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Low field anomaly in magnetization curves

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Abstract. We present a calculation of the isothermal magnetization hysteresis curves appropriate to high T_c superconductors. We discuss the nature of the low field anomaly as one goes from this strong pinning case to the weak pinning case. We show that the shape of the equilibrium (thermodynamic) magnetization curve is recovered in the limit of J_c approaching zero.

Keywords. High temperature superconductors; lower critical field; magnetization curves; critical currents; hysteresis loops.

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1. Introduction

The two properties of infinite conductivity and of magnetic flux expulsion exhibited by superconductors can be seen in two different magnetization experiments. If the sample is cooled below T_c in a magnetic field, then the observed diamagnetism of this field cooled (FC) sample is a manifestation only of flux expulsion (Meissner effect). When a sample is cooled in the absence of a field and then a magnetic field is applied, the diamagnetism of this zero field cooled (ZFC) sample is a manifestation of shielding caused by perfect conductivity in addition to the Meissner effect. For a type-I superconductor as well for a type-II superconductor in a field below the lower critical field H_{c1}, the property of complete flux expulsion ensures that the FC and ZFC samples have the same magnetization, while this may not be true for a type-II superconductor in a field $H > H_{c1}$. (Throughout this paper we ignore complications because of a finite demagnetization factor.)

For a type-II superconductor the magnetization of the thermodynamic equilibrium state is depicted in figure 1a. Shielding currents account for irreversibilities in magnetization in hard type-II superconductors. Bean (1962) had shown that isothermal magnetization measurements on ZFC sample will not yield this thermodynamic magnetization. The extensively used model of Bean reflects the present understanding of the isothermal magnetization of a hard type-II superconductor. Bean obtained the virgin magnetization curves for various values of H_{c1}, and the magnetization hysteresis curves (Bean 1964) in the approximation of $H_{c1} = 0$. Since Bean's model found extensive application for studying materials at $H \gg H_{c1}$, the approximation $H_{c1} = 0$ was appropriate. Over the last year Bean's model has been extensively used for high T_c superconductors (see the references in Grover *et al* 1988b) where simultaneously large values of H_{c1} were quoted. This prompted us to extend

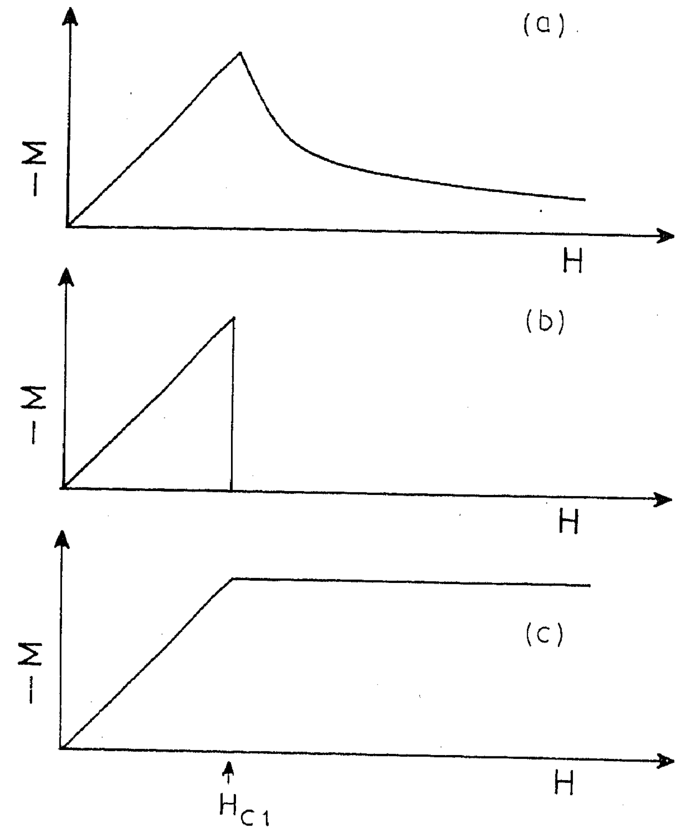


Figure 1. (a) Thermodynamic magnetization curve of a type-II superconductor. The detailed shape depends on the Ginzburg-Landau parameters. (b) is the approximation used by Bean (1962), while (c) is the other extreme approximation possible.

Bean's model and calculate the magnetization hysteresis curves for finite values of H_{c1}. This led us to our prediction (Ravi Kumar and Chaddah 1987; Grover *et al* 1988a) of an anomaly in the hysteresis curves near $H = 0$. In this paper, we present the details of our calculation, and discuss the experimental and intrinsic (material) factors that can influence the slope of this anomaly.

2. Bean's assumptions

Bean (1962) had proposed a model to explain the isothermal magnetization curves of a ZFC hard superconductor. As an external field $H > H_{c1}$ is applied to such a superconducting sample, a shielding current is set up near the surface. The magnitude of the shielding current density is equal to the critical current density J_c, and this current will flow in the sample to a depth necessary to reduce the local field to H_{c1}. (Fields below H_{c1} are screened by the London screening currents in the "soft" superconductor.) With this model Bean calculated virgin magnetization curves for various values of H_{c1}, and of the sample-dependent parametric field $H^* = k\mu_0 J_c a$, where a is the dimension of the sample perpendicular to the field direction and k is a geometrical factor.

Bean made two assumptions in this calculation. He explicitly assumed that J_c is independent of field. He also implicitly assumed (see figure 1 of Bean (1962) for $H^* = 0$)

that the susceptibility of the thermodynamic equilibrium state is zero above H_{c1} and that the magnetization has the form shown in figure 1b. We should point out that both these assumptions could be questioned. Various reports (Farrell *et al* 1987; Senoussi *et al* 1988) show that for the high T_c superconductors J_c decays exponentially with field. We have also shown that for high T_c superconductors J_c decaying exponentially with field yields good agreement with the isothermal magnetization data (Ravi Kumar and Chaddah 1988). We shall, therefore, modify the second assumption of Bean and present the numerical results for J_c decaying exponentially with H, and briefly compare with Bean's assumption of J_c independent of H.

As regards Bean's first assumption, the thermodynamic magnetization actually decreases slowly (the rate actually depends on the Ginzburg-Landau parameters) as the field increases from H_{c1} to H_{c2}. We shall, therefore, use two extreme forms for the thermodynamic magnetization curve to calculate virgin and hysteresis curves.

3. Calculation of magnetization curves

There exists a limited discussion of the hysteresis curve for finite H_{c1} during 1960's (Fietz *et al* 1964; Dunn and Hlawiczka 1968) but none that discusses Bean's model in detail. We shall present a calculation of the hysteresis curves within the framework of Bean's model incorporating the exponential dependence of critical current density on magnetic field as $J_c(H) = J_c(0) \exp(-H/H_0)$. We will also discuss the effect of finite H_{c1} on virgin and hysteresis curves.

The crucial point in Bean's model is that if a region of the sample experiences a field $|H| < H_{c1}$, then no macroscopic shielding currents flow in this region, and there is complete flux expulsion from this region.

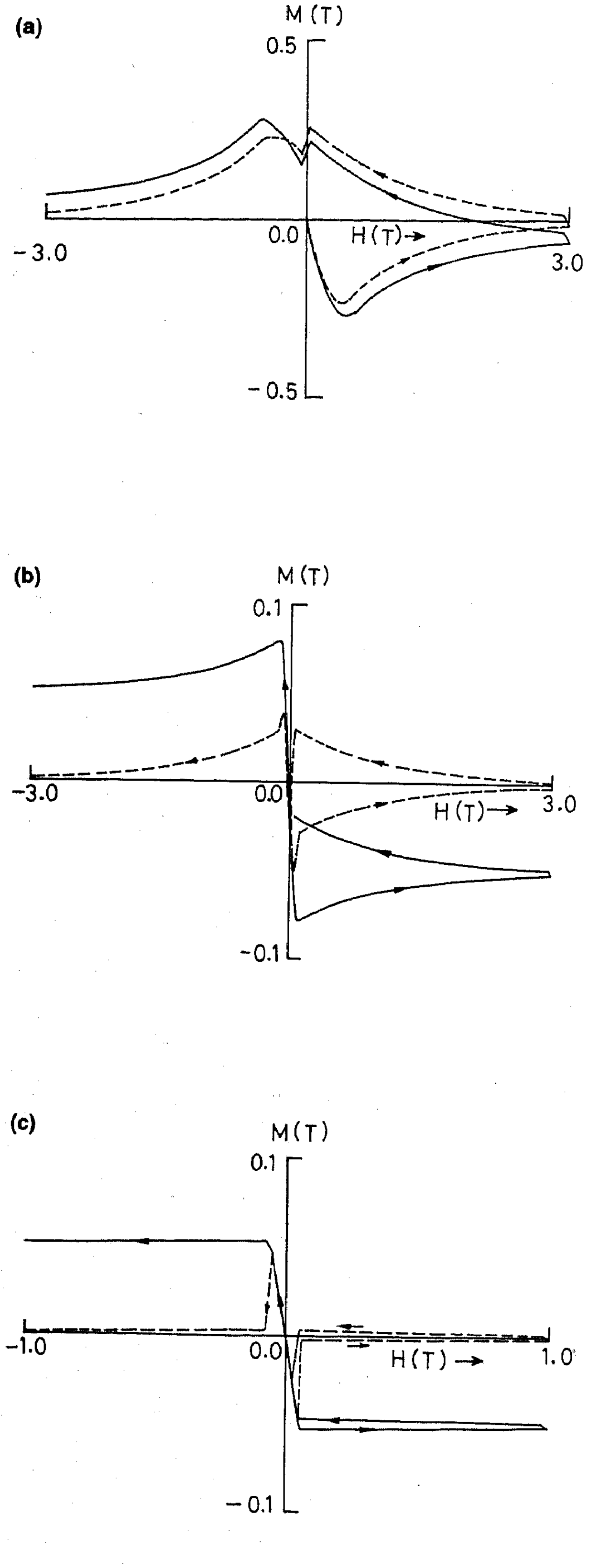
For the actual calculations of the hysteresis curves we assume that the sample is an infinite slab of thickness D. Besides computational simplicity, this geometry obviates problems associated with demagnetization corrections. For the virgin magnetization curve this follows exactly the calculations of Bean (1962) for an infinite cylinder. For an external field $|H_e| < H_{c1}$ the flux expulsion is caused by screening currents within the London penetration depth λ of the surface. For $D \gg \lambda$, the flux expulsion is complete for $|H_e| < H_{c1}$. For $H_e > H_{c1}$ macroscopic shielding currents of density $J_c(H)$ are set up near the surface and the field $H_v(x)$ varies inside the sample as (Bean 1964)

$$dH_v(x)/dx = \mu_0 J_c[H_v(x)]. \tag{1}$$

$H_v(x)$ is obtained by solving this equation with the boundary condition $H_v(\pm D/2) = H_e$. These currents flow up to a depth necessary to reduce the local field to H_{c1}. The field is then screened by the London screening currents. Magnetization is then obtained by summing up the two contributions. The London contribution is just $-H_{c1}v$, where v is the volume fraction enclosed by the surfaces where $H = H_{c1}$. The shielding contribution is the magnetization caused by the macroscopic currents and is obtained as

$$M(H_e) = \frac{\mu_0}{D} \int_{-D/2}^{D/2} x J_c(x)\, dx. \tag{2}$$

The virgin magnetization curve is then obtained. We show the virgin magnetization so



(a)
M (T)
0.5
-0.5
-3.0
0.0
3.0
H (T)→
(b)
M (T)
0.1
-0.1
-3.0
0.0
3.0
H (T)→
(c)
M (T)
0.1
-0.1
-1.0
0.0
1.0
H (T)→

obtained for $H^* = 0{\cdot}63$ tesla and $H_0 = 1$ tesla in figure 2a. We also show the results that are obtained when the thermodynamic M vs H is of the form of figure 1c, instead of Bean's assumption of figure 1b.

As H_e is lowered from H_{max} shielding currents are induced (in accrodance with Lenz's law) in the superconducting slab in such a way as to shield the interior regions from this change. The field $H_r(x)$ and thus the current density $J_{c0}\exp(-|H_r|/H_0)$ are determined by solving

$$dH_r(x)/dx = -\mu_0 J_c(|H_r(x)|) \tag{3}$$

with the boundary condition $H_r(\pm D/2) = H_e$. This equation is solved numerically. Following Lenz's law $H_r(x)$ is calculated up to an x_c where $H_r(x_c) = H_0(x_c)$. $H_0(x)$ is the field profile in the virgin cycle when $H_e = H_{max}$. The shielding current distribution for $x < x_c$ is not affected by the lowering of the applied field. The new field and current density profiles are given by

$$\begin{aligned} H(x) &= H_0(x) \quad \text{for } x < x_c, \\ &= H_r(x) \quad \text{for } x > x_c, \end{aligned}$$

and

$$\begin{aligned} J(x) &= -\operatorname{sign}(x) J_c(|H_0(x)|) \quad \text{for } x < x_c, \\ &= \operatorname{sign}(x) J_c(|H_r(x)|) \quad \text{for } x > x_c, \end{aligned}$$

when $x_c > 0$. For lower values of H_e, equation (3) is solved over the entire slab. Magnetization is obtained by using equation (1) with the current distribution obtained as described above.

The introduction of H_{c1}, however, deserves a detailed description. As H_e is lowered below H_{c1} in the hysteresis cycle, $H_r(x)$ assumes a value less than H_{c1} within a small region near the surface of the slab. In this region where $H_r(x) < H_{c1}$ (Meissner zone or "soft" region) the superconductor expels the field and no shielding currents are induced here (see figures 3a and b). The sample magnetization is again obtained by summing the screening and shielding contributions. Beyond $-H_{c1}$ the Meissner zone moves inwards, but in that region of the sample, the contribution of the shielding currents to magnetization decreases because of the weight factor in equation (1).

We also present results as H^* is decreased in comparison to H_{c1}. In the limiting case of $H^* = 0$ the isothermal magnetization curve should be the same as the thermodynamic magnetization curve. This trend has been confirmed as seen in figure 2c where we take $H^* = 0{\cdot}0063$ tesla. The forms of figure 1b and 1c are closely reproduced in this weak pinning (i.e. $H^* \ll H_{c1}$) limit. In the intermediate pinning limit ($H^* = 0{\cdot}063$ tesla), we show in figure 2b an interesting reversal in magnetization near $H = 0$.

In figure 4 we show the hysteresis curve for strong pinning ($H^* = 0{\cdot}63$ tesla) but with Bean's original assumption of a field-independent J_c. While the sharp decrease in magnetization is still seen near $H = 0$, the shape of the hysteresis curve is very different from that observed in high T_c superconductors.

The difference between hysteresis curves for the thermodynamic magnetization assumed by Bean (figure 1b) and that for the other extreme case of figure 1c is also shown in figures 2a, b and c. The major change is that magnetization remains negative even while H_e is reduced for the case of figure 1c. This negative value is, however, equal

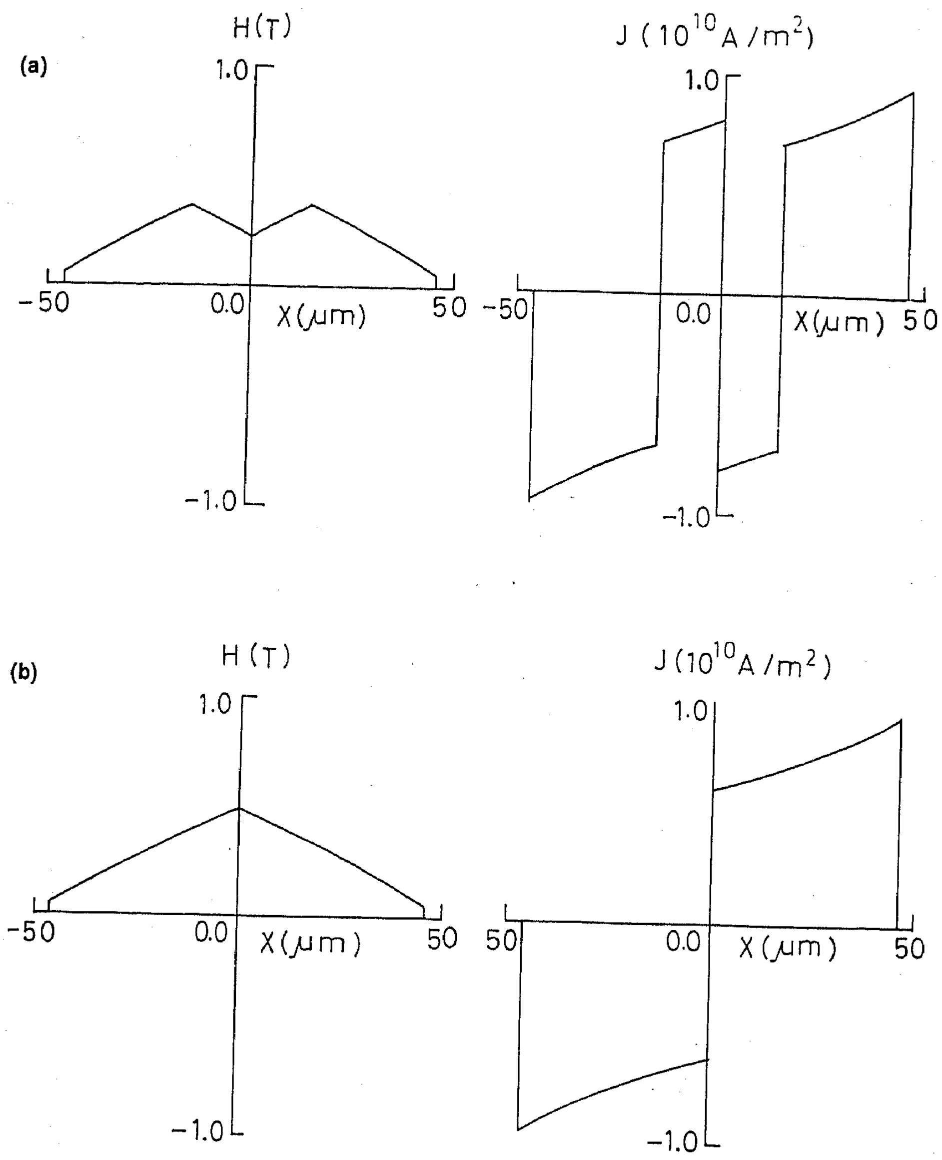


Figure 3. Magnetic field profile and the current density distribution inside the slab when the external field is reversed to zero from **(a)** $H_{max} = 0{\cdot}63$ tesla and **(b)** $H_{max} = 2$ tesla. The values of H^*, H_0 and H_{c1} used are 0·63, 1 and 0·05 tesla respectively.

to H_{c1}. It may not be possible to distinguish between the cases of figure 1b and 1c for high T_c superconductors as $H_{c1} \approx 1$ Oe in these materials (Grover *et al* 1988b). The low H_{c1} also ensures that these samples will exhibit the strong pinning behaviour.

4. Discussion and conclusions

We have shown in figures 2a, b and c the nature of the predicted anomaly (Ravi Kumar and Chaddah 1987) in the hysteresis curve for the cases of strong, intermediate and weak pinning. The existence of an anomaly near $H_e = 0$ has been recognised, in passing, earlier (figure 27 of Campbell and Evetts 1972). We have shown, however, the detailed

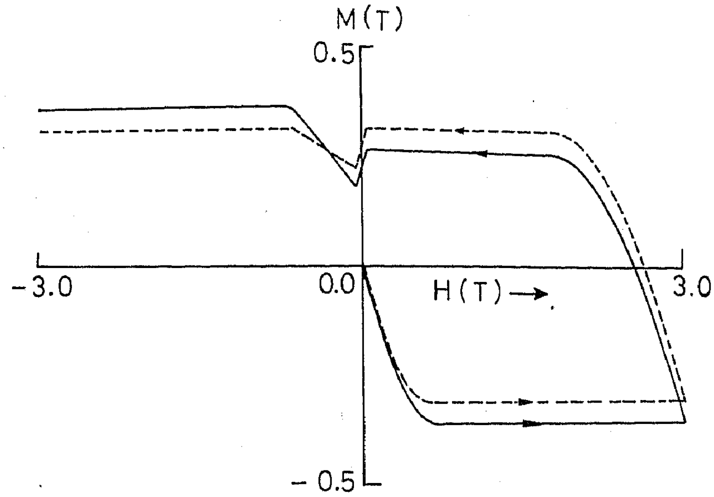


Figure 4. Virgin and hysteresis magnetization curves with critical current density (or H^*) independent of magnetic field (Bean's 1964 approximation).

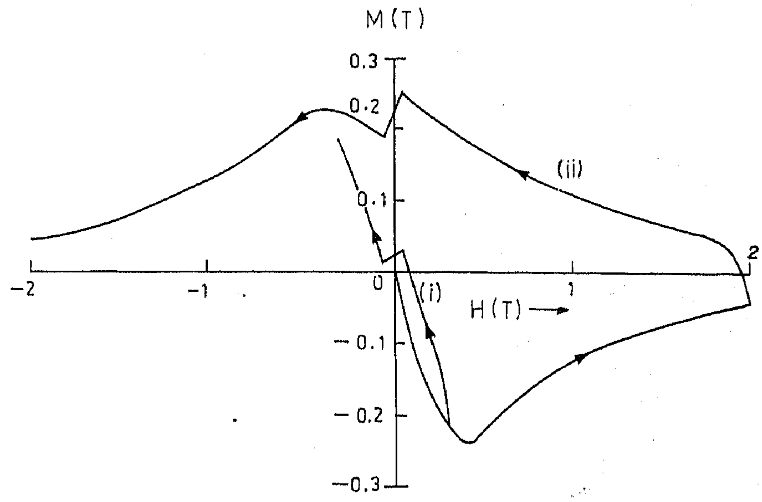


Figure 5. Hysteresis curves with the field reversed from (i) $H_{max} = 0{\cdot}3$ tesla and (ii) $H_{max} = 2$ tesla.

origin of this anomaly and presented an analysis consistent with Bean's work (Bean 1962). Since the anomaly originates because of changes in the shielding current in the strong pinning case, this anomaly should also change if the maximum applied field $H_e < H^*$. This feature has been considered in detail earlier (see figure 15 of Grover *et al* 1988b), but for completeness we show in figure 5 the change in the anomalous region when the maximum applied field is reduced.

The calculations presented in this paper are for a homogeneous sample of specified shape that is computationally tractable. As the geometrical shape of the sample is changed so must the details of the calculation. The physical origin of the low field anomaly, as discussed in § 3, is the growth of the Meissner zone. The low field anomaly must, accordingly, continue to exist in homogeneous samples of arbitrary shape. Such an anomaly has been reported in powder samples (Grover *et al* 1988; Grader *et al* 1988) as well as in single crystal samples (Schneemeyer *et al* 1987; Senoussi *et al* 1988; Van Dover *et al* 1988) of high T_c superconductors.

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