems is well known to be related to the question of convergence of the Brillouin Wigner perturbation expansion for self-energy involving random energy denominators. In this preliminary communication an explicit criterion for the absolute convergence in the probability sense, of a partial series, has been obtained in terms of the width Δ of the rectangular distribution of the random site-energies and the band width W. The partial series considered involves only terms with non-repeating site indices.

CONSIDER a cellularly disordered crystal described by the Anderson Hamiltonian in the i site representation as

$$H = \sum_l \epsilon_l a_l^+ a_l + \sum_{l \neq m} V_{lm} a_m^+ a_l \qquad (1)$$

where a_l^+, a_l are the creation, annihilation fermion operators corresponding to the non-degenerate (Wannier) orbital at site l. The ϵ_l's represent the statistically independent random orbital energies having a rectangular distribution of width Δ centred at zero. V_{lm} is the transfer matrix element.

For simplicity we consider a simple-cubic lattice with nearest-neighbour interactions only. Thus the matrix elements $V_{lm} = V \delta_{l, m+\Delta}$ where Δ spans nearest neighbours. It is well known[1-2], that the problem of localisation is connected with the question of convergence of the Brillouin-Wigner perturbation series for the self-energy[2]:

$$E = \epsilon_0 + \sum_{l_1} V_{0l_1} \frac{1}{E - \epsilon_{l_1}} V_{l_1 0} + \sum_{l_1, l_2} V_{0l_1} E \frac{1}{-\epsilon_{l_1}} V_{l_1 l_2} E \frac{1}{-\epsilon_{l_2}} V_{l_2 0} + \cdots + \sum_{l_1 l_2, \ldots, l_n} \frac{V_{0l_1} V_{l_1 l_2} \cdots V_{l_n 0}}{(E - \epsilon_{l_1})(E - \epsilon_{l_2}) \cdots (E - \epsilon_{l_n})} + \cdots \qquad (2)$$

Anderson[1] has discussed the criterion for the localisation of an electron placed at site '0' in terms of the convergence of an analogous series.

In the present analysis, however, we shall discuss the criterion for the absolute convergence of a partial series consisting of terms with non-repeating indices only. Thus the nth order term of the partial series involves Z^n terms, approximately equal to the number of self-avoiding closed random walks of length n from a given site (Z being the number of nearest neighbours). The absolute value of a typical nth order term in this partial series will be denoted by T_n, where

$$T_n \simeq \prod_{i=1}^{n} |x_i - \epsilon|^{-1}$$

with

$$x_i = \frac{\epsilon_i}{Z|V|}, \quad \epsilon = \frac{E}{Z|V|}.$$

From the hypothesis on $\{\epsilon_i\}$ it follows that the sequence $\{x_i\}$ is a sequence of independently and identically distributed random variables each having a rectangular distribution over the range $(-\Delta/2Z|V|, \Delta/2Z|V|)$. To discuss the convergence of $\sum_n T_n$ we can now use results from modern probability theory.

We write $T_n = \exp S_n$ where $S_n = -\sum_{i=1}^{n} \log |x_i - \epsilon|$. If we set $Y_i = -\log |x_i - \epsilon|$ then it is well known (*see* for instance Feller[3]) that

Prob $\{S_n \geq 0$ for infinitely many $n\}$ = one or zero according as the mean $\bar{Y}_1$ of $Y_1 \geq 0$ or < 0. Thus if $\bar{Y}_1 \geq 0$ the $T_n \geq 1$ for infinitely many n and hence $\sum_n T_n = \infty$ with probability one. If $\bar{Y}_1 < 0$, we use the strong law of large numbers[3] to conclude that $S_n < n\bar{Y}_1/2$ for all large n. Thus if $\bar{Y}_1 < 0$, $T_n = \exp S_n < C^n$ where $0 < C = \exp(\bar{Y}_1/2) < 1$ and hence $\sum_n T_n < \infty$ with probability one.

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We may summarise the above discussion as follows : The series $\sum_n T_n$ converges with probability one if $\bar{Y}_1 < 0$ and diverges with probability one if $Y_1 \geqslant 0$.

It may be verified that

$$\bar{Y}_1 = -\frac{1}{4}\left\{(1+x)\log(1+x) + (1-x) \times \log|1-x| + 2\left(\log\frac{\Delta}{2ZV} - 1\right)\right\}$$

$$= -f(x) \text{ say}$$

where $0 \leqslant x = (2E/\Delta) < \infty$. Of course, the case $-2E/\Delta = x \leqslant 0$ is just the mirror image.

Using the monoticity of $f(x)$, we get the following criterion for convergence or localisation in the different cases as given above in a tabular form.

In conclusion, we may remark that the transition from the delocalised states in the centre of the band to the localised states near the band edges can take place within or outside the band according as $0 < \Delta/VZ\,(\equiv 2\Delta/W) < e$ or $e < 2\Delta/W < 2e$. The first case may imply the existence of mobility edges. Thus the mobility edges move inwards as $\Delta/W \to \infty$. For $2\Delta/W > 2e$ all staates are localised. The relationship of the present work to that of Economou and Cohen[4] is not transparent at the moment.

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Δ/VZ / $2E/\Delta$	$< e$	$e < \frac{2\Delta}{W} < 2e$	$\frac{2\Delta}{W} > 2e$
$\leqslant 1$	Delocalised	$\exists X_0\ 0 \leqslant X_0 \leqslant 1$ ∋ $X \leqslant X_0$ delocalised, $X > X_0$ localised. This X_0 is the unique zero of f in $[0, 1]$	Localised
> 1	$\exists X_0 > 1$ ∋ $X \leqslant X_0 \Rightarrow$ delocalised, $> X_0$ localised. This X_0 is the unique zero of f in $(1, \infty)$	Localised	Localised

1. Anderson, P. W., *Phys. Rev.*, 1958, **109**, 1492.
2. Ziman, J. M., *J. Phy. C. Ser.* 2, 1969, **2**, 1230.
3. Feller, W., *Introduction to the Theory of Probability and Application*, Vol. I, 1967; Vol. II, 1972, John Wiley, New York.
4. Economou, E. N. and Cohen, M. H., *Phys. Rev.*, 1972, 5 (8), 2931.

INDUCED BREEDING OF THE FRESHWATER CATFISH *CLARIAS BATRACHUS* (LINN.) BY USING PITUITARY GLANDS FROM MARINE CATFISH

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INTRODUCTION

THE freshwater catfish, *Clarias batrachus* (Linn.), commonly known as 'magur', has great cultural possibilities. The possession of accessory respiratory organs in this fish renders it suitable for culture in swamps and derelict ponds, where carps cannot be cultured. Because of its hardy nature, the survival rate is quite high, and as high a production as 1,07,500 kg/ha/year has been reported from Thailand by Sidthimunka *et al.* (1966), by stocking at a rate of 20 lakhs of fry per hactare. This compares very favourably with the major carp production of about 4,000 kg/ha/year only in tropical climates (Lakshmanan *et al.*, 1971). Further, the 'magur' is a highly esteemed table fish in the inland areas of our country. In view of these favourable features, investigations on the culture of *Clarias batrachus* have been taken up at the Fisheries College, Mangalore. Since availability of stocking material is a pre-requisite for culture, experiments were conducted successfully to breed the species by hypophysation technique and the results therefrom are reported in this paper. Because

of the paucity of suitable freshwater fishes in the locality, an attempt was made to breed this freshwater fish by using pituitary glands taken from marine fish, which are available locally in good numbers.

Material and Methods

The adult fish used for breeding experiments were collected from local wells during February to April, 1972 and maintained in plastic-lined pools and cement cisterns. They were fed with artificial pelleted feed daily. The pituitary glands used for breeding were collected from mature *Clarias batrachus,* and marine catfish (*Tachysurus* sp.) in May and preserved in glass vials in absolute alcohol. The marine catfish from which pituitary glands were collected were immature. Each pituitary gland was weighed in an electrically operated mono-pan balance. The weights of pituitary glands collected from *Clarias* ranged from 1·7 mg to 2·0 mg, while those of *Tachysurus* sp. varied from 8·6 mg to 16·3 mg.

The breeding experiments were conducted in glass aquarium tanks, measuring 75 × 40 × 40 cm, in which a 30 cm depth of water was maintained. In each tank, one pair of the species (one female and one male) of comparable size was introduced. Both the female and the male fish were given the same dose of pituitary extract, varying from 12 to 30 mg/kg weight of fish. The pituitary extract was administered in two injections, a provocative dose of 5–10 mg/kg and a final dose of 8 to 20 mg/kg, with a 5–6 hour interval, as is usually done for breeding Indian major carps (Alikunhi *et al.*, 1960; Chaudhuri, 1960). The pituitary extract prepared in distilled water was administered by intramuscular injections, slightly above the lateral line in the posterior region of the body.

The fishes used for breeding ranged from 75 to 200 g in weight. The water temperature in aquarium tanks varied from 24° C to 28° C. The experiments were conducted during June–July, 1972.

Results

Clarias batrachus was successfully bred by both homoplastic and heteroplastic pituitary extract injections. In the latter case, pituitary gland taken from a marine catfish, *Tachysurus* sp., was used, at a higher dose of 30 mg/kg. There appears to be no earlier record of breeding a freshwater fish by injecting pituitary extract from a purely marine fish.

Eighty per cent of the fish injected responded and spawned. Breeding took place within 8–10 hours after the second injection. Spawning was complete in all the cases. The male fish chased and injured the female with its pectoral spines during the spawning act. In a few cases, the badly injured female fish died within a day or two.

The eggs were siphoned out from the aquarium tank after removing the spent fish. The percentage of fertilization, which was estimated by examining 100 eggs under a dissection microscope, was found to be uniformly high, ranging from 77 to 99. The number of eggs released by individual fish ranged from 1968 to 7,380, the number per ml being 250. The average outer and inner diameters of developing egg at yolk-plug stage were 1·74 mm and 1·63 mm respectively. The eggs were hatched in enamel trays and in double hatching hapas fixed in cement cisterns. The percentage of hatching was higher in the latter case, probably due to better aeration. The eggs hatched out within 26 to 32 hours after breeding, with the water temperature ranging from 24 to 28° C. As the diameter of the developing eggs was only about 1·8 mm, a few of them passed through the round meshed inner hatching hapa. The hatchling had a bulky yolk sac and were lying at the bottom on their sides for most of the time. With the progression of the absorption of yolk, they became more and more active. The yolk sac was completely absorbed on the fifth day after hatching. At this stage, they were fed on zooplankton. The hatchlings stocked in a manured cement nursery tank have grown quite fast, attaining a size of 25 mm in 6 days.

Discussion

Sundararaj and Goswami (1969) have reported successful induced breeding of *Clarias batrachus* by administering intraperitoneal homoplastic pituitary extract injections. As they have calculated the dosage of pituitary in terms of number of glands injected, and not by weight, the dosages used by us are not comparable. Tonsanga *et al.* (1962) and Sidthimunka *et al.* (1966) have succeeded in induced breeding of *Clarias macrocephalus* by using 13 to 39 mg of pituitary glands on fishes ranging in size from 100 to 190 g. These dosages are much higher than those used by the present workers on *C. batrachus.* Carreon (1972) reported induced breeding of *C. batrachus* by hypophyseal treatment at the Institute of Fisheries Development and Research, of the University of Philippines. The number of fertilized eggs in his experiments ranged from 2,000 to 7,000, which is very similar to the present studies. The percentage of fertilization

observed by him was also very high (80–95), as observed by us.

The successful use of a marine catfish pituitary glands for induced breeding of a freshwater catfish is of great significance. The shortage of pituitary glands is often quoted as an important factor holding up large scale production of quality fish seed of Indian major carps by induced breeding. As different species of marine catfish are landed in large numbers along both the coasts of India, utilization of pituitary glands from these for induced breeding of Indian major carps, if proved successful, will substantially help the quality fish seed production programme of the country.

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1. Alikunhi, K. H., Vijayalakshmanan, M. A. and Ibrahim, K. H., *Indian J. Fish.*, 1960, 1 (1), 1.
2. Carreon, J. A., "Induced breeding of catfish," *FAO, Aquaculture Bulletin*, 1972, 4 (2), 3.
3. Chaudhuri, H., *Indian J. Fish.*, 1960, 7 (1), 20.
4. Lakshmanan, M. A. V., Sukumaran, K. K., Murthy, D. S., Chakraborty, D. P. and Philipose, M. T., *J. Inland Fish. Soc. India*, 1971, 3, 1.
5. Sidthimunka, A., Sanglert, J. and Pawapootanon, O., "The culture of catfish (*Clarias* spp.) in Thailand," *FAO World Symposium on Warm-Water Pond Fish Culture*, 1966, **44** (5), 196.
6. Sundararaj, B. I. and Goswami, S. V., "Techniques of hypophysation of catfishes," *FAO/UNDP Regional Seminar on Induced Breeding of Cultivated Fishes, FRi/IBCF/* 15, 1969, p. 23.
7. Tonsanga, S., Sidthimunka, A. and Menasaveta, D., "Induced spawning of catfish (*Clarias macrocephalus* Gunter) by hormone injection," *IPFC.*, FRO, Bangkok, Thailand.