Modulation Instability of Perturbative Nonlinear Schrödinger Equation with Higher Order Dispersion

B.Kalithasan and K.Porsezian

Abstract— ***Modulation instability conditions for the generation of femtosecond pulses represented by perturbative nonlinear Schrödinger equation with higher order dispersion and nonlinear effects is investigated. It is observed that modulation instability occurs even in the normal dispersion regime due to the presence of fourth order dispersion.***

I. INTRODUCTION

Extensive research has been carried out in the field of soliton propagation in optical fibers [1-3]. In the picosecond regime, the nonlinear Schrödinger equation describes the bright / dark solitons in the anomalous / normal dispersion regime and the plane wave shows the modulation instability. In the case of dispersion shifted fibers operating in the femtosecond regime, the constant guroup velocity dispersion term becomes relatively small and as a result, higher order corrections are called for. The major higher order terms come from the dispersion slope, the dispersive effect of the Kerr coefficient and the self-induced Raman scattering (SRS). Taking into account of these higher order terms the nonlinear Schrödinger equation is perturbative to the higher order nonlinear Schrödinger equation (HNLSE) [2-5]. In the case of optical solitons, when femtosecond pulses are concerned, the effect of nonlinear dispersion is so significant that it can no longer be treated as a perturbation term. Perturbative nonlinear Schrödinger equation has been integrated by using the lie transform technique [6]. The effect of nonlinear dispersion term can be studied exactly while the effect of other perturbation terms has to be studied with the aid of soliton perturbation theory. This was done by Shchesnovich and Doktorov based on the Riemann-Hillbert problem [7] .

On par with the soliton propagation, continuous wave propagation in optical fibers have also demanded special attention [1-3]. A continuous wave with a cubic nonlinearity in an anomalous dispersion regime is known to develop instability with respect to small modulations in amplitude or in phase in the presence of noise or any other weak perturbation, called modulation instability (MI) [1-3]. Generally, the perturbation has its origin from quantum noise or from a frequency shifted signal wave [3]. The MI phenomenon was discovered in fluids [8], in nonlinear optics [9] and in plasmas [10]. MI of a light wave in an optical fiber was suggested by Hasegawa and Brinkman [11] as a means to generate a far infrared light source and since then has attracted extensive attention for both its fundamental and applied interests [8, 9]. As regards applications, MI provides a natural means of generating ultra short pulses at ultra-high repetition rates and is thus potentially useful for the development of high speed optical communication systems in future and hence has been exploited a great deal in many theoretical and experimental studies for the realization of laser sources adapted to ultrahigh bit-rate optical transmissions [11, 12]. When breakup of continuous wave and quasi-continuous wave radiates into a train of picosecond and femtosecond pulses in the fiber, higher-order nonlinear effects such as self-steepening (SS), self-induced Raman scattering (SRS) and higher order dispersion effects such as third and fourth order dispersion should also be taken into account [1-3]. The influence of self-phase modulation (SPM), higher order nonlinear effects and higher order dispersion effects on MI in the anomalous dispersion regime have been obtained in Ref. [1-3] and the following conclusion have been arrived at: The instability conditions that govern the generation of ultrashort pulses in the anomalous dispersion regime are not affected irrespective of the presence or absence of the dimensionless third order dispersion coefficient. The effect of SRS on MI is such that for comparatively small values of the perturbation frequency, group velocity dispersion and self-phase modulation terms dominate whereas for comparatively large values the perturbation frequency, the gain spectrum increases linearly with the result that the region of MI is widened due to SRS. Moreover the self-steepening effect reduces the maximum gain and bandwidth. This article purports to the investigation of MI condition for the generation of femtosecond pulses represented by perturbative HNLSE. The remainder of the article is arranged as follows: In sec. II, the theoretical model is discussed. Linear stability analysis of the governing equation is performed in Sec. III. Results and discussions are discussed in Sec. IV. Section V comprises the the conclusion.

B.Kalithasan and K.Porsezian (corresponding author) are with Department of Physics, Pondicherry University, Pondicherry 605 014. e-mail: ponzsol@yahoo.com. KP wishes to thank the CSIR, DST, UGC (Research Award) and DAE-BRNS, Government of India, for the financial support through projects.

II. THEORETICAL MODEL

The governing equation depicting femtosecond soliton propagation in nonlinear optical fibers with higher order linear effects such as third and fourth order dispersion effects and higher order nonlinear effects such as SS, SRS, nonlinear gain/loss etc., is represented by the perturbative higher order nonlinear Schrödinger equation and is given by [14]

$$\begin{aligned} iq_z - \frac{\rho}{2}q_{tt} + \alpha|q|^2 q = i\varepsilon(\lambda(|q|^2 q)_t + \nu q(|q|^2)_t) - \gamma q_{ttt} \\ + iq_{tttt} - i\beta(q^2 q_t^*)_t - i\Gamma q_t^2 q^* - i\eta q^* q_{tt}^2 + \delta|q|^{2m} q), \end{aligned} \quad (1)$$

where q is the slowly varying envelope, z is the propagation direction, t is the retarded time, ρ is the group velocity dispersion (GVD) coefficient, α is the self-phase modulation (SPM) coefficient and ε is the width of the spectrum. The right hand side of the governing equation comprises of the perturbative terms where λ is the self steepening coefficient, ν is the self-induced Raman scattering (SRS) coefficient, γ is the third order dispersion (TOD) coefficient, σ is the fourth order dispersion (FOD) coefficient. The terms β, Γ and η arise in the context of quasi-solitons in fiber optics [14,15] and δ is the nonlinear damping or amplification depending on its sign and m could be $0, 1, 2$. The nonlinear damping term δ, $m = 0$, is commonly known as attenuation, while for $m = 1$ or 2 we get the cubic or the quintic saturation, respectively. Here, we have considered the case for $m = 0$.

Using the following transform given by

$$q(z,t) = Q(z,t)e^{\varepsilon\delta z}, \tag{2}$$

into equation (1), we get

$$iQ_z - \frac{\rho}{2}Q_{tt} + \alpha f|Q|^2Q = i\varepsilon(f(\lambda(|Q|^2Q)_t + \nu Q(|Q|^2)_t)$$
$$-\gamma Q_{ttt} + iQ_{tttt} - if(\beta(Q^2Q_t^*)_t + \Gamma Q_t^2Q^* + \eta Q^*Q_{tt}^2)), \tag{3}$$

where $f = exp(2\varepsilon\delta z)$.

We consider equation (3) as the governing equation and obtain the MI condition for the generation of femtosecond pulses.

III. LINEAR STABILITY ANALYSIS

Steady state solution of equation (3) can be written as

$$Q(z,t) = \sqrt{P_0}e^{i\phi(z)}, \tag{4}$$

where P_0 is the input power and $\phi(z)$ is the nonlinear phase shift. Substituting equation (4) into equation (3), we get

$$\phi(z) = \alpha P_0\frac{(e^{2\varepsilon\delta z} - 1)}{2\varepsilon\delta}. \tag{5}$$

The linear-stability of the steady state can be examined by introducing a perturbed field of the form

$$Q(z,t) = (\sqrt{P_0} + a(z,t))e^{i\phi(z)}, \tag{6}$$

where P_0 is the incident pump power and $|a(z,t)|^2 << P_0$. We assume for the perturbation $a(z,t)$ the following ansatz with frequency detuning from the pump Ω:

$$a(z,t) = u(z)e^{-i\Omega t} + v(z)e^{i\Omega t}, \tag{7}$$

where $u(z)$ and $v(z)$ are the complex perturbation fields. By substituting equation (6) into equation (3) and collecting the linear terms in $u(z)$ and $v(z)$, we obtain the equation for the perturbed field as

$$i\frac{du(z)}{dz} = M_{11}u(z) + M_{12}v^*(z), \tag{8}$$

$$i\frac{dv^*(z)}{dz} = M_{21}u(z) + M_{22}v^*(z), \tag{9}$$

where

$$\begin{aligned}
M_{11} &= e^{2\varepsilon\delta z}P_0(-\alpha + 2\varepsilon\lambda\Omega + \varepsilon\nu\Omega - 2\varepsilon\eta\Omega^2) \\
&\quad -\frac{\rho\Omega^2}{2} + \varepsilon\gamma\Omega^3 - \varepsilon\sigma\Omega^4, \\
M_{12} &= e^{2\varepsilon\delta z}P_0(-\alpha + \varepsilon\lambda\Omega + \varepsilon\nu\Omega - \varepsilon\beta\Omega^2), \\
M_{21} &= e^{2\varepsilon\delta z}P_0(\alpha + \varepsilon\lambda\Omega + \varepsilon\nu\Omega + \varepsilon\beta\Omega^2), \\
M_{22} &= e^{2\varepsilon\delta z}P_0(\alpha + 2\varepsilon\lambda\Omega + \varepsilon\nu\Omega + 2\varepsilon\eta\Omega^2) \\
&\quad +\frac{\rho\Omega^2}{2} + \varepsilon\gamma\Omega^3 + \varepsilon\sigma\Omega^4.
\end{aligned} \tag{10}$$

Equations (8) and (9) can be written as

$$i\frac{d}{dz}\begin{bmatrix} u(z) \\ v^*(z) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} u(z) \\ v^*(z) \end{bmatrix}. \tag{11}$$

The eigenvalues of the matrix M determine the wave number K of the perturbation. The modulation instability occurs when K possesses a nonzero imaginary part. The eigenvalues are given by dispersion relation

$$K = \frac{1}{2}(M_{11} + M_{22} + \sqrt{(M_{11} - M_{22})^2 + 4M_{12}M_{21}}\,). \tag{12}$$

The importance of modulation instability is measured by a power gain defined by

$$G(\Omega) = 2|Im(K)|, \tag{13}$$

where $Im(K)$ represents the imaginary part of K.

IV. RESULT AND DISCUSSION

We have observed that the TOD order dispersion term has no influence on the MI condition but the second order dispersion and FOD terms have influence in the MI condition.The MI condition. MI gain increases due to the influence of the SRS effect. From the MI gain spectrum, space between the two sidebands is more when compared with zero and anomalous dispersion regimes.

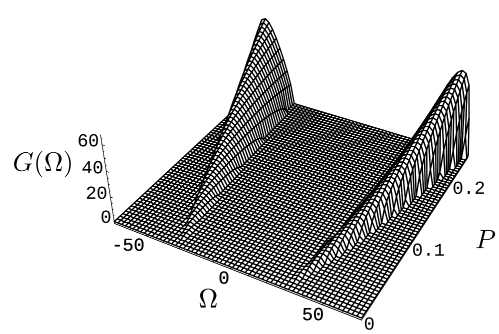

Fig. 1: MI Gain spectrum in the normal dispersion region for various values of $\rho = 0.0114,\ \alpha = 0.7,\ \gamma = 1.18 \times 10^{-4},\ \sigma = -6.44 \times 10^{-7},\ \delta = -0.04,\ \varepsilon = 1,\ \lambda = 0.63,\ \beta = 0.04,\ z = 0.2,\ \eta = 0.03,\ \nu = 0.07.$
figure

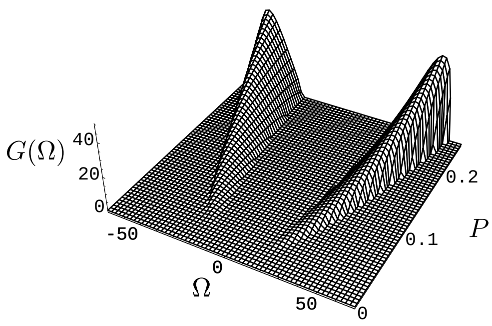

Fig. 2: MI Gain spectrum in the zero dispersion region for various values of $\rho = 0,\ \alpha = 0.7,\ \gamma = 1.18 \times 10^{-4},\ \sigma = -6.44 \times 10^{-7},\ \delta = -0.04,\ \varepsilon = 1,\ \lambda = 0.63,\ \beta = 0.04,\ z = 0.2,\ \eta = 0.03,\ \nu = 0.07.$
figure

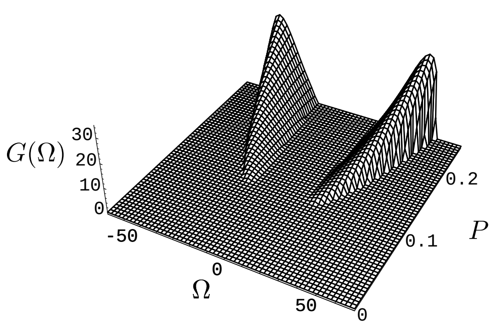

Fig. 3: MI Gain spectrum in the anomalous dispersion region for various values of $\rho = -0.0114,\ \alpha = 0.7,\ \gamma = 1.18 \times 10^{-4},\ \sigma = -6.44 \times 10^{-7},\ \delta = -0.04,\ \varepsilon = 1,\ \lambda = 0.63,\ \beta = 0.04,\ z = 0.2,\ \eta = 0.03,\ \nu = 0.07.$
figure

V. CONCLUSION

We have investigated the modulation instability of perturbative nonlinear Schrödinger equation with higher order dispersion in normal, anomalous and zero dispersion regimes. We have observed that while TOD dispersion term doesn't influence the MI, second order dispersion and FOD terms do influence. The MI is dependant on the signs of the second order dispersion and FOD terms. In normal dispersion regime, MI occurs only for negative values of the higher order dispersion coefficient. We have observed the maximum gain in normal dispersion regime when compared with zero and anomalous dispersion regimes.

REFERENCES

[1] A. Hasegawa in Optical Solitons-Theoretical and Experimental Challenges (Eds. K. Porsezian and V. C. Kuriakose, Springer Verlag, Berlin, 2003).

[2] G. P. Agrawal, 'Nonlinear Fiber Optics', Academic Press, San Diego (2001).

[3] G. P. Agrawal, 'Applications of Nonlinear Fiber Optics', Second Edition, Academic Press, New York (2001).

[4] Y. S. Kivshar and G. P. Agrawal, "Optical solitons -From fibers to photonic crystals", Academic Press, New York (2003).

[5] A. Hasegawa and M. Matsumoto, Optical Solitons in Fibers, Springer Verlag, Germany (2003).

[6] A. Hasegava and Y. Kodama, Solitons in Optical Communications,Oxford University Press (1995).

[7] V.S. Shchesnovich and E.V. Doktorov, Physica D. **129**, 115 (1998).

[8] M. Nakazawa, K. Suzuki, H. Kubota and H. A. Haus , Phys. Rev. A**39**, 5768 (1989)

[9] L. A. Ostrovskii, Sov. Phys. JETP **24**, 797 (1967).

[10] V. I. Karpman, JETP Lett. **6**, 277 (1967).

[11] A. Hasegawa and W. F. Brinkman, IEEE J. Quantum Electron. **16**, 694 (1980).

[12] G. Millot, P. Tchofo Dinda, E. Seve, and S. Wabnitz, Optical Fiber Technology **7**, 170 (2001).

[13] G. Millot, S. Pitois, J. M. Dudley and M. Haelterman in Optical Solitons-Theoretical and Experimental Challenges (Eds. K. Porsezian and V. C. Kuriakose, Springer Verlag, Berlin, 2003).

[14] A. Biswas, Journal of Electromagnetic Waves and Applications **14**, 95 (2000).

[15] V.E. Zakharov,E.A. Kuznetsov, Journal of Experimental and Theoretical Physics **86**, 1035 (1998).

[16] F.Kh. Abdullaev, S.A. Darmanyan, S. Bischoff, P.L. Christiansen and M.P. Sorensem, Opt. Comm. **108**, 60 (1994).

[17] S.B. Cavalcanti, J.C. Cressoni, H.R. da Cruz, A.S. Gouveia-Neto, Phys.Rev. A **43**, 6162 (1991).

[18] F. Ito, K.-i. Kitayama, H. Yoshinaga, Appl.Phys.Lett. **54**, 2503 (1989).