Spatial Solitons in Bulk Cubic-Quintic Media with Multiphoton Ionization Effect

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Abstract—**Propagation of a high intensity cylindrically symmetric beam in a material characterized by cubic-quintic nonlinearity is studied both analytically and numerically. In this case we have to consider the self-defocussing effect of free electrons caused due to plasma formation. Variational method is used to study the system analytically. Finite Difference Beam Propagation Method is used for the numerical analysis. Stable $(2+1)D$ spatial solitons are observed. The analytical results are in very good agreement with the numerical results.**

Keywords—**finite difference beam propagation method, spatial solitons, variational method**

I. Introduction

The idea that an optical beam can induce a waveguide and guide itself in it was first suggested by Askaryan in 1962. A light beam travelling in either vacuum or in a medium always broadens because of the light's natural wave property of diffraction. But if the beam is shone into a bulk nonlinear material, such as silica glass, it changes the material's refractive index. The consequent variation of the light's velocity across the beam's wavefront focusses the beam as if it were passing through a lens. The earliest experimental observation of the self-focussing of optical beams was in 1964. If the beam's diffraction can be compensated by beams self-focussing, we get the so called spatial solitons. This effect was discovered in 1964 [1].

The idea of controlling light with light by taking advantages of nonlinear optical effects, is a topic of interest to many researchers and scientists. The fundamental benefit is in the possibility of avoiding any opto-electronic conversion process, and hence increasing the device speed and efficiency. It is in this scenario, the self-guided beams called "spatial solitons" find importance. The areas of application include all optical switching devices [2], optical computing, all optical polarization modulators [3] logic gating etc.

The prospect of forming all-optical switches and logic gates presents promise for generations of novel optical interconnects for computing and communications. The perfect balance between diffraction and self-focussing that exists in spatial solitons has been found to occur in either one or two transverse dimensions, and the solitons are named $(1D+1)$ or $(2D+1)$ accordingly. These spatial solitons have been found to occur in a variety of materials like Kerr materials, photorefractive materials, liquid crystals etc. Recently, Peccianti et al [4] set up an experiment to demonstrate all-optical switching and logic gate blocks using spatial solitons in liquid crystals.

Usually, the nonlinear refractive index of the material deviates from the linear (Kerr) dependence for larger light intensities. Nonideality of the nonlinear optical response is known for semiconductor-doped glasses, PTS and various π conjugated polymers. In this types of materials the refractive index, n, is of the form

$$n = n_0 + n_2 I + n_4 I^2 \tag{1}$$

where, I is the beam intensity n_0, n_2 and n_4 are nonlinear coefficients with $n_2 > 0$ and $n_4 < 0$, i.e., the higher order nonlinearity is of the saturating kind. The propagation of spatial solitons in a PTS like medium has been studied by various groups [5] - [7]. In the present work, we have studied the propagation of a high intensity laser beam through a PTS like medium. In this high energy regime, we have to consider the phenomenon of plasma generation through multiphoton absorption. Multiphoton absorption is a nonlinear process, in contrast with the one-photon absorption process. It has a self defocussing effect on the material. The study of spatial solitons in a bulk Kerr medium with multiphoton ionization has been carried out by Hermann et al [8].

The refractive index now takes the form, $n = n_0 + n_2 I + n_4 I^2 - N_e/2n_0 N_{cr}$, N_e is the number density of free electrons and N_{cr} is the critical plasma density.

The beam evolution is studied using the cubic-quintic nonlinear Schrodinger Equation with the effect of multiphoton ionization. We analyzed the problem using both numerical and analytical methods.

We used the variational method [9] with a Gaussian ansatz for the analytical analysis. The Finite Difference Beam Propagation Method (FD-BPM) was used for the numerical analysis [10]. Stability of the solitons are studied by numerical and analytical methods and they are found to be stable.

II. Analytical Approach

The dynamics of the amplitude Ψ of a high intensity laser beam in a PTS like medium is governed by the cubic-quintic nonlinear Schrodinger Equation with additional term for the multiphoton ionization and has the form,

$$\begin{aligned} & i2k\frac{\partial A}{\partial z} + \nabla^2 A + 2kk_0 n_2 \left|A\right|^2 A + 2kk_0 n_4 \left|A\right|^4 A \\ & -\rho a A \int_{-\infty}^{\eta} \left|A(t')\right|^{2n} dt' = 0 \end{aligned} \tag{2}$$

where, $k = \omega/c$, $k_0 = n_0 k$, n the number of quanta necessary to ionize the molecules, $\eta = t - z/v$ is the time of the moving frame of the pulse maximum, and ρ and a are constants.

Here, we are considering the propagation of the beam along the z direction and variation along the radial direction. So we will take cylindrical coordinates for our analysis.

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$$i2k\frac{\partial A}{\partial z} = -\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)\right) + 2k\lambda_1 |A|^2 A + 2k\lambda_2 |A|^4 A + \rho a A\int_{-\infty}^{\eta} |A(t')|^{2n} dt \quad (3)$$

Here $\lambda_1 = -k_0 n_2$ and $\lambda_2 = -k_0 n_4$

The time dependence of the beam is taken into account by the ansatz, $A(z,r,\eta) = B(z,r)T(\eta)$. $T(\eta)$ is the normalized input shape.

The Lagrangian is then given by,

$$L = i\frac{r}{2}\left(B\frac{\partial B^*}{\partial z} - B^*\frac{\partial B}{\partial z}\right)T + \frac{r}{2k}\frac{\partial B}{\partial r}\frac{\partial B^*}{\partial r}T + r\frac{\lambda_1}{2}|B|^4 T^3 + r\frac{\lambda_2}{3}|B|^6 T^4 + r\frac{\rho a}{2k}\frac{|B|^{2n+2}}{n+1}Tg(\eta) \quad (4)$$

where,

$$g(\eta) = \int_{-\infty}^{\eta} T^{2n} dt$$

For the solution of this problem, let us assume a trial solution of the form,

$$B(z,r) = C(z)\exp[-\frac{r^2}{2w(z)^2} + ib(z)r^2] \quad (5)$$

Where $C(z)$ is the maximum amplitude, $b(z)$ is the curvature parameter, $w(z)$ is the beam radius. Ideally, the trial function should include the possibility to model the dynamically varying radial shape function of the beam. But that will make the variational analysis more complicated.

The reduced Lagrangian is then given by,

$$\langle L\rangle = \int_0^{\infty} Lrdr \quad (6)$$

$$\langle L\rangle = i\frac{T}{2}\left(C\frac{\partial C^*}{\partial z} - C^*\frac{\partial C}{\partial z}\right)w^3\frac{\sqrt{\pi}}{4} + b_z|C|^2 Tw^5\frac{\sqrt{\pi}}{8} + \frac{|C|^2 T}{2k}\{\frac{1}{w^4} + 4b^2\}w^5\frac{\sqrt{\pi}}{8} + \frac{\lambda_1}{2}|C|^4 T^3 w^3\frac{\sqrt{\pi}}{8\sqrt{2}} + \frac{\lambda_2}{3}|C|^6 T^4 w^3\frac{\sqrt{\pi}}{12\sqrt{3}} + \frac{\rho a}{2k}\frac{|C|^{2n+2}}{(n+1)^{5/2}}Tg(\eta)w^3\frac{\sqrt{\pi}}{4} \quad (7)$$

Now we can find the variation of $\langle L\rangle$ with respect to the various Gaussian parameters $C(z)$, $C(z)^*$, $w(z)$ and $b(z)$.

$$C\frac{\partial\langle L\rangle}{\partial C} = i\frac{T}{2}CC_z^* w^3 + b_z|C|^2 T\frac{w^5}{2} + \frac{|C|^2 T}{2k}\{\frac{1}{w^4} + 4b^2\}\frac{w^5}{2} + \frac{\lambda_1}{2\sqrt{2}}|C|^4 T^3 w^3 + \frac{\lambda_2}{3\sqrt{3}}|C|^6 T^4 w^3 + \frac{\rho a}{2k}\frac{|C|^{2n+2}}{(n+1)^{5/2}}Tg(\eta)w^3 \quad (8)$$

and

$$C^*\frac{\partial\langle L\rangle}{\partial C^*} = -i\frac{T}{2}C^*C_z w^3 + b_z|C|^2 T\frac{w^5}{2} + \frac{|C|^2 T}{2k}\{\frac{1}{w^4} + 4b^2\}\frac{w^5}{2} + \frac{\lambda_1}{2\sqrt{2}}|C|^4 T^3 w^3 + \frac{\lambda_2}{3\sqrt{3}}|C|^6 T^4 w^3 + \frac{\rho a}{2k}\frac{|C|^{2n+2}}{(n+1)^{5/2}}Tg(\eta)w^3 \quad (9)$$

Subtracting (9) from (8), we get

$$\frac{iT}{2}CC_z^* w^3 + \frac{iT}{2}C^*C_z w^3 = 0$$

$=> |C|^2 = y$ (a constant)

$$=> w(0)^2|C(0)|^2 = w(z)^2|C(z)|^2 = E_0 \quad (10)$$

Adding (8) and (9) we obtain,

$$i(CC_z^* - C^*C_z) = -2b_z|C|^2 w^2 - \frac{2|C|^2}{2k}\{\frac{1}{w^4} + 4b^2\}w^2 - \frac{2\lambda_1}{\sqrt{2}}|C|^4 T^2 - \frac{4\lambda_2}{3\sqrt{3}}|C|^6 T^3 + \frac{4\rho a}{2k}\frac{g(\eta|C|^{2n+2}}{(n+1)^{5/2}} \quad (11)$$

Now, the variation of $\langle L\rangle$ with respect to $w(z)$ is given by

$$\frac{\partial\langle L\rangle}{\partial w} = \frac{3}{2}iT(CC_z^* - C^*C_z) + 5b_z|C|^2 T\frac{w^4}{2} + \frac{|C|^2 T}{4k} + 10|C|^2 Tb^2\frac{w^4}{2k} + \frac{3\lambda_1}{4\sqrt{2}}|C|^4 T^3 w^2 + \frac{\lambda_2}{3\sqrt{3}}|C|^6 T^4 w^2 + \frac{3\rho a}{2k}\frac{|C|^{2n+2}}{(n+1)^{5/2}}Tg(\eta)w^2 \quad (12)$$

and

$$\frac{\partial\langle L\rangle}{\partial b} = \frac{1}{2k}4b|C|^2 w^5 = 0 = \frac{d}{dz}(w^5|C|^2) \quad (13)$$

From this we can write,

$$\frac{dw}{dz} = \frac{4bw}{2k} \quad (14)$$

This implies:

$$b(z) = \frac{k}{2}\frac{d\ln w}{dz} \quad (15)$$

Comparing (11) and (12) we obtain,

$$b_z w^2 + \frac{5}{2kw^2} + \frac{4b^2w^2}{2k} + \frac{9\lambda_1}{2\sqrt{2}}|C|^2 T^2 + \frac{10\lambda_2}{3\sqrt{3}}|C|^4 T^3 + \frac{6(2n+1)\rho a}{2k(n+1)}\frac{|C|^{2n}}{(n+1)^{5/2}}g(\eta) = 0 \quad (16)$$

Now, combining it with the derivative form of (14), we obtain

$$\frac{d^2w}{dz^2} = -\frac{20}{(2k)^2w^3} - \frac{36\lambda_1 T^2 E_0}{4k\sqrt{2}w^3} - \frac{40\lambda_2 T^3 E_0^2}{6k\sqrt{3}w^5} - \frac{24(2n+1)\rho a g(\eta)E_0^n}{(2k)^2(n+1)^{5/2}w^{2n+1}} \quad (17)$$

Here $|C|^2$ has been eliminated by using the fact that $w^2 |C|^2 = E_0$

Now, on integrating the above equation, we get an equation for the variation of $w(z)$ as

$$\frac{1}{2}(\frac{dw}{dz})^2 + \Pi(w) = 0 \tag{18}$$

This is analogous to the equation of a particle moving in a potential well. The potential $\Pi(w)$ is given by

$$\Pi(w) = -\frac{10}{(2k)^2w^2} - \frac{18\lambda_1 T^2 E_0}{4k\sqrt{2}w^2} - \frac{10\lambda_2 T^3 E_0^2}{6k\sqrt{3}w^4} - \frac{24(2n+1)\rho a g(\eta)E_0^n}{2n(2k)^2(n+1)^{5/2}w^{2n}} + c \tag{19}$$

The phase $\phi(z)$ of $C(z)$ $(C(z) = |C(z)|\, exp[i\phi(z)])$ is obtained from (11) and also using (16).

$$\frac{d\phi}{dz} = \frac{4}{2kw^2} + \frac{7\lambda_1}{2\sqrt{2}}|C|^2 T^2 + \frac{8\lambda_2}{3\sqrt{3}}|C|^4 T^3 + \frac{2(7n+4)\rho a}{(n+1)2k}\frac{g(\eta\,|C|^{2n}}{(n+1)^{5/2}} \tag{20}$$

For further analysis of (18) and (19), we introduce the normalization $w(z)/w_0 = y(z)$. Substituting in (18) we get, $\frac{1}{2}(\frac{dy}{dz})^2 + \Pi(y) = 0$ where,

$$\Pi(y) = \frac{\mu}{y^2} + \frac{\nu}{y^4} + \frac{\xi}{y^{2n}} + K \tag{21}$$

$$\mu = -\frac{10}{4k^2w_0^4} - \frac{18\lambda_1 T^2 E_0}{4k\sqrt{2}w_0^4}$$

$$\nu = -\frac{10\lambda_2 T^3 E_o^2}{6k\sqrt{3}w_0^6}$$

$$\xi = -\frac{24(2n+1)\rho a g(\eta)E_0^n}{2n(2k)^2(n+1)^{5/2}w_0^{2n+2}}$$

$$K = \frac{c}{a_0^2}$$

Now, assume that the beam at z = 0 has $a(0) = a_0$ and $[\frac{da(z)}{dz}]_{z=0} = 0$. This gives $K = -(\mu + \nu + \xi)$.

Depending on the values of μ, ν and ξ we can identify four different regimes:

1) $\frac{\mu+\nu}{\xi} > 0$ This condition implies defocussing due to both third and fifth order nonlinearity as well as the nonlinearity due to the multiphoton effect. We can clearly see from figure 1 that the beam diffracts faster than in the purely linear case.

2) $-1 < \frac{\mu+\nu}{\xi} < 0$ This condition implies focussing due to the third order nonlinearity and defocussing due to a weak fifth order nonlinearity. The multiphoton effect is also of defocussing nature. We can see (figure 2) that the nonlinearity is trying to focus the beam.

3) $-2.5 < \frac{\mu+\nu}{\xi} < -1$ In this case the third order nonlinearity is of the focussing case and there is a strong fifth order nonlinearity. A potential well has been created. The spreading of the beam is stopped at the zeros of the potential function (figure 3).

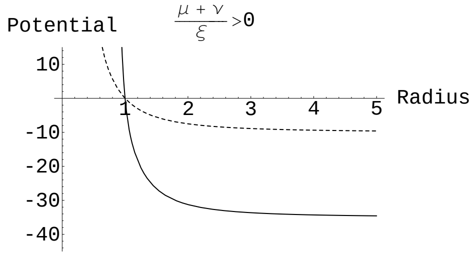

Fig. 1. Qualitative plot of the potential function when all the nonlinearities are of defocussing nature. Dotted line represents the linear case.

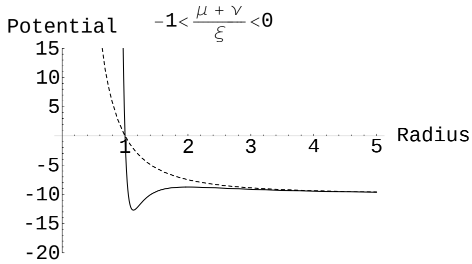

Fig. 2. Qualitative plot of the potential function when third order nonlinearity is of focussing nature and all other nonlinearities are of defocussing nature (weak fifth order). Dotted line represents the linear case.

4) $\frac{\mu+\nu}{\xi} = -2.5$ This is the limit case. The potential well has degenerated into a single point. The diffraction of the beam is exactly compensated by the focussing effect of the nonlinearity and beam propagates without any change in its shape (figure 4).

III. NUMERICAL ANALYSIS

Equation (2) is numerically studied using the Finite Difference Beam Propagation Method (FD-BPM). It is a cylindrical partial differential equation that can be "integrated" forward in z by a number of standard techniques. In this approach, the field in the transverse plane is represented only at discrete points on a grid, and at discrete planes along the propagation direction z. Given the field at one z plane, we can find the field at the next z plane. This is then repeated to determine the field throughout the structure.

Let Ψ_i^{s+1} denote the field at transverse grid point i and longitudinal plane s_i , and assume that the grid points and planes

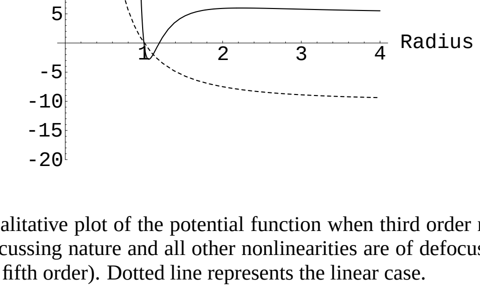

Fig. 3. Qualitative plot of the potential function when third order nonlinearity is of focussing nature and all other nonlinearities are of defocussing nature (strong fifth order). Dotted line represents the linear case.

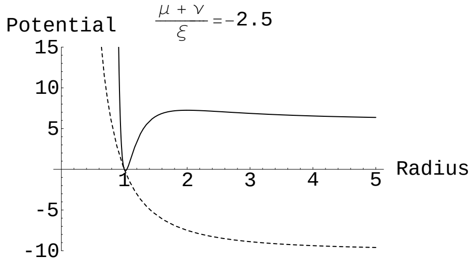

Fig. 4. Qualitative plot of the potential function when focussing due to the third order nonlinearity is completely balanced by the defocussing due to the fifth order nonlinearity and multiphoton ionization. This is the limit case. Dotted line represents the linear case.

are equally spaced by Δr and Δz apart, respectively. The radial and longitudinal dimensions are discretized by the values r_i and z_s according to the relations

$r_i = i\Delta r$ and $z_s = s\Delta z$

We get a tridiagonal matrix of the form,

$$-c_1\Psi_{i+1}^{s+1} + d\Psi_i^{s+1} - c_3\Psi_{i-1}^{s+1} = c_1\Psi_{i+1}^{s} + c_2\Psi_i^{s} + c_3\Psi_{i-1}^{s}$$

This can be easily solved using Thomas Algorithm [11]. Once the field at s is known, we can determine the field at $s+1$ and so on.

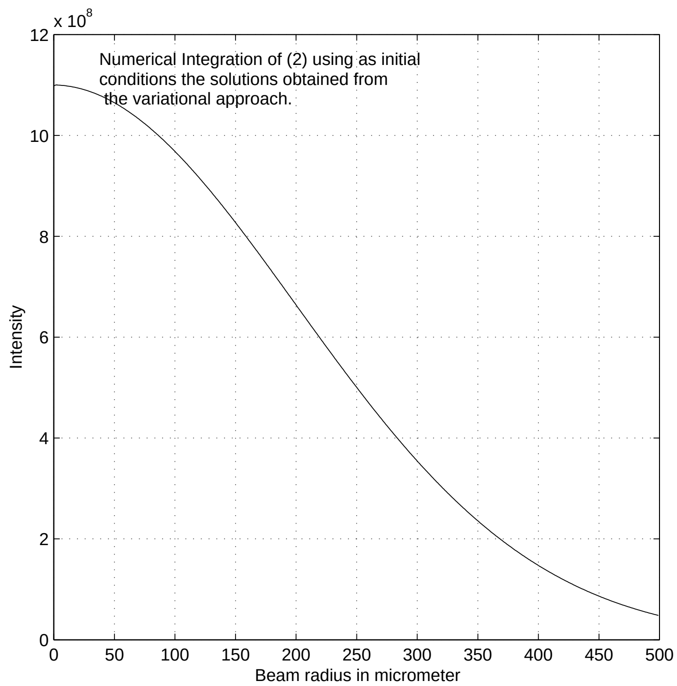

Fig. 5. Numerical Integration of (2).

We integrated (2) using the result obtained from the variational analysis as initial condition. The numerical parameters of the simulation has been choosen so as to fit the usual experimental configurations. Here, we have choosen $n_0 = 1.6755$, $n_2 = 2.2\times10^{-12}cm^2/W$ and $n_4 = -8\times10^{-22}cm^4/W^2$ which are the nonlinear coefficients of PTS at wavelength $1600nm$ [12]. Similarly, for AlGaAs, with $n_0 = 3$, $n_2 = 1.5\times10^{-13}cm^2/W$, $n_4 = -5\times10^{-23}cm^4/W^2$ at wavelength $1550nm$ [12]. The outcome of these simulations (see figure(6)) agrees very well with that obtained from the variational approach. The beam propagates without any change in shape.

IV. Results and Conclusion

In this work we have studied, both analytically and numerically, the propagation of a high energy laser beam through a medium characterized by both third and fifth order nonlinearity. We have to consider the self-defocussing effect caused by plasma generation through multiphoton ionization. Approximated solutions are obtained using the variational formulation. All the basic parameters of the self trapped beam are calculated. The multiphoton ionization helps in containing the catastrophic breakdown of the beam and helps in forming a stable soliton. We could show both numerically and analytically the formation of stable solitons.

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