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PAINLEVÉ ANALYSIS OF THE INHOMOGENEOUS KADOMTSEV-PETVIASHVILI EQUATION

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The complete integrability aspects of a variable coefficient Kadomtsev-Perviashvili equation is analysed from the point of view of the Painlevé analysis. The associated Bäcklund transformation is constructed from the Painlevé analysis.

PAINLEVÉ PROPERTY

In recent years, there has been considerable interest in the study of the complete integrability of the inhomogeneous nonlinear evolution equations (NEEs), since it gives more realistic model for many physical systems. So far, the complete integrability of only a few inhomogeneous nonlinear equations have been analysed[1] when compared with the homogeneous nonlinear equations.

Singularity structure analysis admitting the Painlevé property advocated by Ablowitz *et al.*[2] for ordinary differential equations (ODEs) and extended by Weiss *et al.*[3] to partial differential equations play a key role in investigating the integable nonlinear dynamical systems. The remarkable feature of the Painlevé (P-) analysis, particularly for soliton equations, is that a natural connection exists in relation to the Bäcklund transformation and other integrability aspects. This analysis aims to identify the conditions under which a given system is free from movable singular manifolds. The definition is that a PDE has the Painlevé property if its general solution is single valued about the non characteristic movable singular manifold. In other words, if the singular manifold is defined by

$$\phi\ (x, y, t) = 0 \qquad \dots (1)$$

and $u = u(x, y, t) = 0$ is the solution of the PDE, we require that

$$u = \phi^{\alpha} \sum_{j=0}^{\infty} u_j \phi^j \qquad \dots (2)$$

where $u_0 \neq 0, \phi = \phi\ (x, y, t)$, $u_j = u_j\ (x, y, t)$ are analytic functions of (x, y, t) in the neighbourhood of the manifold and α is a negative integer. This Painlevé analysis has been successfully performed for a class of nonlinear evolution equations (NEEs) with inhomogenities[7, 8]. In this paper, we are dealing with the inhomogeneous Kadomtsev-Petviashvili equation of the form :

$$u_t = h(u_{xxx} + 6uu_x + 3\alpha^2 w_{yy}) + b_1 u_x - k(xu_x + 2u + 2yu_y) - \alpha b_1 xu_y - 2\alpha b_1 w_y,$$

$$w_x = u. \qquad \ldots (3)$$

Equations (3) contains many integrable systems. In the case $h = -1$ and $k = 0$, it reduces to the standard KP equation[4]. In the case $h = -1$ and u, w being independent of y, it reduces to the $(h - t)$ KdV equation[5].

The Painlevé test for equation (3) consists of assuming the leading orders of the above equation in the form

$$u = u_0 \phi^\alpha \quad \text{and} \quad w = w_0 \phi^\beta. \qquad \ldots (4)$$

Substituting (4) in (3) and balancing the most dominant terms, we obtain

$$\alpha = -2 \quad \text{and} \quad \beta = -1 \qquad \ldots (5a)$$

with $$u_0 = -2\phi_x^2 \quad \text{and} \quad w_0 = \phi_x. \qquad \ldots (5b)$$

For finding the resonances, we substitute

$$u = \sum_{j=0}^{\infty} u_j \phi^{j-2}, \qquad w = \sum_{j=0}^{\infty} w_j \phi^{j-1} \qquad \ldots (6)$$

in eqns. (3) and equate the coefficients of (ϕ^{j-6}, ϕ^{j-3}). The resonances are found to be

$$j = -1,\ 1,\ 4 \text{ and } 6.$$

The resonance $j = -1$ corresponds to the arbitrary singularity manifold $(\phi = 0)$. In order to check the existence of sufficient number of arbitary function at the other resonance values, we substitute the full Laurent expansions (6) in (3) and collecting the coefficients of (ϕ^{-4}, ϕ^{-1}) and solving we get,

$$u_1 = 2\phi_{xx}. \qquad \ldots (8)$$

From eqn. (8), it is obvious that one of the functions, say w_1, is arbitrary which corresponds to the resonance value at $j = 1$. In a similar way, proceeding further by collecting the coefficients of (ϕ^{-3}, ϕ^0) and solving, we obtain the following expressions for u_2 and w_2

$$u_2 = \{4\phi_x^2 \phi_t + 6hu_{0xx}\phi_x + 8h\phi_x^2 \phi_{xxx} + 36h\phi_x \phi_{xx}^2 - 12h\alpha^2 \phi_x \phi_y^2 - 4b_1 \phi_x^3 + 4kx\phi_x^3 + 4\alpha b_1 x \phi_x^2 \phi_y + 8ky\phi_x^2 \phi_y\}/24h\phi_x^3$$

and

$$w_2 = 24h\, w_{1x}\, \phi_x^3 - 4\phi_x^2\, \phi_t - 6hu_{0xx}\, \phi_x - 8h\, \phi_x^2\, \phi_{xxx} - 36h\, \phi_x\, \phi_{xx}^2$$

$$+ 12h\alpha^2\, \phi_x\, \phi_y^2 + 4b_1\, \phi_x^3 - 4kx\, \phi_x^3 - 4\alpha\, b_1\, x\, \phi_x^2\, \phi_y$$

$$- 8ky\, \phi_x^2\, \phi_y\}/(-24h\, \phi_x^3). \quad \ldots (9)$$

Further collecting the coefficient of (ϕ^{-2}, ϕ^1) and solving, we have

$$u_3 = \{-4\, \phi_x\, \phi_t - 2\phi_{xx}\, \phi_t - h\, u_{0xxx} + 3h\, u_{1xx}\, \phi_x + 12\, hu_{2x}\, \phi_x^2 - 16h\, \phi_{xx}\, \phi_{xxx}$$

$$+ 36h\, u_2\, \phi_x\, \phi_x + 12h\alpha^2\, \phi_{xy}\, \phi_y + 6\alpha^2\, \phi_x\, \phi_{yy} + 6b_1\, \phi_x\, \phi_{xx} - 6kx\, \phi_x\, \phi_{xx}$$

$$- 4k\, \phi_x^2 - 8ky\, \phi_x\, \phi_y - 4ky\, \phi_{xx}\, \phi_y - 4\alpha\, b_1\, x\, \phi_x\, \phi_y - 2\alpha b_1 x\, \phi_{xx}\, \phi_y$$

$$- 4\alpha\, b_1\, \phi_x\, \phi_y\}/12h\, \phi_x^3$$

and

$$w_3 = \frac{w_{2x}\,(12\, \phi_x^3 - 1)}{-24\, \phi_x^4}. \quad \ldots (10)$$

In order to prove the existence of arbitrary function at $j = 4$, we collect the coefficient of (ϕ^{-1}, ϕ^2) and after careful analysis we find that u_4 is arbitrary and satisfying the resonance condition at $j = 4$. Thus the general solution $\{u(x, y, t), w(x, y, t)\}$ of equation (3) admits the required number arbitrary functions without the introduction of any movable critical manifold and hence satisfying Painlevé property. Thus, we except that eqns. (3) are integrable inhomogeneous Kadomtsev- Petviashvili equation.

BÄCKLUND TRANSFORMATION (BT)

Since eqns. (3) satisfy the required condition for integrability, our next aim is to construct the integrability properties. For this purpose, we truncate the Laurent series (6) at the constant level term by setting $u_j = 0$, $j \geq 3$ and $w_j = 0$, $j \geq 2$.

$$u = u_0\, \phi^{-2} + u_1\, \phi^{-1} + u_2, \quad w = w_0\, \phi^{-1} + w_1. \quad \ldots (11)$$

Here (u, u_2), and (w, w_2) are functions satisfying eqns (3). Substituting eqn. (11) in (3) and equating the coefficients of different powers of ϕ we obtain a set of different equations with u_1 and w satisfying eqns. (3) without loss of generality. Next, to construct the BT of eqns. (3), we put $u_2 = w_1 = 0$[9-12]. Then we have

$$\left.\begin{aligned} u &= u_0\, \phi^{-2} + u_1\, \phi^{-1} \\ w &= w_0\, \phi^{-1}. \end{aligned}\right\} \quad \ldots (12)$$

Substituting the values of u_0, u_1 and w_0, we have

$$\left.\begin{aligned} u &= 2\frac{\partial^2}{\partial x^2}(\log\phi) \\ u &= 2\frac{\partial}{\partial x}(\log\phi). \end{aligned}\right\} \qquad \ldots (13)$$

Equations (13) are the bilinear transformations for eqns. (3). Once the Bäcklund transformation is known, then one can construct the bilinear form using the properties of the Hirota's operator. Thus, in this paper, we investigated the complete integrability of inhomogeneous KP type equation. From our analysis, we concluded that the inhomogeneous KP equation possesses the Painlevé property and so satisfies a necessary condition to be integrable. From this, we also constructed the Bäcklund transformation for the above equation.

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