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Static Deformation due to Two-Dimensional Seismic Sources Embedded in an Isotropic Half-Space in Welded Contact with an Orthotropic Half-Space

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Closed-form analytic expressions for the displacements and stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space caused by two-dimensional seismic sources located in the isotropic half-space are obtained. The method consists of first finding the integral expressions for two half-spaces in welded contact from the corresponding expressions for an unbounded medium by applying suitable boundary conditions at the interface and then evaluating the integrals analytically. Numerical computations indicate that the deformation field due to a source in an isotropic half-space in welded contact with an anisotropic half-space may differ substantially from the deformation field when both the half-spaces are isotropic.

1. Introduction

Maruyama (1966) calculated the Green's functions for two-dimensional elastic dislocations in a semi-infinite medium. Freund and Barnett (1976) obtained two-dimensional surface deformation due to dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable. Rybicki (1971) obtained closed-form analytic solution for the elastic residual field due to a long strike-slip fault in a half-space in the presence of a horizontal or a vertical discontinuity. In a subsequent paper, Rybicki (1978) derived closed-form analytic expressions for the surface displacement field due to a long strike-slip fault in a half-space in the presence of two parallel vertical discontinuities.

Singh and Garg (1986) obtained the integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional seismic sources. Beginning with these expressions, Rani *et al.* (1991) obtained the integral expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space due to various two-dimensional sources by applying the traction-free boundary conditions at the surface of the half-space. The integrals were then evaluated analytically, obtaining closed-form expressions for the Airy stress function, the

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displacements and the stresses at any point of the half-space caused by two-dimensional buried sources. Singh *et al.* (1991) followed a similar procedure to obtain closed-form analytic expressions for the displacements and stresses at any point of either of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to two-dimensional sources.

In this paper, we study the static deformation caused by various two-dimensional seismic sources located in a homogeneous, isotropic, perfectly elastic half-space lying over a homogeneous, anisotropic, perfectly elastic half-space with which it is in welded contact. Most anisotropic media of interest in seismology have, at least approximately, a horizontal plane of elastic symmetry. The most general system with one plane of elastic symmetry is the monoclinic system. A material having three mutually perpendicular planes of elastic symmetry at a point is said to possess orthotropic or orthorhombic symmetry. This symmetry is exhibited by olivine and orthopyroxenes, the principal rock-forming minerals of the deep crust and upper mantle. Therefore, we assume that the lower half-space is orthotropic.

In an orthotropic material, there are nine elastic constants. The results for a tetragonal material with six elastic constants, for a transversely isotropic material with five elastic constants and for a cubic material with three elastic constants can be derived as particular cases. We have verified that the results of Singh *et al.* (1991) for two isotropic half-spaces in welded contact follow from the results of the present paper when the lower orthotropic half-space is replaced by an isotropic half-space.

Wu and Chou (1982) applied the generalized method of images to obtain the elastic field of an in-plane line force acting in a two-phase orthotropic medium. Singh (1986), Garg and Singh (1987), and Pan (1989 a) studied the static deformation of a transversely isotropic multilayered half-space by surface loads. The problem of the static deformation of a transversely isotropic multilayered half-space by buried sources has been discussed by Pan (1989 b). Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads has been investigated by Garg *et al.* (1991).

2. Theory

Let the Cartesian co-ordinates be denoted by $(x, y, z) \equiv (x_1, x_2, x_3)$ with z-axis vertically upwards. Consider two homogeneous, perfectly elastic half-spaces which are welded along the plane $z=0$. The upper half-space ($z>0$) is called medium I and the lower half-space ($z<0$) is called medium II. Medium I is assumed to be isotropic with stress-strain relation

$$p_{ij}=\lambda\delta_{ij}e_{kk}+2\mu e_{ij}\,. \tag{1}$$

Medium II is assumed to be orthotropic with stress-strain relation

$$\begin{bmatrix} p_{11} \\ p_{22} \\ p_{33} \\ p_{23} \\ p_{31} \\ p_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2e_{23} \\ 2e_{31} \\ 2e_{12} \end{bmatrix} . \tag{2}$$

We consider a two-dimensional approximation in which the displacement components (u_1, u_2, u_3) are independent of x so that $\partial/\partial x \equiv 0$. Under this assumption the plane-strain problem $(u_1=0)$ and the antiplane-strain problem $(u_2=u_3=0)$ are decoupled and, therefore, can be solved separately. The plane-strain problem for an isotropic medium can be solved in terms of the Airy stress function U such that

$$p_{22}=\frac{\partial^2 U}{\partial z^2}, \quad p_{33}=\frac{\partial^2 U}{\partial y^2}, \quad p_{23}=-\frac{\partial^2 U}{\partial y \partial z}, \tag{3}$$

$$\nabla^2\nabla^2 U=0 . \tag{4}$$

The plane-strain problem for an orthotropic medium can be solved in terms of the Airy stress function U^* such that (Garg *et al.*, 1991)

$$p_{22}=\frac{\partial^2 U^*}{\partial z^2}, \quad p_{33}=\frac{\partial^2 U^*}{\partial y^2}, \quad p_{23}=-\frac{\partial^2 U^*}{\partial y \partial z}, \tag{5}$$

$$\left(a^2\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(b^2\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)U^*=0 , \tag{6}$$

where

$$\begin{gathered} a^2+b^2=(c_{22}c_{33}-c_{23}^2-2c_{23}c_{44})/(c_{33}c_{44}) , \\ a^2b^2=c_{22}/c_{33} . \end{gathered} \tag{7}$$

For an isotropic medium

$$\begin{aligned} c_{11}&=c_{22}=c_{33}=\lambda+2\mu , \\ c_{12}&=c_{13}=c_{23}=\lambda , \\ c_{44}&=c_{55}=c_{66}=\mu . \end{aligned} \tag{8}$$

This yields $a^2=b^2=1$ and Eq. (6) reduces to Eq. (4).

Let there be a line source parallel to the x-axis passing through the point $(0, 0, h)$ of the upper half-space $z>0$. As shown by Singh and Garg (1986), the Airy stress function U_0 for a line source parallel to the x-axis passing through the point $(0, 0, h)$ in an unbounded, isotropic medium, with Lamé constants λ_1, μ_1, can be expressed in the form

$$U_0=\int_0^{\infty}[(L_0+M_0k|z-h|)\sin ky+(P_0+Q_0k|z-h|)\cos ky]k^{-1}e^{-k|z-h|}dk\,, \quad (9)$$

where the source coefficients L_0, M_0, P_0, Q_0 are independent of k. Singh and Garg (1986) have obtained these source coefficients for various seismic sources. These are listed in Table 1 for ready reference. We use the notation of Ben-Menahem and Singh (1981) for labeling various sources. Thus, (23) denotes the single couple in the x_2x_3-plane with forces in the x_2-direction, and F_{23} is the corresponding moment; (23)+(32) denotes the double couple in the x_2x_3-plane; (32)−(23) denotes the center of rotation in the x_2x_3-plane; (22) denotes the dipole in the x_2-direction of strength F_{22}; (22)+(33) denotes the center of dilatation and (33)−(22) denotes the double couple whose forces bisect the angles between the dipoles (22) and (33). In Table 1, we have included the source coefficients for a tensile dislocation as well. A tensile dislocation in the x_2-direction is equivalent to a dipole in the x_2-direction with

Table 1. Source coefficients for various sources.

Source	L_0	M_0	P_0	Q_0
Single couple (23)	$\mp\frac{F_{23}}{2\pi}$	$\pm\frac{\alpha_1 F_{23}}{2\pi}$	0	0
Single couple (32)	$\pm\frac{F_{32}}{2\pi}$	$\pm\frac{\alpha_1 F_{32}}{2\pi}$	0	0
Double couple (23)+(32) $F_{23}=F_{32}=D_{23}$	0	$\pm\frac{\alpha_1 D_{23}}{\pi}$	0	0
Center of rotation (32)−(23) $F_{23}=F_{32}=R_{23}$	$\pm\frac{R_{23}}{\pi}$	0	0	0
Dipole (22)	0	0	$(1-\alpha_1)\frac{F_{22}}{2\pi}$	$-\frac{\alpha_1}{2\pi}F_{22}$
Dipole (33)	0	0	$(1-\alpha_1)\frac{F_{33}}{2\pi}$	$\frac{\alpha_1}{2\pi}F_{33}$
Center of dilatation (22)+(33) $F_{22}=F_{33}=C_0$	0	0	$(1-\alpha_1)\frac{C_0}{\pi}$	0
Double couple (33)−(22) $F_{22}=F_{33}=D'_{23}$	0	0	0	$\frac{\alpha_1}{\pi}D'_{23}$
Tensile dislocation in x_2-direction	0	0	$\frac{\alpha_1 T_0}{\pi}$	$-\frac{\alpha_1 T_0}{\pi}$
Tensile dislocation in x_3-direction	0	0	$\frac{\alpha_1 T_0}{\pi}$	$\frac{\alpha_1 T_0}{\pi}$

The upper sign is for $z>h$ and the lower sign for $z<h$. $[\alpha_1=(\lambda_1+\mu_1)/(\lambda_1+2\mu_1)]$.

$$F_{22}=(\lambda_1+2\mu_1)\bar{b}ds$$

together with a dipole in the x_3-direction with

$$F_{33}=\lambda_1\bar{b}ds\,,$$

where $\bar{b}$ is the slip and ds is the width of the dislocation plane. Similar relations hold for a tensile dislocation in the x_3-direction. In Table 1, we have used the notation $T_0=\mu_1\bar{b}ds$ in the source coefficients for a tensile dislocation.

For a line source parallel to the x-axis acting at the point $(0, 0, h)$ of medium I ($z>0$) which is in welded contact with medium II ($z<0$), the Airy stress function in medium I is a solution of Eq. (4) and may be taken to be of the form

$$U^{(1)}=U_0+\int_0^\infty [(L_1+M_1kz)\sin ky+(P_1+Q_1kz)\cos ky]\mathrm{e}^{-kz}k^{-1}\mathrm{d}k\,. \tag{10}$$

The Airy stress function in medium II is a solution of Eq. (6) and is of the form (assuming $a\neq b$)

$$U^{(2)}=\int_0^\infty [(L_2\mathrm{e}^{akz}+M_2\mathrm{e}^{bkz})\sin ky+(P_2\mathrm{e}^{akz}+Q_2\mathrm{e}^{bkz})\cos ky]k^{-1}\mathrm{d}k\,. \tag{11}$$

The superscript (1) denotes quantities related to medium I and the superscript (2) denotes quantities related to medium II. The constants L_1, M_1, L_2, M_2 etc. are to be determined from the boundary conditions.

Since the half-spaces are assumed to be in welded contact along the plane $z=0$, the boundary conditions are

$$\begin{aligned} p_{23}^{(1)}&=p_{23}^{(2)}\,, \quad p_{33}^{(1)}=p_{33}^{(2)}\,,\\ u_2^{(1)}&=u_2^{(2)}\,, \quad u_3^{(1)}=u_3^{(2)}\,, \end{aligned} \tag{12}$$

at $z=0$. The stresses and the displacements for the isotropic medium I in terms of the Airy stress function $U^{(1)}$ are given by (Rani *et al.*, 1991)

$$p_{22}^{(1)}=\frac{\partial^2 U^{(1)}}{\partial z^2}\,, \quad p_{23}^{(1)}=-\frac{\partial^2 U^{(1)}}{\partial y\partial z}\,, \quad p_{33}^{(1)}=\frac{\partial^2 U^{(1)}}{\partial y^2}\,, \tag{13}$$

$$\begin{aligned} 2\mu_1 u_2^{(1)}&=-\frac{\partial U^{(1)}}{\partial y}+\frac{1}{2\alpha_1}\int(p_{22}^{(1)}+p_{33}^{(1)})\mathrm{d}y\,,\\ 2\mu_1 u_3^{(1)}&=-\frac{\partial U^{(1)}}{\partial z}+\frac{1}{2\alpha_1}\int(p_{22}^{(1)}+p_{33}^{(1)})\mathrm{d}z\,, \end{aligned} \tag{14}$$

where

$$\alpha_1=(\lambda_1+\mu_1)/(\lambda_1+2\mu_1)\,. \tag{15}$$

The stresses and the displacements for the orthotropic medium II are given by (Garg *et al.*, 1991)

$$p_{22}^{(2)}=\frac{\partial^2 U^{(2)}}{\partial z^2}\,, \quad p_{23}^{(2)}=-\frac{\partial^2 U^{(2)}}{\partial y \partial z}\,, \quad p_{33}^{(2)}=\frac{\partial^2 U^{(2)}}{\partial y^2}\,, \tag{16}$$

$$u_2^{(2)}=\Delta^{-1}\int(c_{33}p_{22}^{(2)}-c_{23}p_{33}^{(2)})\mathrm{d}y\,,$$

$$u_3^{(2)}=\Delta^{-1}\int(c_{22}p_{33}^{(2)}-c_{23}p_{22}^{(2)})\mathrm{d}z\,, \tag{17}$$

where

$$\Delta=c_{22}c_{33}-c_{23}^2\,. \tag{18}$$

From Eqs. (9), (10), (13), and (14), we obtain

$$p_{22}^{(1)}=\int_0^\infty[(L_0-2M_0+M_0k|z-h|)\mathrm{e}^{-k|z-h|}+(L_1-2M_1+M_1kz)\mathrm{e}^{-kz}]\sin ky\,k\mathrm{d}k$$
$$+\int_0^\infty[(P_0-2Q_0+Q_0k|z-h|)\mathrm{e}^{-k|z-h|}+(P_1-2Q_1+Q_1kz)\mathrm{e}^{-kz}]\cos ky\,k\mathrm{d}k\,, \tag{19}$$

$$p_{23}^{(1)}=\int_0^\infty[\pm(L_0-M_0+M_0k|z-h|)\mathrm{e}^{-k|z-h|}+(L_1-M_1+M_1kz)\mathrm{e}^{-kz}]\cos ky\,k\mathrm{d}k$$
$$+\int_0^\infty[\mp(P_0-Q_0+Q_0k|z-h|)\mathrm{e}^{-k|z-h|}-(P_1-Q_1+Q_1kz)\mathrm{e}^{-kz}]\sin ky\,k\mathrm{d}k\,, \tag{20}$$

$$p_{33}^{(1)}=-\int_0^\infty[(L_0+M_0k|z-h|)\mathrm{e}^{-k|z-h|}+(L_1+M_1kz)\mathrm{e}^{-kz}]\sin ky\,k\mathrm{d}k$$
$$-\int_0^\infty[(P_0+Q_0k|z-h|)\mathrm{e}^{-k|z-h|}+(P_1+Q_1kz)\mathrm{e}^{-kz}]\cos ky\,k\mathrm{d}k\,, \tag{21}$$

$$2\mu_1u_2^{(1)}=\int_0^\infty[(-L_0+M_0/\alpha_1-M_0k|z-h|)\mathrm{e}^{-k|z-h|}$$
$$-(L_1-M_1/\alpha_1+M_1kz)\mathrm{e}^{-kz}]\cos ky\,\mathrm{d}k$$
$$+\int_0^\infty[(P_0-Q_0/\alpha_1+Q_0k|z-h|)\mathrm{e}^{-k|z-h|}$$
$$+(P_1-Q_1/\alpha_1+Q_1kz)\mathrm{e}^{-kz}]\sin ky\,\mathrm{d}k\,, \tag{22}$$

$$2\mu_1u_3^{(1)}=\int_0^\infty[\pm(L_0-M_0+M_0/\alpha_1+M_0k|z-h|)\mathrm{e}^{-k|z-h|}$$

$$+(L_1-M_1+M_1/\alpha_1+M_1kz)e^{-kz}]\sin ky\,dk$$

$$+\int_0^\infty [\pm(P_0-Q_0+Q_0/\alpha_1+Q_0k|z-h|)e^{-k|z-h|}$$

$$+(P_1-Q_1+Q_1/\alpha_1+Q_1kz)e^{-kz}]\cos ky\,dk\,. \tag{23}$$

In Eqs. (20) and (23), the upper sign is for $z>h$ and the lower sign is for $0<z<h$.

Similarly, from Eqs. (11), (16), and (17), we obtain

$$p_{22}^{(2)}=\int_0^\infty [(a^2L_2e^{akz}+b^2M_2e^{bkz})\sin ky+(a^2P_2e^{akz}+b^2Q_2e^{bkz})\cos ky]kdk\,, \tag{24}$$

$$p_{23}^{(2)}=\int_0^\infty [-(aL_2e^{akz}+bM_2e^{bkz})\cos ky+(aP_2e^{akz}+bQ_2e^{bkz})\sin ky]kdk\,, \tag{25}$$

$$p_{33}^{(2)}=-\int_0^\infty [(L_2e^{akz}+M_2e^{bkz})\sin ky+(P_2e^{akz}+Q_2e^{bkz})\cos ky]kdk\,, \tag{26}$$

$$u_2^{(2)}=\int_0^\infty [-(r_1L_2e^{akz}+r_2M_2e^{bkz})\cos ky+(r_1P_2e^{akz}+r_2Q_2e^{bkz})\sin ky]dk\,, \tag{27}$$

$$u_3^{(2)}=-\int_0^\infty [(s_1L_2e^{akz}+s_2M_2e^{bkz})\sin ky+(s_1P_2e^{akz}+s_2Q_2e^{bkz})\cos ky]dk\,, \tag{28}$$

where

$$r_1=(c_{33}a^2+c_{23})/\Delta\,,\quad r_2=(c_{33}b^2+c_{23})/\Delta\,, \tag{29}$$

$$s_1=(c_{23}a+c_{22}/a)/\Delta\,,\quad s_2=(c_{23}b+c_{22}/b)/\Delta\,, \tag{30}$$

and Δ is given by Eq. (18).

It is noticed from Table 1 that the coefficients L_0, M_0, P_0, and Q_0 might have different values for $z \gtrless h$; let L^-, M^-, P^-, and Q^- be the values of L_0, M_0, P_0, and Q_0, respectively, valid for $z<h$. The boundary conditions (12) give the following system of equations:

$$\begin{gathered}
-(L^--M^-+M^-kh)e^{-kh}+L_1-M_1=-(aL_2+bM_2)\,,\\
-(P^--Q^-+Q^-kh)e^{-kh}+P_1-Q_1=-(aP_2+bQ_2)\,,\\
(L^-+M^-kh)e^{-kh}+L_1=L_2+M_2\,,\\
(P^-+Q^-kh)e^{-kh}+P_1=P_2+Q_2\,,\\
(L^--M^-/\alpha_1+M^-kh)e^{-kh}+L_1-M_1/\alpha_1=2\mu_1(r_1L_2+r_2M_2)\,,\\
(P^--Q^-/\alpha_1+Q^-kh)e^{-kh}+P_1-Q_1/\alpha_1=2\mu_1(r_1P_2+r_2Q_2)\,,\\
-(L^--M^-+M^-/\alpha_1+M^-kh)e^{-kh}+L_1-M_1+M_1/\alpha_1=-2\mu_1(s_1L_2+s_2M_2)\,,\\
-(P^--Q^-+Q^-/\alpha_1+Q^-kh)e^{-kh}+P_1-Q_1+Q_1/\alpha_1=-2\mu_1(s_1P_2+s_2Q_2)\,.
\end{gathered} \tag{31}$$

Solving the system for L_1, M_1, P_1, Q_1, L_2, M_2, P_2, Q_2, we get

$$\begin{aligned}
L_1 &= [X_1(L^- + M^- kh) + 2(D-C)M^-]e^{-kh}, \\
M_1 &= [2X_2(L^- + M^- kh) + X_3 M^-]e^{-kh}, \\
L_2 &= [2A(L^- + M^- kh) - 2CM^-]e^{-kh}, \\
M_2 &= [2B(L^- + M^- kh) + 2DM^-]e^{-kh}, \\
P_1 &= [X_1(P^- + Q^- kh) + 2(D-C)Q^-]e^{-kh}, \\
Q_1 &= [2X_2(P^- + Q^- kh) + X_3 Q^-]e^{-kh}, \\
P_2 &= [2A(P^- + Q^- kh) - 2CQ^-]e^{-kh}, \\
Q_2 &= [2B(P^- + Q^- kh) + 2DQ^-]e^{-kh},
\end{aligned} \tag{32}$$

where

$$\begin{aligned}
X_1 &= 2(A+B) - 1, \\
X_2 &= A(1+a) + B(1+b) - 1, \\
X_3 &= 2D(1+b) - 2C(1+a) + 1, \\
A &= \alpha_1(1 - b + 2\mu_1 s_2 - 2\mu_1 r_2)/W, \\
B &= \alpha_1(a - 1 + 2\mu_1 r_1 - 2\mu_1 s_1)/W, \\
C &= (1 + b - b\alpha_1 + 2\mu_1\alpha_1 s_2)/W, \\
D &= (1 + a - a\alpha_1 + 2\mu_1\alpha_1 s_1)/W, \\
W &= (1 + a - \alpha_1 + 2\mu_1\alpha_1 r_1)(1 + b - b\alpha_1 + 2\mu_1\alpha_1 s_2) \\
&\quad - (1 + a - a\alpha_1 + 2\mu_1\alpha_1 s_1)(1 + b - \alpha_1 + 2\mu_1\alpha_1 r_2).
\end{aligned} \tag{33}$$

Putting the values of the constants L_1, M_1, P_1, etc. in Eqs. (10) and (11), we get the integral expressions for the Airy stress function in the two media. These integrals can be evaluated analytically using the formulae given in the Appendix. The displacements and stresses can be obtained similarly. Using the notation ($z \neq h$, $az \neq h$, $bz \neq h$)

$$\begin{aligned}
R^2 &= y^2 + (z-h)^2, \quad T^2 = y^2 + (h - az)^2, \\
S^2 &= y^2 + (z+h)^2, \quad H^2 = y^2 + (h - bz)^2,
\end{aligned} \tag{34}$$

the final results are given below.

For the isotropic medium I,

$$U^{(1)} = L_0 \tan^{-1}\left(\frac{y}{|z-h|}\right) + \frac{M_0 y|z-h|}{R^2} - P_0 \ln R$$

$$+ \frac{Q_0(z-h)^2}{R^2} + L^-\left[X_1 \tan^{-1}\left(\frac{y}{z+h}\right) + \frac{2X_2 yz}{S^2}\right]$$

$$+M^-\left[2(D-C)\tan^{-1}\left(\frac{y}{z+h}\right)+\frac{(X_1h+X_3z)y}{S^2}+\frac{4X_2hyz(z+h)}{S^4}\right]$$

$$+P^-\left[-X_1\ln S+\frac{2X_2z(z+h)}{S^2}\right]$$

$$+Q^-\left[-2(D-C)\ln S+\frac{(X_1h+X_3z)(z+h)-2X_2hz}{S^2}+\frac{4X_2hz(z+h)^2}{S^4}\right]; \quad (35)$$

$$p_{22}^{(1)}=\frac{2L_0y|z-h|}{R^4}+\frac{2M_0y|z-h|}{R^4}\left[-3+\frac{4(z-h)^2}{R^2}\right]$$

$$+\frac{P_0}{R^2}\left[-1+\frac{2(z-h)^2}{R^2}\right]+\frac{2Q_0}{R^2}\left[1-\frac{5(z-h)^2}{R^2}+\frac{4(z-h)^4}{R^4}\right]$$

$$+\frac{2L^-y}{S^4}\left[X_1(z+h)-2X_2(3z+2h)+\frac{8X_2z(z+h)^2}{S^2}\right]$$

$$+\frac{2M^-y}{S^4}\left[2(D-C)(z+h)+(4X_2-X_1)h-X_3(3z+2h)\right.$$

$$\left.+\frac{4(z+h)^2\{(X_1-4X_2)h+X_3z\}}{S^2}-\frac{24X_2hz(z+h)}{S^2}+\frac{48X_2hz(z+h)^3}{S^4}\right]$$

$$+\frac{P^-}{S^2}\left[4X_2-X_1-\frac{2(z+h)\{6X_2z-(X_1-4X_2)(z+h)\}}{S^2}+\frac{16X_2z(z+h)^3}{S^4}\right]$$

$$+\frac{2Q^-}{S^2}\left[X_3-D+C+\frac{(z+h)\{2(D-C)(z+h)-3(X_1-4X_2)h-X_3(5z+2h)\}}{S^2}\right.$$

$$\left.+\frac{6X_2hz}{S^2}+\frac{4(z+h)^3\{(X_1-4X_2)h+X_3z\}-48X_2hz(z+h)^2}{S^4}+\frac{48X_2hz(z+h)^4}{S^6}\right];$$

$$(36)$$

$$p_{23}^{(1)}=\mp\frac{L_0}{R^2}\left[1-\frac{2(z-h)^2}{R^2}\right]\pm\frac{M_0}{R^2}\left[1-\frac{8(z-h)^2}{R^2}+\frac{8(z-h)^4}{R^4}\right]$$

$$\mp\frac{2P_0y|z-h|}{R^4}\pm\frac{4Q_0y|z-h|}{R^4}\left[1-\frac{2(z-h)^2}{R^2}\right]$$

$$+\frac{L^-}{S^2}\left[2X_2-X_1+\frac{2(z+h)\{(X_1-2X_2)(z+h)-6X_2z\}}{S^2}+\frac{16X_2z(z+h)^3}{S^4}\right]$$

$$+\frac{M^-}{S^2}\left[X_3+2C-2D+\frac{2(z+h)\{2(D-C)(z+h)-X_3(4z+h)-3(X_1-2X_2)h\}}{S^2}\right.$$

$$+\frac{12X_2hz}{S^2}+\frac{8(z+h)^2\{(X_1h+X_3z-2X_2h)(z+h)-12X_2hz\}}{S^4}+\frac{96X_2hz(z+h)^4}{S^6}\Bigg]$$

$$+\frac{2P^-y}{S^4}\left[2X_2(2z+h)-X_1(z+h)-\frac{8X_2z(z+h)^2}{S^2}\right]$$

$$+\frac{2Q^-y}{S^4}\Bigg[(X_1-2X_2)h+X_3(2z+h)-2(D-C)(z+h)+\frac{24X_2hz(z+h)}{S^2}$$

$$-\frac{4(z+h)^2\{(X_1-2X_2)h+X_3z\}}{S^2}-\frac{48X_2hz(z+h)^3}{S^4}\Bigg]; \tag{37}$$

$$p_{33}^{(1)}=-\frac{2L_0y|z-h|}{R^4}+\frac{2M_0y|z-h|}{R^4}\left[1-\frac{4(z-h)^2}{R^2}\right]+\frac{P_0}{R^2}\left[1-\frac{2(z-h)^2}{R^2}\right]$$

$$+\frac{2Q_0(z-h)^2}{R^4}\left[3-\frac{4(z-h)^2}{R^2}\right]+\frac{2L^-y}{S^4}\left[2X_2z-X_1(z+h)-\frac{8X_2z(z+h)^2}{S^2}\right]$$

$$+\frac{2M^-y}{S^4}\Bigg[X_1h+X_3z-2(D-C)(z+h)-\frac{4(z+h)\{(X_1h+X_3z)(z+h)-6X_2hz\}}{S^2}$$

$$-\frac{48X_2hz(z+h)^3}{S^4}\Bigg]+\frac{P^-}{S^2}\left[X_1+\frac{2(z+h)\{6X_2z-X_1(z+h)\}}{S^2}-\frac{16X_2z(z+h)^3}{S^4}\right]$$

$$+\frac{2Q^-}{S^2}\Bigg[D-C-\frac{6X_2hz}{S^2}+\frac{(z+h)\{3(X_1h+X_3z)-2(D-C)(z+h)\}}{S^2}$$

$$-\frac{4(z+h)^2\{(X_1h+X_3z)(z+h)-12X_2hz\}}{S^4}-\frac{48X_2hz(z+h)^4}{S^6}\Bigg]; \tag{38}$$

$$2\mu_1u_2^{(1)}=-\frac{L_0|z-h|}{R^2}+\frac{M_0|z-h|}{R^2}\left[1+\frac{1}{\alpha_1}-\frac{2(z-h)^2}{R^2}\right]+\frac{P_0y}{R^2}+\frac{Q_0y}{R^2}\left[-\frac{1}{\alpha_1}+\frac{2(z-h)^2}{R^2}\right]$$

$$+\frac{L^-}{S^2}\left[2X_2z-(X_1-2X_2/\alpha_1)(z+h)-\frac{4X_2z(z+h)^2}{S^2}\right]$$

$$+\frac{M^-}{S^2}\Bigg[2(C-D)(z+h)+(X_1-2X_2/\alpha_1+X_3/\alpha_1)h+X_3z\left(1+\frac{1}{\alpha_1}\right)$$

$$-\frac{2(z+h)^2\{(X_1-2X_2/\alpha_1)h+X_3z\}}{S^2}+\frac{12X_2hz(z+h)}{S^2}-\frac{16X_2hz(z+h)^3}{S^4}\Bigg]$$

$$+\frac{P^-y}{S^2}\left[X_1-2X_2/\alpha_1+\frac{4X_2z(z+h)}{S^2}\right]$$

$$+\frac{Q^-y}{S^2}\Bigg[2(D-C)-X_3/\alpha_1+\frac{2(z+h)\{(X_1-2X_2/\alpha_1)h+X_3z\}}{S^2}$$

$$-\frac{4X_2hz}{S^2}+\frac{16X_2hz(z+h)^2}{S^4}\Bigg]; \tag{39}$$

$$\begin{aligned}2\mu_1 u_3^{(1)} = &\pm\frac{L_0 y}{R^2}\pm\frac{M_0 y}{R^2}\left[\frac{1}{\alpha_1}-1+\frac{2(z-h)^2}{R^2}\right]+\frac{P_0(z-h)}{R^2}\\
&+\frac{Q_0(z-h)}{R^2}\left[\frac{1}{\alpha_1}-2+\frac{2(z-h)^2}{R^2}\right]+\frac{L^- y}{S^2}\left[X_1-2X_2+\frac{2X_2}{\alpha_1}+\frac{4X_2 z(z+h)}{S^2}\right]\\
&+\frac{M^- y}{S^2}\Bigg[2(D-C)-X_3(1-1/\alpha_1)-\frac{4X_2h(2z+h)}{S^2}\\
&+\frac{2(z+h)(X_3z+X_1h+2X_2h/\alpha_1)}{S^2}+\frac{16X_2hz(z+h)^2}{S^4}\Bigg]\\
&+\frac{P^-}{S^2}\left[(X_1+2X_2/\alpha_1)(z+h)-2X_2(2z+h)+\frac{4X_2z(z+h)^2}{S^2}\right]\\
&+\frac{Q^-}{S^2}\Bigg[\{2D-2C-X_3(1-1/\alpha_1)\}(z+h)-X_3z-(X_1-2X_2+2X_2/\alpha_1)h\\
&-\frac{12X_2hz(z+h)}{S^2}+\frac{2(z+h)^2\{(X_1-2X_2+2X_2/\alpha_1)h+X_3z\}}{S^2}+\frac{16X_2hz(z+h)^3}{S^4}\Bigg].\end{aligned} \tag{40}$$

For the orthotropic medium II,

$$\begin{aligned}U^{(2)}=&2L^-\left[A\tan^{-1}\left(\frac{y}{h-az}\right)+B\tan^{-1}\left(\frac{y}{h-bz}\right)\right]\\
&+2M^-\left[-C\tan^{-1}\left(\frac{y}{h-az}\right)+D\tan^{-1}\left(\frac{y}{h-bz}\right)+hy(A/T^2+B/H^2)\right]\\
&-2P^-[A\ln T+B\ln H]+2Q^-\left[C\ln T+\frac{Ah(h-az)}{T^2}-D\ln H+\frac{Bh(h-bz)}{H^2}\right];\end{aligned} \tag{41}$$

$$\begin{aligned}p_{22}^{(2)}=&4L^- y\left[\frac{Aa^2(h-az)}{T^4}+\frac{Bb^2(h-bz)}{H^4}\right]\\
&+4M^- y\Bigg[-\frac{a^2}{T^4}\{Ah+C(h-az)\}+\frac{4Aa^2h(h-az)^2}{T^6}\\
&-\frac{b^2}{H^4}\{Bh-D(h-bz)\}+\frac{4Bb^2h(h-bz)^2}{H^6}\Bigg]\end{aligned}$$

$$+2P^{-}\left[-\frac{Aa^2}{T^2}+\frac{2Aa^2(h-az)^2}{T^4}-\frac{Bb^2}{H^2}+\frac{2Bb^2(h-bz)^2}{H^4}\right]$$

$$+2Q^{-}\left[\frac{Ca^2}{T^2}-\frac{2a^2(h-az)\{C(h-az)+3Ah\}}{T^4}+\frac{8Aa^2h(h-az)^3}{T^6}\right.$$

$$\left.-\frac{Db^2}{H^2}+\frac{2b^2(h-bz)\{-3Bh+D(h-bz)\}}{H^4}+\frac{8Bb^2h(h-bz)^3}{H^6}\right]; \tag{42}$$

$$p_{23}^{(2)}=2L^{-}\left[\frac{Aa}{T^2}-\frac{2Aa(h-az)^2}{T^4}+\frac{Bb}{H^2}-\frac{2Bb(h-bz)^2}{H^4}\right]$$

$$+2M^{-}\left[-\frac{Ca}{T^2}+\frac{2a(h-az)\{3Ah+C(h-az)\}}{T^4}-\frac{8Aah(h-az)^3}{T^6}\right.$$

$$\left.+\frac{Db}{H^2}+\frac{2b(h-bz)\{3Bh-D(h-bz)\}}{H^4}-\frac{8Bbh(h-bz)^3}{H^6}\right]$$

$$+4P^{-}y\left[\frac{Aa(h-az)}{T^4}+\frac{Bb(h-bz)}{H^4}\right]$$

$$+4Q^{-}y\left[-\frac{a\{Ah+C(h-az)\}}{T^4}+\frac{4Aah(h-az)^2}{T^6}\right.$$

$$\left.-\frac{b\{Bh-D(h-bz)\}}{H^4}+\frac{4Bbh(h-bz)^2}{H^6}\right]; \tag{43}$$

$$p_{33}^{(2)}=-4L^{-}y\left[\frac{A(h-az)}{T^4}+\frac{B(h-bz)}{H^4}\right]$$

$$+4M^{-}y\left[\frac{Ah+C(h-az)}{T^4}-\frac{4Ah(h-az)^2}{T^6}+\frac{Bh-D(h-bz)}{H^4}-\frac{4Bh(h-bz)^2}{H^6}\right]$$

$$+2P^{-}\left[\frac{A}{T^2}-\frac{2A(h-az)^2}{T^4}+\frac{B}{H^2}-\frac{2B(h-bz)^2}{H^4}\right]$$

$$+2Q^{-}\left[-\frac{C}{T^2}+\frac{2(h-az)\{3Ah+C(h-az)\}}{T^4}-\frac{8Ah(h-az)^3}{T^6}\right.$$

$$\left.+\frac{D}{H^2}+\frac{2(h-bz)\{3Bh-D(h-bz)\}}{H^4}-\frac{8Bh(h-bz)^3}{H^6}\right]; \tag{44}$$

$$u_2^{(2)}=-2L^{-}\left[\frac{Ar_1(h-az)}{T^2}+\frac{Br_2(h-bz)}{H^2}\right]$$

$$+2M^{-}\left[\frac{r_1\{Ah+C(h-az)\}}{T^2}-\frac{2Ar_1h(h-az)^2}{T^4}+\frac{r_2\{Bh-D(h-bz)\}}{H^2}\right.$$

$$
\left. - \frac{2Br_2h(h-bz)^2}{H^4} \right]
$$

$$
+2P^- y\left[\frac{Ar_1}{T^2}+\frac{Br_2}{H^2}\right]
$$

$$
+2Q^- y\left[-\frac{Cr_1}{T^2}+\frac{2Ar_1h(h-az)}{T^4}+\frac{Dr_2}{H^2}+\frac{2Br_2h(h-bz)}{H^4}\right]; \tag{45}
$$

$$
u_3^{(2)}=-2L^- y\left[\frac{As_1}{T^2}+\frac{Bs_2}{H^2}\right]
$$

$$
+2M^- y\left[\frac{Cs_1}{T^2}-\frac{2As_1h(h-az)}{T^4}-\frac{Ds_2}{H^2}-\frac{2Bs_2h(h-bz)}{H^4}\right]
$$

$$
-2P^-\left[\frac{As_1(h-az)}{T^2}+\frac{Bs_2(h-bz)}{H^2}\right]
$$

$$
+2Q^-\left[\frac{s_1\{Ah+C(h-az)\}}{T^2}-\frac{2As_1h(h-az)^2}{T^4}\right.
$$

$$
\left.+\frac{s_2\{Bh-D(h-bz)\}}{H^2}-\frac{2Bs_2h(h-bz)^2}{H^4}\right]. \tag{46}
$$

3. Dip-Slip Dislocation

As shown by Maruyama (1966), the double couple (23)+(32) is equivalent to a vertical dip-slip line source such that

$$
D_{23}=\mu_1\bar{b}ds\,, \tag{47}
$$

where $\bar{b}$ is the slip and ds is the width of the dislocation plane. Similarly, the double couple (33)−(22) is equivalent to dip-slip on a 45° dipping line source such that

$$
D'_{23}=\mu_1\bar{b}ds\,. \tag{48}
$$

The Airy stress function due to dip-slip on an inclined plane can be expressed in terms of the Airy stress functions due to these two double-couples:

$$
U=\mu_1\bar{b}ds\left[\frac{\cos 2\delta}{D_{23}}U_{(23)+(32)}+\frac{\sin 2\delta}{D'_{23}}U_{(33)-(22)}\right], \tag{49}
$$

where δ is the dip angle and $U_{(23)+(32)}$ and $U_{(33)-(22)}$ are obtained from Eq. (35) or (41) on inserting the values of the source coefficients L_0, M_0, P_0, and Q_0 from Table 1 corresponding to the sources (23)+(32) and (33)−(22), respectively. This yields

$$
U^{(1)}=(\alpha_1\mu_1\bar{b}ds/\pi)\left[\cos 2\delta\left\{\pm\frac{y|z-h|}{R^2}-2(D-C)\tan^{-1}\left(\frac{y}{z+h}\right)\right.\right.
$$

$$-\frac{(X_1h+X_3z)y}{S^2}-\frac{4X_2hyz(z+h)}{S^4}\bigg\}+\sin 2\delta\bigg\{-2(D-C)\ln S$$
$$+\frac{(z-h)^2}{R^2}+\frac{(X_1h+X_3z)(z+h)-2X_2hz}{S^2}+\frac{4X_2hz(z+h)^2}{S^4}\bigg\}\bigg]; \tag{50}$$

$$U^{(2)}=(2\alpha_1\mu_1\bar{b}ds/\pi)\bigg[-\cos 2\delta\bigg\{-C\tan^{-1}\left(\frac{y}{h-az}\right)$$
$$+D\tan^{-1}\left(\frac{y}{h-bz}\right)+hy(A/T^2+B/H^2)\bigg\}$$
$$+\sin 2\delta\bigg\{C\ln T-D\ln H+\frac{Ah(h-az)}{T^2}+\frac{Bh(h-bz)}{H^2}\bigg\}\bigg]. \tag{51}$$

Similar expressions can be obtained for the stresses and the displacements.

4. Discussion

Equations (36)–(40) and (42)–(46) constitute very general results. These yield closed-form expressions for the displacements and stresses at any point of a two-phase medium consisting of an isotropic half-space lying over an orthotropic half-space due to an arbitrary two-dimensional source in the isotropic half-space in terms of the source coefficients L_0, M_0, P_0, Q_0. The values of these source coefficients for a single couple, a double couple, a dipole, a center of dilatation, a center of rotation and a tensile dislocation are given in Table 1.

We have derived the results when an isotropic half-space (medium I) lies over an orthotropic half-space (medium II). The results when medium II is tetragonal can be obtained on putting

$$c_{22}=c_{11}\,,\quad c_{23}=c_{13}\,,\quad c_{55}=c_{44}\,. \tag{52}$$

The results when medium II is transversely isotropic follow by taking

$$c_{22}=c_{11}\,,\quad c_{23}=c_{13}\,,\quad c_{55}=c_{44}\,,\quad c_{66}=(c_{11}-c_{12})/2\,. \tag{53}$$

Similarly, the results when medium II is cubic are obtained on taking

$$c_{22}=c_{33}=c_{11}\,,\quad c_{12}=c_{13}=c_{23}\,,\quad c_{44}=c_{55}=c_{66}\,. \tag{54}$$

When medium II is isotropic,

$$\begin{aligned}&c_{11}=c_{22}=c_{33}=\lambda_2+2\mu_2\,,\\&c_{12}=c_{13}=c_{23}=\lambda_2\,,\\&c_{44}=c_{55}=c_{66}=\mu_2\,.\end{aligned} \tag{55}$$

This is a degenerate case for which $a=b=1$ (see Eq. (7)). However, we have verified that, when medium II is replaced by an isotropic medium, the results of the present

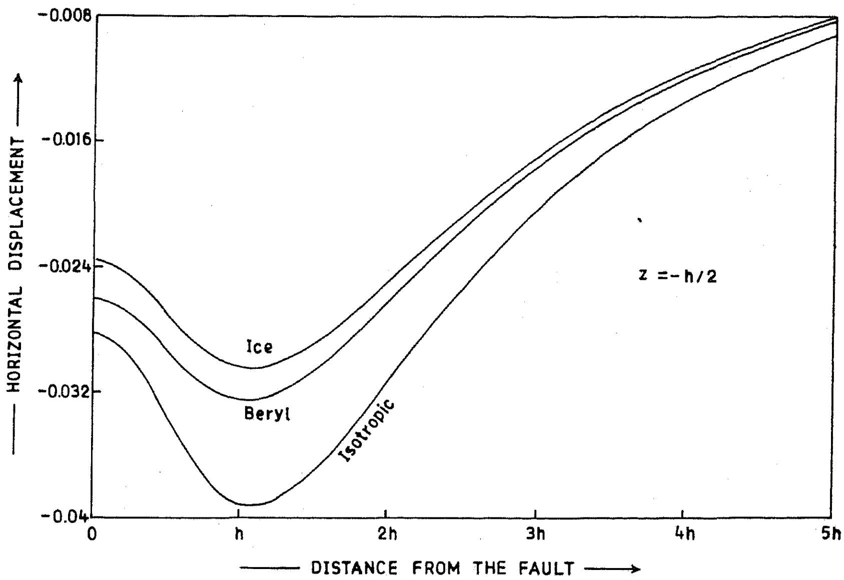

Fig. 1. Variation of the horizontal displacement (u_2) with the distance (y) from a vertical dip-slip fault for $z=-h/2$. u_2 is measured in units of $\bar{b}ds/h$.

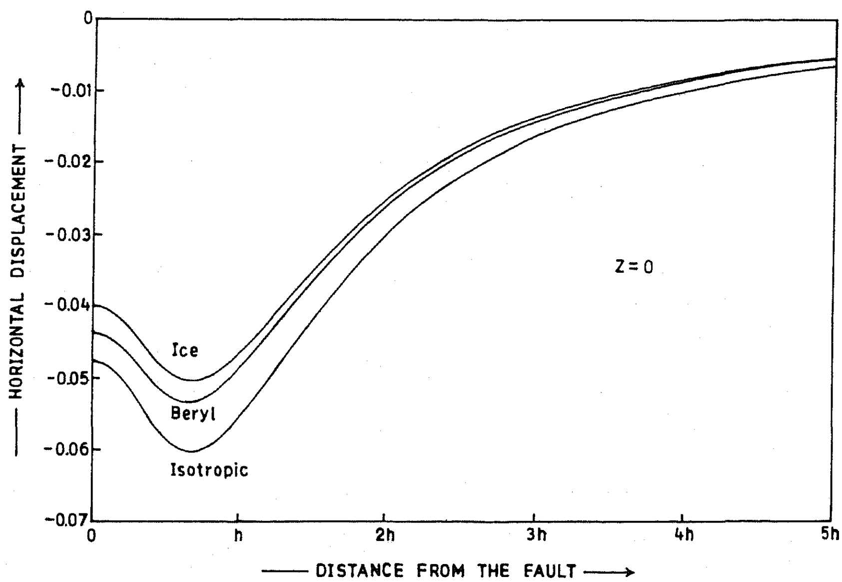

Fig. 2. Variation of u_2 with y for $z=0$.

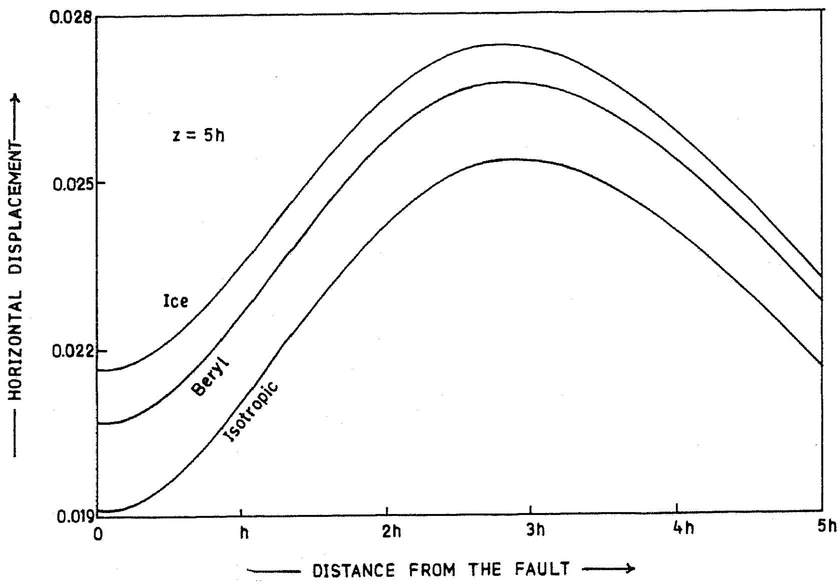

Fig. 3. Variation of u_2 with y for $z=5h$.

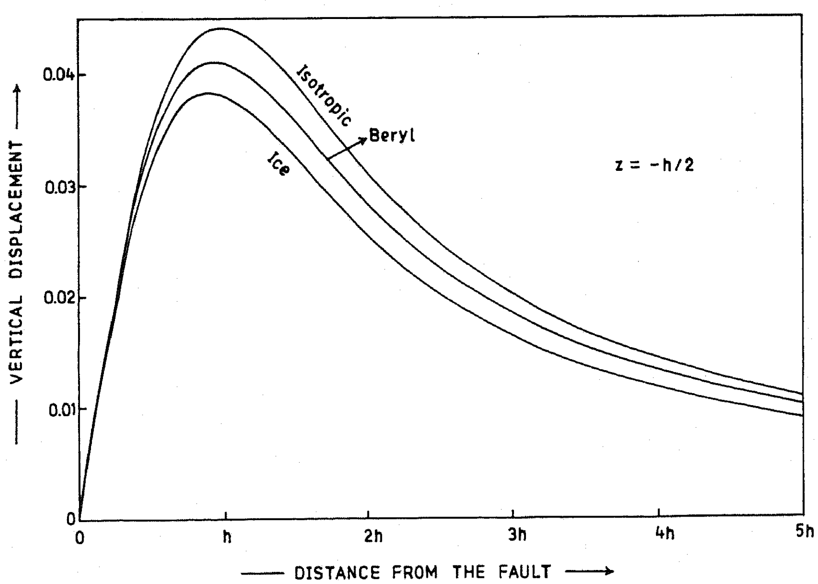

Fig. 4. Variation of the vertical displacement (u_3) with y for $z=-h/2$.

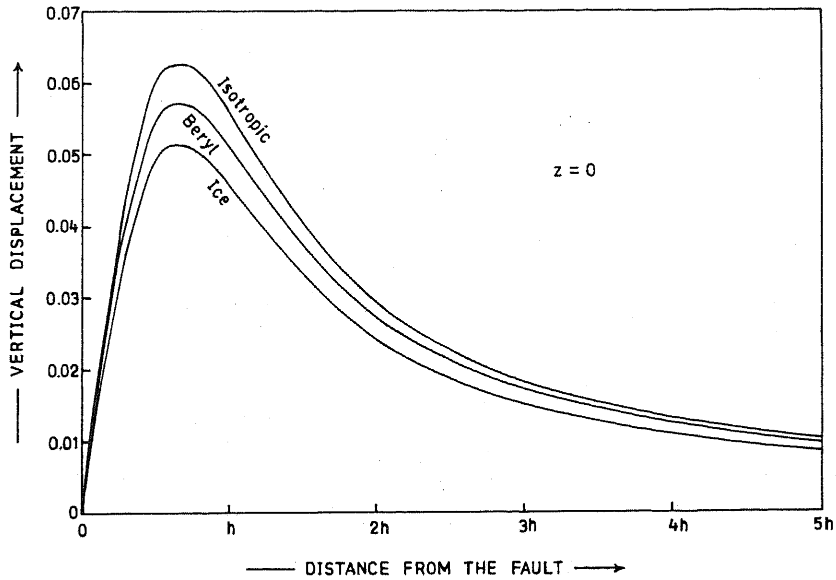

Fig. 5. Variation of u_3 with y for $z=0$.

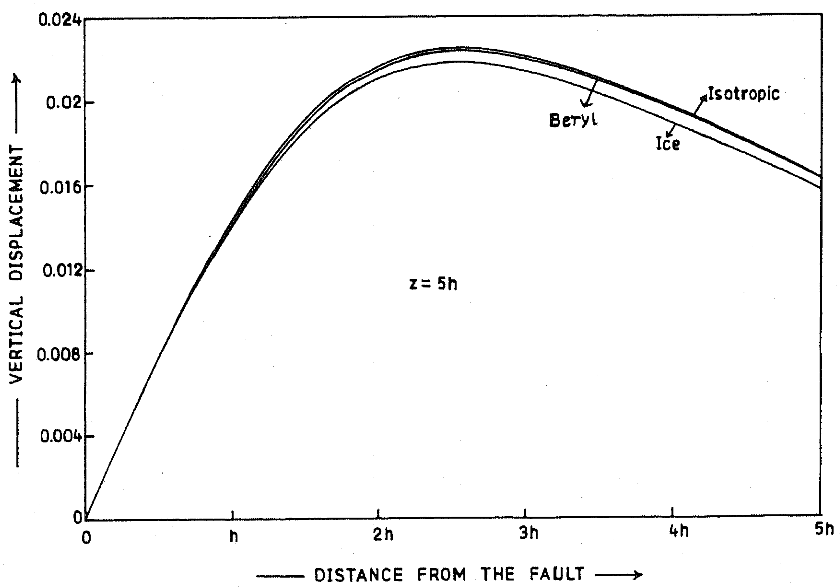

Fig. 6. Variation of u_3 with y for $z=5h$.

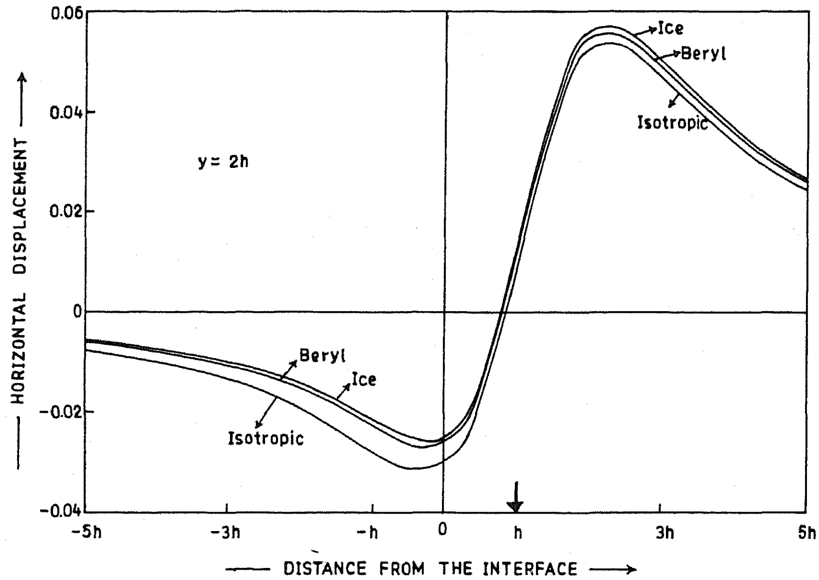

Fig. 7. Variation of u_2 with the distance from the interface (z) for $y=2h$. The arrow at $z=h$ indicates the source depth.

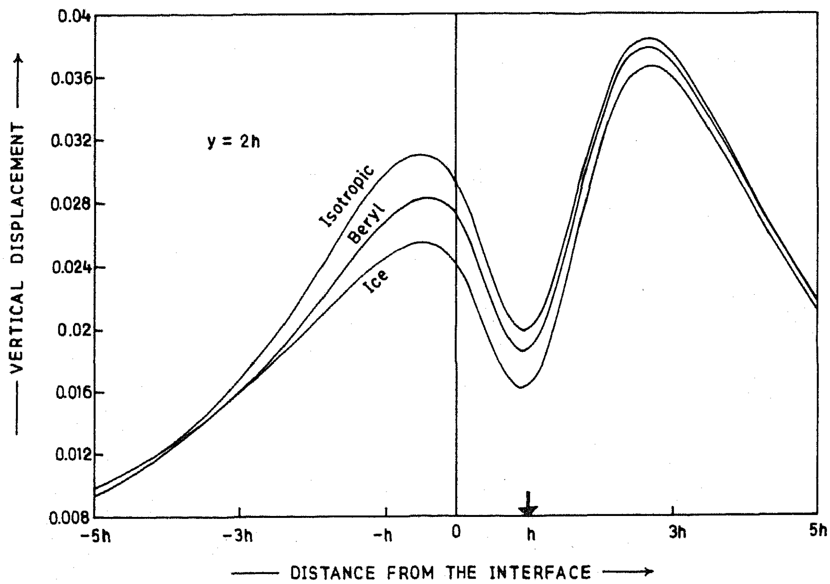

Fig. 8. Variation of u_3 with z for $y=2h$.

paper, in the limit, coincide with the results of Singh *et al.* (1991) for two isotropic half-spaces in welded contact.

For numerical calculations we assume that medium II is transversely isotropic and use the values of the elastic constants given by Anderson (1961). For beryl,

$$c_{11}/c_{44}=4.13\,,\quad c_{33}/c_{44}=3.62,\quad c_{12}/c_{44}=1.47\,,\quad c_{13}/c_{44}=1.01\,.$$

This yields $a=1.7018$, $b=0.6276$. For ice,

$$c_{11}/c_{44}=4.70\,,\quad c_{33}/c_{44}=4.96\,,\quad c_{12}/c_{44}=2.27\,,\quad c_{13}/c_{44}=1.60\,,$$

and $a=1.8019$, $b=0.5402$. For the isotropic medium I, we assume that $\lambda_1=\mu_1$. We further assume that $c_{44}/\mu_1=2$. When medium II is also isotropic, we take $\lambda_2=\mu_2$ for numerical work.

Figure 1 shows the variation of the horizontal displacement due to a vertical dip-slip dislocation with the distance from the fault for $z=-h/2$. The displacements are given in units of $\bar{b}ds/h$. Figures 2 and 3 are for $z=0$ and $z=5h$, respectively. When $z=0$, the receiver is at the interface. When $z=-h/2$, the receiver is in the transversely isotropic medium II and the source is in the isotropic medium I. When $z=5h$, the receiver is also in medium I. The variation of the vertical displacement with the distance from the fault is exhibited in Figs. 4–6. The variation with the distance from the interface is shown in Figs. 7 and 8. Positive values of the distance from the interface imply that the point lies in the isotropic medium I, in which the source lies, whereas negative values imply that the point lies in the transversely isotropic medium II. The arrow at $z=h$ indicates the source depth. These figures show that the displacement field for the anisotropic case can differ significantly from the corresponding field for the isotropic case.

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APPENDIX

$$\int_0^\infty \frac{1}{k} \mathrm{e}^{-kz} \sin ky \, \mathrm{d}k = \tan^{-1} \frac{y}{z}$$

$$\int_0^\infty \frac{1}{k} \mathrm{e}^{-kz} \cos ky \, \mathrm{d}k = -\frac{1}{2} \ln(y^2+z^2)$$

$$\int_0^\infty \mathrm{e}^{-kz} k^n \sin ky \, \mathrm{d}k = n! \left[\frac{z}{y^2+z^2}\right]^{n+1} \sum_{0 \le 2m \le n} (-1)^m \binom{n+1}{2m+1} \left(\frac{y}{z}\right)^{2m+1} \qquad (n=0, 1, 2, 3)$$

$$\int_0^\infty \mathrm{e}^{-kz} k^n \cos ky \, \mathrm{d}k = n! \left[\frac{z}{y^2+z^2}\right]^{n+1} \sum_{0 \le 2m \le n+1} (-1)^m \binom{n+1}{2m} \left(\frac{y}{z}\right)^{2m} \qquad (n=0, 1, 2, 3)$$