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ON TWO-DIMENSIONAL DYNAMICAL DISLOCATIONS: THEORETICAL SEISMOGRAMS

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The problem of a two-dimensional dynamical dislocation in an isotropic, homogeneous, unbounded, elastic medium has been discussed by the present authors in a recent paper (SINGH and SIKKA, 1988), assuming harmonic time-dependence. The aim of the present paper is to generalize the results to arbitrary time-dependence. Explicit theoretical expressions for the displacement and the stress components for a two-dimensional dislocation source have been obtained. These expressions can be used to calculate the theoretical seismograms due to a two-dimensional dislocation in an unbounded medium.

1. *Introduction*

The problem of a two-dimensional dynamical dislocation in a uniform unbounded medium has been discussed by several investigators (see, e.g., BOORE *et al.*, 1971; NIAZY, 1973; GELLER, 1974; BOORE and ZOBACK, 1974). In a recent paper (SINGH and SIKKA, 1988; hereinafter referred to as Paper I), we considered in great detail the problem of a two-dimensional dynamical dislocation in infinite and semi-infinite media, assuming harmonic time-dependence. In the present study, we extend the results of Paper I to arbitrary time-dependence of the source.

2. *Basic Equations*

We consider a homogeneous, isotropic, unbounded, elastic medium of density ρ and elastic parameters λ, μ. Assuming a harmonic time-dependence $\exp(-i\omega t)$, the equations of motion in two-dimensions may be expressed as follows:

$$(\lambda+\mu)\partial_\alpha\partial_\beta u_\beta+\mu\nabla^2 u_\alpha+f_\alpha+\rho\omega^2 u_\alpha=0\,, \qquad \text{(Plane strain)} \tag{1}$$

$$\mu\nabla^2 u_3+f_3+\rho\omega^2 u_3=0\,, \qquad \text{(Antiplane strain)} \tag{2}$$

where

$$\partial/\partial x_3\equiv 0\,, \qquad \partial_i\equiv\partial/\partial x_i\,, \qquad \nabla^2\equiv\partial_\alpha\partial_\alpha\,. \tag{3}$$

The summation over repeated suffixes is understood. The Greek subscripts can assume the values 1 and 2 only while the Latin subscripts can assume the values

1, 2, and 3, f_i is the body force per unit volume, and u_i are the displacement components.

Let $G_\beta^\alpha(Q, P)$ be the x_α-component of the displacement at the point $Q(x_1, x_2)$ caused by a line force of unit magnitude acting at the point $P(y_1, y_2)$ in the x_β-direction. $G_3^3(Q, P)$ is similarly defined. Then, displacement field due to a long displacement dislocation can be expressed as line integrals (see Paper I):

Dip-slip

$$u_\gamma(Q)=\int_L b\left[G_{12}^\gamma \cos 2\delta+\frac{1}{2}(G_{22}^\gamma-G_{11}^\gamma) \sin 2\delta\right] \mathrm{d}s\,, \tag{4}$$

Strike-slip

$$u_3(Q)=\int_L b(G_{32}^3 \cos \delta-G_{31}^3 \sin \delta)\, \mathrm{d}s\,. \tag{5}$$

In Eqs. (4) and (5), L is the fault section, b is the magnitude of the dislocation, δ is the dip angle and

$$G_{\alpha\beta}^\gamma(Q, P)=-\lambda\delta_{\alpha\beta}\partial_\varepsilon G_\varepsilon^\gamma-\mu(\partial_\alpha G_\beta^\gamma+\partial_\beta G_\alpha^\gamma)\,, \tag{6}$$

$$G_{3\beta}^3(Q, P)=-\mu\partial_\beta G_3^3\,. \tag{7}$$

For the stress field, we have the following results:

Dip-slip

$$p_{\gamma\varepsilon}(Q)=\int_L b\left[G_{12}^{\gamma\varepsilon} \cos 2\delta+\frac{1}{2}(G_{22}^{\gamma\varepsilon}-G_{11}^{\gamma\varepsilon}) \sin 2\delta\right] \mathrm{d}s\,, \tag{8}$$

Strike-slip

$$p_{3\alpha}(Q)=\int_L b(G_{32}^{3\alpha} \cos \delta-G_{31}^{3\alpha} \sin \delta)\, \mathrm{d}s\,. \tag{9}$$

In Eqs. (8) and (9),

$$G_{\alpha\beta}^{\gamma\varepsilon}=\lambda\delta_{\gamma\varepsilon}\partial_\theta G_{\alpha\beta}^\theta+\mu(\partial_\gamma G_{\alpha\beta}^\varepsilon+\partial_\varepsilon G_{\alpha\beta}^\gamma)\,, \tag{10}$$

$$G_{3\beta}^{3\alpha}=\mu\partial_\alpha G_{3\beta}^3\,. \tag{11}$$

Explicit expressions for $G_{\alpha\beta}^\gamma$, etc., in terms of the Hankel functions of the first kind $H_m(x)=H_m^{(1)}(x)$ have been obtained in Paper I.

3. *Theoretical Seismograms*

The expressions for the Green's functions G_α^β, G_3^3, $G_{\alpha\beta}^\gamma$, $G_{3\beta}^3$, $G_{\alpha\beta}^{\gamma\varepsilon}$, and $G_{3\beta}^{3\alpha}$ given in Paper I are valid for a harmonic time-dependence, $\exp(-i\omega t)$. We next generalize these results to an arbitrary time-dependence. In the following, if $g(\omega)$ is the harmonic solution, the corresponding time-domain solution is denoted by $g(t\,|\,f)$, assuming the time-dependence of the source to be $f(t)$. Let

$$g_{\mathrm{m}}(R,\, c_\alpha;\, \omega)=\frac{i}{4}\,\mathrm{e}^{m\pi i/2}H_{\mathrm{m}}(K_\alpha R)\mathrm{e}^{-i\omega t},\ K_\alpha=\omega/c_\alpha\,. \tag{12}$$

However, we know that (MAGNUS *et al.*, 1966; p. 81)

$$\pi H_{\mathrm{m}}(z)=-2i\mathrm{e}^{-m\pi i/2}\int_0^\infty \mathrm{e}^{iz\cosh u}\cosh(mu)\,\mathrm{d}u\,. \qquad (0<\arg z<\pi)$$

Therefore, assuming that ω is complex with very small imaginary part which is positive, we obtain

$$\mathrm{e}^{m\pi i/2}H_{\mathrm{m}}(K_\alpha R)=\frac{2}{\pi i}\int_0^\infty \mathrm{e}^{iK_\alpha R\cosh u}\cosh mu\,\mathrm{d}u \tag{13}$$

$$=\frac{2}{\pi i}\int_R^\infty \frac{\mathrm{e}^{iK_\alpha S}}{(S^2-R^2)^{1/2}}\cosh[m\cosh^{-1}(S/R)]\,\mathrm{d}S\,, \tag{14}$$

where $S=R\cosh u$. Using the integral representation (14) of the Hankel function of the first kind and order m, we have

$$g_{\mathrm{m}}(R,\, c_\alpha;\, \omega)=\frac{1}{2\pi}\int_R^\infty \frac{\mathrm{e}^{-i\omega(t-S/c_\alpha)}}{(S^2-R^2)^{1/2}}\cosh[m\cosh^{-1}(S/R)]\,\mathrm{d}S\,. \tag{15}$$

It is now apparent that corresponding to the harmonic solution $g_{\mathrm{m}}(R, c_\alpha; \omega)$, the time-domain solution is given by

$$g_{\mathrm{m}}(R,\, c_\alpha;\, t\,|\,f)=\frac{1}{2\pi}\int_R^\infty \frac{f(t-S/c_\alpha)}{(S^2-R^2)^{1/2}}\cosh[m\cosh^{-1}(S/R)]\,\mathrm{d}S\,. \tag{16}$$

Putting $S/c_\alpha=t'$, Eq. (16) becomes

$$g_{\mathrm{m}}(R,\, c_\alpha;\, t\,|\,f)=\frac{1}{2\pi}\int_{R/c_\alpha}^\infty \frac{f(t-t')}{(t'^2-R^2/c_\alpha^2)^{1/2}}\cosh[m\cosh^{-1}(c_\alpha t'/R)]\mathrm{d}t'\,. \tag{17}$$

Therefore, $g_{\mathrm{m}}(R, c_\alpha; t\,|\,f)$ can be regarded as the convolution of $f(t)$ with the function

$$\frac{1}{2\pi}H\left(t-\frac{R}{c_\alpha}\right)\frac{\cosh[m\cosh^{-1}(c_\alpha t/R)]}{(t^2-R^2/c_\alpha^2)^{1/2}}\,, \tag{18}$$

where $H(t)$ denotes the Heaviside unit-step function.

If the source acts only for a finite time, so that $f(t)=0$ except in the interval $0<t<\tau$, Eq. (17) reveals that, for $t-\dfrac{R}{c_\alpha}\gg\tau$,

$$g_{\mathrm{m}}(R,\, c_\alpha;\, t\,|\,f)=\frac{1}{2\pi}H\left(t-\frac{R}{c_\alpha}\right)\frac{\cosh[m\cosh^{-1}(c_\alpha t/R)]}{(t^2-R^2/c_\alpha^2)^{1/2}}\int_0^\tau f(T)\,\mathrm{d}T\,. \tag{19}$$

Equation (19) gives the well-known "tail" typical to two-dimensional wave propagation.

If $f(t)=0$ for $t<0$, Eq. (16) should be replaced by

$$g_m(R, c_\alpha; t\,|\,f)=\frac{1}{2\pi}H\left(t-\frac{R}{c_\alpha}\right)\int_R^{c_\alpha t}\frac{f(t-S/c_\alpha)}{(S^2-R^2)^{1/2}}\cosh[m\cosh^{-1}(S/R)]\,\mathrm{d}S\,. \tag{20}$$

The time-domain expressions for the displacement components for a line force, G_α^β and G_3^3, the displacement components for a displacement dislocation, $G_{\alpha\beta}^\gamma$ and $G_{3\beta}^3$, and the stress components for a displacement dislocation, $G_{\alpha\beta}^{\gamma\varepsilon}$ and $G_{3\beta}^{3\alpha}$ can now be obtained from Eqs. (24)–(29) of Paper I. We find

$$\begin{aligned}G_\alpha^\beta(t\,|\,f)=\frac{1}{2\mu}\Bigg[&\left(\frac{c_2}{c_1}\right)^2\{\delta_{\alpha\beta}g_0(R, c_1; t\,|\,f)\\&-(\delta_{\alpha\beta}-2X_\alpha X_\beta)g_2(R, c_1; t\,|\,f)\}\\&+\delta_{\alpha\beta}g_0(R, c_2; t\,|\,f)\\&+(\delta_{\alpha\beta}-2X_\alpha X_\beta)g_2(R, c_2; t\,|\,f)\Bigg]\,,\end{aligned} \tag{21}$$

$$G_3^3(t\,|\,f)=\frac{1}{\mu}g_0(R, c_2; t\,|\,f)\,, \tag{22}$$

$$\begin{aligned}G_{\alpha\beta}^\gamma(t\,|\,f)=\frac{1}{2c_2}\Bigg[&\left(\frac{c_2}{c_1}\right)^3\left\{\left(2\frac{2A-1}{1-A}\delta_{\alpha\beta}X_\gamma+\sum\delta_{\beta\gamma}X_\alpha\right)g_1(R, c_1; t\,|\,f')\right.\\&\left.-(\sum\delta_{\beta\gamma}X_\alpha-4X_\alpha X_\beta X_\gamma)g_3(R, c_1; t\,|\,f')\right\}\\&+(\sum\delta_{\beta\gamma}X_\alpha-2\delta_{\alpha\beta}X_\gamma)g_1(R, c_2; t\,|\,f')\\&+(\sum\delta_{\beta\gamma}X_\alpha-4X_\alpha X_\beta X_\gamma)g_3(R, c_2; t\,|\,f')\Bigg]\,,\end{aligned} \tag{23}$$

$$G_{3\beta}^3(t\,|\,f)=\frac{1}{c_2}X_\beta g_1(R, c_2; t\,|\,f')\,, \tag{24}$$

$$\begin{aligned}G_{\alpha\beta}^{\gamma\varepsilon}(t\,|\,f)=\frac{\mu}{6c_2^2}\Bigg[&-3\left(\frac{c_2}{c_1}\right)^4\left\{2\frac{2A-1}{1-A}\left(\frac{c_1}{c_2}\right)^2\delta_{\alpha\beta}\delta_{\gamma\varepsilon}\right.\\&\left.+\sum\delta_{\alpha\beta}\delta_{\gamma\varepsilon}\right\}g_0(R, c_1; t\,|\,f'')\\&+4\left(\frac{c_2}{c_1}\right)^4\left\{3\frac{2A-1}{1-A}(\delta_{\alpha\beta}\delta_{\gamma\varepsilon}-\delta_{\alpha\beta}X_\gamma X_\varepsilon-\delta_{\gamma\varepsilon}X_\alpha X_\beta)\right.\\&\left.+\sum\delta_{\alpha\beta}\delta_{\gamma\varepsilon}-\sum\delta_{\alpha\beta}X_\gamma X_\varepsilon\right\}g_2(R, c_1; t\,|\,f'')\end{aligned}$$

$$-\left(\frac{c_2}{c_1}\right)^4(\sum\delta_{\alpha\beta}\delta_{\gamma\varepsilon}-4\sum\delta_{\alpha\beta}X_\gamma X_\varepsilon+24X_\alpha X_\beta X_\gamma X_\varepsilon)g_4(R,c_1;t\,|\,f'')$$
$$-3(\sum\delta_{\alpha\beta}\delta_{\gamma\varepsilon}-2\delta_{\alpha\beta}\delta_{\gamma\varepsilon})g_0(R,c_2;t\,|\,f'')$$
$$+2(\sum\delta_{\alpha\beta}\delta_{\gamma\varepsilon}-3\delta_{\alpha\beta}\delta_{\gamma\varepsilon}+3\delta_{\alpha\beta}X_\gamma X_\varepsilon+3\delta_{\gamma\varepsilon}X_\alpha X_\beta$$
$$-\sum\delta_{\alpha\beta}X_\gamma X_\varepsilon)g_2(R,c_2;t\,|\,f'')$$
$$\left.+(\sum\delta_{\alpha\beta}\delta_{\gamma\varepsilon}-4\sum\delta_{\alpha\beta}X_\gamma X_\varepsilon+24X_\alpha X_\beta X_\gamma X_\varepsilon)g_4(R,c_2;t\,|\,f'')\right], \qquad (25)$$

$$G^{3\alpha}_{3\beta}(t\,|\,f)=\frac{\mu}{2c_2^2}[-\delta_{\alpha\beta}g_0(R,c_2;t\,|\,f'')+(\delta_{\alpha\beta}-2X_\alpha X_\beta)g_2(R,c_2;t\,|\,f'')], \qquad (26)$$

where a prime denotes differentiation with respect to the argument. Moreover,

$$\delta_{\alpha\beta}=\text{Kronecker delta},$$
$$R^2=(x_1-y_1)^2+(x_2-y_2)^2,$$
$$X_\alpha=(x_\alpha-y_\alpha)/R,$$
$$c_1, c_2=\text{P and S wave-velocities},$$
$$A=(\lambda+\mu)/(\lambda+2\mu).$$

The symbols $\Sigma\delta_{\alpha\beta}\delta_{\gamma\varepsilon}$, etc., are defined in Paper I.

In the derivation of Eqs. (23)–(26), we have made use of the results that the time-domain solution corresponding to $-i\omega g(R, c_\alpha; \omega)$ is $g(R, c_\alpha; t\,|\,f')$ and the time-domain solution corresponding to

$$-\omega^2 g(R, c_\alpha; \omega) \text{ is } g(R, c_\alpha; t\,|\,f'').$$

For vertical dip-slip and vertical strike-slip dislocations, Eqs. (20), (23), and (24) yield the following explicit expressions representing displacements:

$$G^1_{12}(t\,|\,f)=\frac{1}{4\pi c_2}\left[\left(\frac{c_2}{c_1}\right)^3\sin\theta H\left(t-\frac{R}{c_1}\right)\int_R^{c_1t}\frac{f'(t-S/c_1)}{(S^2-R^2)^{1/2}}\left(\frac{S}{R}\right)\mathrm{d}S\right.$$
$$+\left(\frac{c_2}{c_1}\right)^3\sin 3\theta H\left(t-\frac{R}{c_1}\right)\int_R^{c_1t}\frac{f'(t-S/c_1)}{(S^2-R^2)^{1/2}}\left(4\frac{S^3}{R^3}-3\frac{S}{R}\right)\mathrm{d}S$$
$$+\sin\theta H\left(t-\frac{R}{c_2}\right)\int_R^{c_2t}\frac{f'(t-S/c_2)}{(S^2-R^2)^{1/2}}\left(\frac{S}{R}\right)\mathrm{d}S$$
$$\left.-\sin 3\theta H\left(t-\frac{R}{c_2}\right)\int_R^{c_2t}\frac{f'(t-S/c_2)}{(S^2-R^2)^{1/2}}\left(4\frac{S^3}{R^3}-3\frac{S}{R}\right)\mathrm{d}S\right], \qquad (27\,\text{a})$$

$$G^2_{12}(t\,|\,f)=\frac{1}{4\pi c_2}\left[\left(\frac{c_2}{c_1}\right)^3\cos\theta H\left(t-\frac{R}{c_1}\right)\int_R^{c_1t}\frac{f'(t-S/c_1)}{(S^2-R^2)^{1/2}}\left(\frac{S}{R}\right)\mathrm{d}S\right.$$

$$-\left(\frac{c_2}{c_1}\right)^3 \cos 3\theta H\left(t-\frac{R}{c_1}\right)\int_R^{c_1 t}\frac{f'(t-S/c_1)}{(S^2-R^2)^{1/2}}\left(4\frac{S^3}{R^3}-3\frac{S}{R}\right)\mathrm{d}S$$

$$+\cos\theta H\left(t-\frac{R}{c_2}\right)\int_R^{c_2 t}\frac{f'(t-S/c_2)}{(S^2-R^2)^{1/2}}\left(\frac{S}{R}\right)\mathrm{d}S$$

$$+\cos 3\theta H\left(t-\frac{R}{c_2}\right)\int_R^{c_2 t}\frac{f'(t-S/c_2)}{(S^2-R^2)^{1/2}}\left(4\frac{S^3}{R^3}-3\frac{S}{R}\right)\mathrm{d}S\Bigg], \tag{27 b}$$

$$G_{31}^3(t\,|\,f)=\frac{1}{2\pi c_2}\cos\theta H\left(t-\frac{R}{c_2}\right)\int_R^{c_2 t}\frac{f'(t-S/c_2)}{(S^2-R^2)^{1/2}}\left(\frac{S}{R}\right)\mathrm{d}S, \tag{28}$$

where

$$X_1=\cos\theta, \qquad X_2=\sin\theta.$$

It can be easily seen that

$$G_0^\gamma=G_{11}^\gamma+G_{22}^\gamma=G_{\alpha\alpha}^\gamma$$

corresponds to the displacement field due to a dilatational line source. From Eqs. (20) and (23), we find

$$G_0^R(t\,|\,f)=G_0^1\cos\theta+G_0^2\sin\theta$$

$$=\frac{A}{\pi c_1}H\left(t-\frac{R}{c_1}\right)\int_R^{c_1 t}\frac{f'(t-S/c_1)}{(S^2-R^2)^{1/2}}\left(\frac{S}{R}\right)\mathrm{d}S, \tag{29}$$

$$G_0^\theta(t\,|\,f)=G_0^2\cos\theta-G_0^1\sin\theta=0.$$

Equation (29) represents P waves with purely radial displacements.

Consider the particular case in which $f(t)=H(t)$. Equations (27)–(29) then yield

$$G_{12}^1(t\,|\,H)=\frac{1}{4\pi}\left[\left(\frac{c_2}{c_1}\right)^2 H\left(t-\frac{R}{c_1}\right)\frac{c_1 t}{R}(c_1^2t^2-R^2)^{-1/2}\left\{\sin\theta+\left(4\frac{c_1^2t^2}{R^2}-3\right)\sin 3\theta\right\}\right.$$

$$\left.+H\left(t-\frac{R}{c_2}\right)\frac{c_2 t}{R}(c_2^2t^2-R^2)^{-1/2}\left\{\sin\theta-\left(4\frac{c_2^2t^2}{R^2}-3\right)\sin 3\theta\right\}\right], \tag{30 a}$$

$$G_{12}^2(t\,|\,H)=\frac{1}{4\pi}\left[\left(\frac{c_2}{c_1}\right)^2 H\left(t-\frac{R}{c_1}\right)\frac{c_1 t}{R}(c_1^2t^2-R^2)^{-1/2}\left\{\cos\theta-\left(4\frac{c_1^2t^2}{R^2}-3\right)\cos 3\theta\right\}\right.$$

$$\left.+H\left(t-\frac{R}{c_2}\right)\frac{c_2 t}{R}(c_2^2t^2-R^2)^{-1/2}\left\{\cos\theta+\left(4\frac{c_2^2t^2}{R^2}-3\right)\cos 3\theta\right\}\right], \tag{30 b}$$

$$G_{31}^3(t\,|\,H)=\frac{1}{2\pi}H\left(t-\frac{R}{c_2}\right)\frac{c_2 t}{R}(c_2^2t^2-R^2)^{-1/2}\cos\theta, \tag{31}$$

$$G_0^R(t\,|\,H)=\frac{A}{\pi}H\left(t-\frac{R}{c_1}\right)\frac{c_1 t}{R}(c_1^2t^2-R^2)^{-1/2}, \tag{32}$$

where we have used the relations

$$H'(t-S/c_\alpha)=\delta(t-S/c_\alpha)=c_\alpha\delta(S-c_\alpha t)\,.$$

Taking the limit as $t\to\infty$, Eqs. (30)–(32) give

$$G^1_{12}=\frac{1}{4\pi R}[(2-A)\sin\theta+A\sin 3\theta]\,, \tag{33}$$

$$G^2_{12}=\frac{1}{4\pi R}[(2-A)\cos\theta-A\cos 3\theta]\,, \tag{34}$$

$$G^3_{31}=\frac{1}{2\pi R}\cos\theta\,,\qquad G^R_0=\frac{A}{\pi R}\,. \tag{35}$$

Equations (33)–(35) coincide with the corresponding statical results.

The displacement $G^3_{31}(t\,|\,H)$ in Eq. (31) falls from ∞ at $t=R/c_2$ and approaches the static limit $(1/2\pi R)\cos\theta$ at $t\to\infty$. Writing

$$t=\frac{R}{c_\alpha}+\tau_\alpha\qquad(\tau_\alpha>0)\,, \tag{36}$$

we get, for small values of $s_2=c_2\tau_2/R$,

$$G^3_{31}(t\,|\,H)\sim\frac{\cos\theta}{2\pi R}H(\tau_2)\left[\frac{1}{\sqrt{2s_2}}\left(1+\frac{3}{4}s_2\right)\right]. \tag{37}$$

The displacement consists of a sudden jerk at $t=R/c_2$, followed by a gradual recovery, which is incomplete.

For small values of $s_1=c_1\tau_1/R$, Eq. (30 a) yields, for the P pulse,

$$G^1_{12}(t\,|\,H)\sim\frac{1-A}{4\pi R}H(\tau_1)\left[\frac{1}{\sqrt{2s_1}}\left(1+\frac{3}{4}s_1\right)\sin\theta\right.$$
$$\left.+\frac{1}{\sqrt{2s_1}}\left(1+\frac{35}{4}s_1\right)\sin 3\theta\right]. \tag{38}$$

Similarly, for small values of $s_2=c_2\tau_2/R$, we find, for the S pulse,

$$G^1_{12}(t\,|\,H)\sim\frac{1}{4\pi R}H(\tau_2)\left[\frac{1}{\sqrt{2s_2}}\left(1+\frac{3}{4}s_2\right)\sin\theta-\frac{1}{\sqrt{2s_2}}\left(1+\frac{35}{4}s_2\right)\sin 3\theta\right]. \tag{39}$$

In this case, the displacement consists of a sudden jerk at $t=R/c_1$ (P pluse), another sudden jerk at $t=R/c_2$ (S pulse), followed by a gradual recovery, which is incomplete.

Figure 1 shows the variation of G^3_{31} with t for $\theta=0°$. Figures 2, 3, and 4 show the variation of G^2_{12} with t for $\theta=0°$, 30°, and 60°, respectively, for the Poisson case $c_1=\sqrt{3}\,c_2$. When $\theta=0°$, G^2_{12} increases from 0 to $-\sqrt{6}/3\pi R$ as t increases

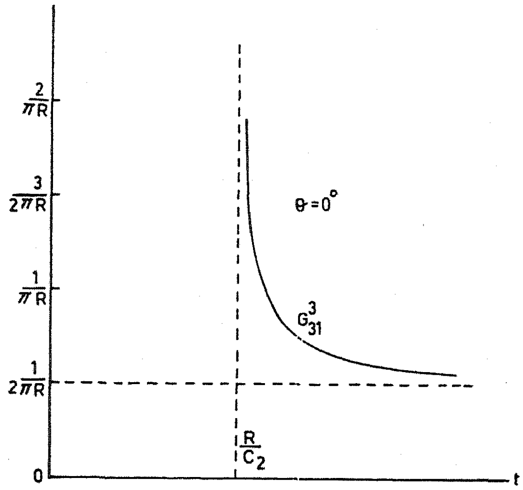

Fig. 1. Variation of G_{31}^3 with t for $\theta=0°$.

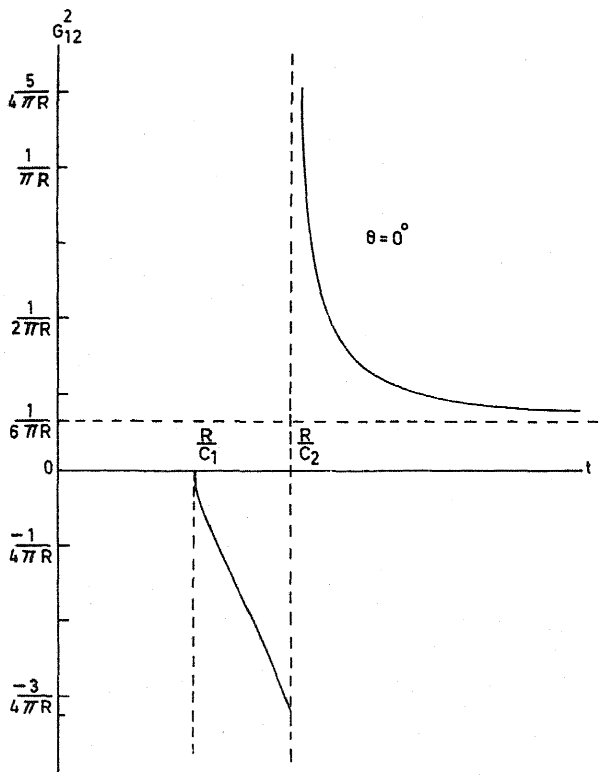

Fig. 2. Variation of G_{12}^2 with t for $\theta=0°$.

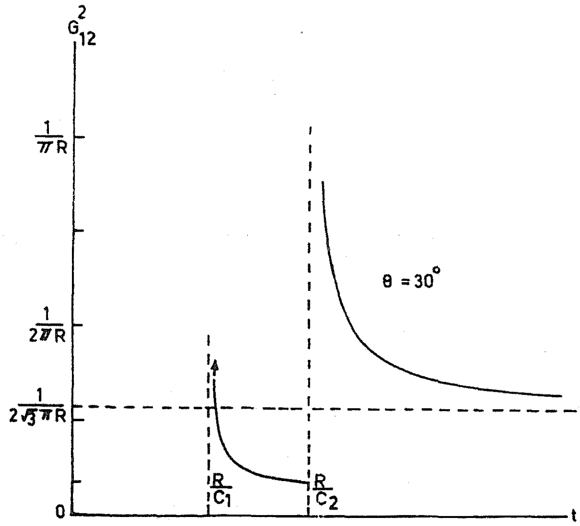

Fig. 3. Variation of G_{12}^2 with t for $\theta=30°$.

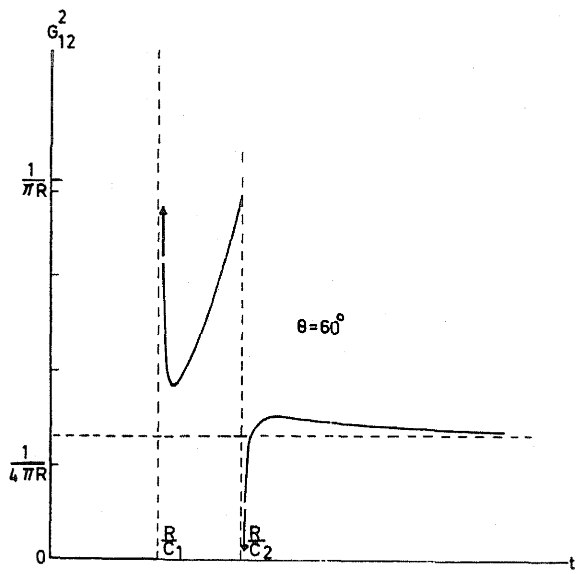

Fig. 4. Variation of G_{12}^2 with t for $\theta=60°$.

from R/c_1 to R/c_2. There is a sudden jerk at R/c_2 followed by a gradual recovery to the static limit $1/6\pi R$. For $\theta=30°$, there is a sudden jerk at R/c_1 followed by another jerk at R/c_2. For $\theta=60°$, $G_{12}^2 \to -\infty$ as $t \to R/c_2+0$. Thus, we notice that the behaviour of G_{12}^2 strongly depends upon the angle θ. The variation of G_{12}^1 is similar to the variation of G_{12}^2. The following relations follow from Eqs. (30a) and (30b).

$$G_{12}^1(\theta=30^\circ)=G_{12}^2(\theta=60^\circ)\,,$$
$$G_{12}^1(\theta=60^\circ)=G_{12}^2(\theta=30^\circ)\,,$$
$$G_{12}^1(\theta=90^\circ)=G_{12}^2(\theta=0^\circ)\,.$$

Neglecting $1/R^{3/2}$ and higher powers of $1/R$, we have the following far-field approximations obtained from Eqs. (30a, b):

$$G_{12}^1(t\,|\,H)=\frac{(1-A)H(\tau_1)}{2\pi(2c_1\tau_1R)^{1/2}}\cos\theta\sin2\theta-\frac{H(\tau_2)}{2\pi(2c_2\tau_2R)^{1/2}}\sin\theta\cos2\theta\,, \tag{40}$$

$$G_{12}^2(t\,|\,H)=\frac{(1-A)H(\tau_1)}{2\pi(2c_1\tau_1R)^{1/2}}\sin\theta\sin2\theta+\frac{H(\tau_2)}{2\pi(2c_2\tau_2R)^{1/2}}\cos\theta\cos2\theta\,. \tag{41}$$

Therefore,

$$G_{12}(t\,|\,H)=e_1G_{12}^1(t\,|\,H)+e_2G_{12}^2(t\,|\,H)=\frac{(1-A)H(\tau_1)}{2\pi(2c_1\tau_1R)^{1/2}}\sin2\theta\,e_R+\frac{H(\tau_2)}{2\pi(2c_2\tau_2R)^{1/2}}\cos2\theta\,e_\theta\,.$$

The first term in Eq. (42) corresponds to P waves and the second term corresponds to S waves. The far-field displacements decay with distance as $R^{-1/2}$. Moreover, the ratio of the far-field S wave amplitude to the far-field P wave amplitude is about $(c_1/c_2)^{5/2}$, which is roughly equal to 4, if $\lambda=\mu$.

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