Engines at molecular scales

Raishma Krishnan and A. M. Jayannavar*
Institute of Physics, Sachivalaya Marg, Bhubaneswar-751005, India

In recent literature there has been a lot of interest in the phenomena of noise induced transport in the absence of an average bias occurring in spatially periodic systems far from equilibrium. One of the main motivations in this area is to understand the mechanism behind the operation of biological motors at molecular scale. These molecular motors convert chemical energy available during the hydrolysis of ATP into mechanical motion to transport cargo and vesicles in living cells with very high reliability, adaptability and efficiency in a very noisy environment. The basic principle behind such a motion, namely the Brownian ratchet principle, has applications in nanotechnology as novel nanoparticle separation devices. Also, the mechanism of ratchet operation finds applications in game theory. Here, we briefly focus on the physical concepts underlying the constructive role of noise in assisting transport at a molecular level. The nature of particle currents, the energetic efficiency of these motors, the entropy production in these systems and the phenomenon of resonance/coherence are discussed.

PACS numbers: 05.40.-a,05.60.cd,02.50.Ey.,05.70.lw
Keywords: Ratchets, Molecular motors, energetics, transport coherence

I. INTRODUCTION

Noise or fluctuations, that arise either due to the coupling of the system with external incompletely described system or from the bath is traditionally thought of as an unwanted effect. Recently, much work has been done the outcome of which reveal clearly the constructive role of noise in many systems [1, 2, 3, 4, 5, 6, 7]. Particularly, biological systems provide an important motivation to study the physics of active processes. In the molecular scale, these systems transduce chemical energy obtained from chemical reactions out of equilibrium into mechanical work, generating net motion in a very noisy environment. In terms of magnitude, the particle is acted upon by a noise power of about $8-9$ orders of magnitude greater than the chemical power available to drive the motion. Even then the molecular motors, for instance, are able to move and transfer cargo from one point to another and sometimes against the potential gradient. They perform this useful work with high efficiency and reliability even when the environmental conditions are changing all the time.

Examples of molecular motors include cytoskeletal motor proteins namely kinesin, dynein, etc., which move on the microtubules. Also molecular pumps, for example, sodium or potassium pumps etc., maintain active transport across membranes against a concentration gradient. What distinguishes these machines from their macroscopic counterparts or heat engines is the fact that they operate in a highly viscous medium which is characterised by low Reynolds number and are subjected to strong thermal fluctuations due to which their motion is stochastic. Hence these motors are termed as Brownian motors or rectifiers. Also, they operate at isothermal conditions. They work by harnessing the force of random motion in the surrounding medium in the absence of a conventional energy source and use it for creating

*Electronic address: jayan@iopb.res.in

directed motion. Here we focus mainly on some general physical principles behind such phenomena without going into the details of its biological implications.

Any system which is in equilibrium with a thermal bath at temperature T has the presence of noise in it. Though these thermal noise/fluctuations are ubiquitous, the validity of the second law of thermodynamics forbids the harnessing of noise for useful purposes without spending any energy from the external sources. A Brownian particle executes a random motion in a liquid without any preferential direction. The principle of detailed balance, which essentially means that the rate of forward motion is equal to the rate in the backward direction, forbids current in any preferential direction and hence one cannot extract useful work. In other words, one can extract energy only when the system is driven away from equilibrium. This has been very well demonstrated by Feynman in his 'Feynman Lectures on Physics' [8] by introducing a mechanical ratchet and pawl subjected to thermal fluctuations to demonstrate the impossibility of the violation of the second law of thermodynamics. Hence building a motor that uses thermal energy from a single heat bath to do mechanical work is not possible.

II. CONDITIONS FOR THE EFFECT

The model to understand such noise induced active transport in a fluctuating environment is provided by the so called Brownian ratchets. These are systems with an underlying spatially asymmetric periodic potential, that exploit the nonequilibrium fluctuations, present in the medium, to generate a directed flow. The effect of the thermal environment is modeled by considering randomly fluctuating force $\xi(t)$ and a concomitant viscous (frictional) force with a friction coefficient η. η and random noise $\xi(t)$ with $<\xi(t)>=0$ are related through the fluctuation-dissipation theorem, i.e., $<\xi(t)\xi(s)>=2\eta k_B T\delta(t-s)$. Physical models like flashing ratchets, rocking ratchets, time asymmetric ratchets, inhomogeneous (frictional) ratchets etc., have been proposed to achieve essential nonequilibrium conditions for net motion in a periodic system. As long as the system is left alone and remains in thermodynamic equilibrium particles in a ratchet cannot diffuse in any preferential direction, in spite of the spatial asymmetry in the potential. But in the presence of additive or multiplicative nonequilibrium fluctuations the particles in general start to move in one direction as the principle of detailed balance does not hold in this case. Thus both nonequilibrium fluctuations and spatial or temporal asymmetry in potential conspire to generate a unidirectional flow in the absence of bias. In the following we briefly discuss the essential ideas behind some of the ratchet models.

III. DIFFERENT TYPES OF RATCHETS

A. Flashing ratchets

This is a simple model that closely resembles the mode of operation of protein motors. In this model the periodic potential is allowed to fluctuate with finite time correlation between two states characterised by different barrier heights. For example, the overdamped Brownian particles are subjected to two potential states periodically, i.e., V_{on} which corresponds to an asymmetric saw tooth like potential for a time τ_{on} and V_{off} which corresponds to zero potential (flat) state for a time interval τ_{off} as is shown in Fig. 1. During the period when the

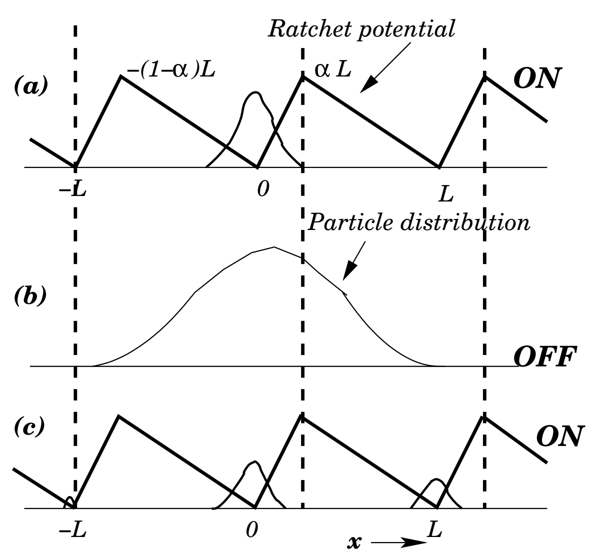

FIG. 1: Flashing ratchet model. Here L is the period of the potential and α is the asymmetry parameter.

potential is *on* the particles will slide down the potential slope to the bottom of the potential minima due to which there is a peak in the probability density of particles at these minima. Switching the potential *off* allows the particles to diffuse freely and the density of particles spread into a Gaussian curve centered around the minima as shown in Fig. 1b. At the end of τ_{off} the potential is again put back to the *on* state for an interval τ_{on} and the particles will again slide down along the direction of local force to the nearest minima, Fig. 1c. This process is continued indefinitely resulting in a net current in one direction because of spatial asymmetry in the potential within a period.

The main point to be taken care of is the time interval between switching *on* and *off* periods of the potential. If τ_{off} is adjusted such that by the end of τ_{off} the diffusive motion takes the particle out of the earlier existing potential minima in the steeper slope (smaller distance) direction of the saw tooth potential but fails to do so in as much proportion in the gentler slope (larger distance) direction, then in the next *on* interval of the potential the particles will slide along the gentler slope to the adjacent minimum of the saw tooth potential. This process of sequential flipping of the potential between *on* and *off* states is continued and in the long time limit one gets a net flow of current to the right side. The system is supplied with the required energy to flip the potential states externally thereby rendering the system nonequilibrium. It is to be emphasized that thermal fluctuations are necessary for the working of flashing ratchets. Moreover, no macroscopic bias is applied to the system.

1. *Equivalence to Carnot engine*

The flashing ratchet model where the potential is flipped between *on* and *off* states is analogous to the case where the particle is coupled to two temperature baths. From statistical mechanics it is well known that the probability to cross a barrier is governed by the factor $\exp -(V/k_BT)$ where V is the potential barrier height. When the temperature T is large the average kinetic energy of the particle is large and so is the violent thermal fluctuations. Due to this the particle hardly feels the presence of potential in comparison with the thermal noise and hence the probability to cross the barriers is large. This is equivalent to the particle being in the *off* state of the flashing ratchet discussed above. In the opposite case when the temperature of the medium is small, the average kinetic energy and the thermal fluctuations are small and the particles will feel the presence of the asymmetric potential. This case is akin to the *on* state in the flashing ratchet. Thus the systematic coupling of the particle randomly to two temperature baths is equivalent to flashing ratchet model where the potential is switched *on* and *off* and in the long time limit one gets unidirectional current. In this spirit the ratchet system has direct equivalence to a Carnot engine which extracts work by making use of two thermal baths held at different temperatures.

The relevant system variables for the ratchets are temperature $T(t)$ and the position $x(t)$ while that for a Carnot engine are pressure and volume. However, there exists qualitative differences between the two. For the case of ratchets, after one periodic variation in time of $T(t)$ or one temperature cycle the particle may or maynot come to the same position or in otherwords there is no synchronization between the relevant system variables. But for a Carnot engine there is a complete synchronization between the relevant system variables along the cycle. The ideal Carnot engine moreover will not be simultaneously in contact with two temperature baths assuring the reversible mode of operation. In contrast, ratchets or molecular motors work in an intrinsically irreversible mode of operation with a very low efficiency. It has to be emphasised that Carnot engine gives high efficiency only in the quasi-static mode of operation and though there is net work done by the engine the net power delivered in the cycle is zero. For the case of molecular motors we may get a higher efficiency in the nonadiabatic regime (i.e. by increasing the frequency of oscillation or cycling) as compared to the values obtained in the adiabatic or quasi-static regime of operation. This behaviour is quite contrary to the case of reversible macroscopic heat engines. The distinguishing factor of a Brownian motor is that noise plays a dominant role and that noise may facilitate energy transduction leading to high efficiency of these molecular engines which is counterintuitive [9].

In these molecular engines noise and associated dispersion of particles are no longer thought of as a hindrance, but are instead incorporated as a part of the design. Moreover, an efficient microscopic engine is not necessarily the microscopic equivalent of an efficient macroscopic engine.

2. *Current reversals and mass separation devices*

With a judicious choice of asymmetric potential the current reverses its sign as a function of suitable system parameter. This phenomenon is called current reversal. Thus, Brownian particles with different friction coefficients, masses or charges move in opposite directions and hence they can be readily separated. This is a new modern method of separating particles at nanometer scale. To illustrate the phenomenon of current reversal consider a potential of the form $V_\lambda(x,t) = \lambda V_2(x,t) + (1-\lambda)V_1(x,t)$ where $V_1(x,t)$ and $V_2(x,t)$ are two fluctuating ratchet potentials such that the unidirectional flow of currents in $V_1(x,t)$ and $V_2(x,t)$ are in opposite directions in the presence of nonequilibrium fluctuations and λ is a parameter between 0 and 1. If $V_2(x,t)$ is a mirror reflection of $V_1(x,t)$, the fluctuating ratchet potential shown in Fig. 1, then under the influence of $V_2(x,t)$ alone current flows in the negative direction. The current has a smooth dependence on mass and the higher the mass the lower is the current. A particle of mass m will move in respective directions depending on the potential to which it is subjected to. Suppose that the particle is subjected to the potential $V_\lambda(x)$ which is a combination of $V_1(x,t)$ and $V_2(x,t)$. The plausible curve for unidirectional current as a function of λ in shown Fig. 2. When λ is zero there will be contribution only from $V_1(x,t)$ and one gets a current which is positive. When $\lambda = 1$ the contribution to the current will be from $V_2(x,t)$ and hence the flow will be in the opposite direction. Thus by continuously deforming one potential into another, i.e., $V_1(x,t)$ into $V_2(x,t)$ there must exist an intermediate potential with the property that the particle current is zero at some finite value of the parameter λ_c. Hence there is a critical λ_c at which the current curve passes with a finite slope through this zero point thereby implying the existence of current inversion as a function of λ.

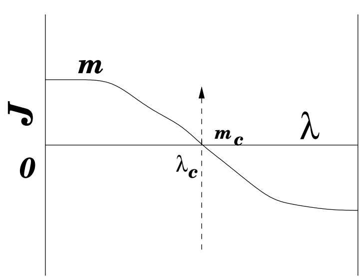

FIG. 2: Figure to illustrate current reversal.

Once a current inversion upon variation of one parameter of the model is established an inversion upon variation of any other parameter can be inferred along the same line of reasoning. Suppose we fix a point, say, the value of $\lambda = \lambda_c$ at which the current for a particular mass, m_c, is zero. Now if we vary the mass around the value m_c one would see that the current as a function of mass will smoothly go through this zero point at $m = m_c$ with a finite slope. This means that a current reversal is obtained as a function of mass. That is, particles of mass greater than m_c and that with mass less than m_c get separated in opposite directions.

The phenomenon of current reversal can play a major role in separation devices. This method of particle separation has many features far superior than the existing methods like electrophoresis, centrifugation, chromatography etc., which rely on the motion caused by long range gradients. In these methods the thermal noise inturn degrades the quality of separation due to the diffusive broadening of the bands. It is also possible to get multiple current reversals by properly choosing the potential as a function of system parameters. By multiple reversals one can separate blocks of particles of different masses with parameters within a characteristic window. It may be noted that for two dimensional ratchets, particles of different masses get separated in different directions.

3. *Parrondo's paradox*

The concept of Brownian ratchets, where there is rectification of fluctuations to give unidirectional current also has its extensions to game theory opening up a new area of paradoxical gambling games under the subject of Parrondo's paradoxes [10]. These games can be thought of as a discrete time version of flashing Brownian ratchets with the interesting consequence that by randomly switching between two losing games one tends to win. To begin with, consider a flashing ratchet model

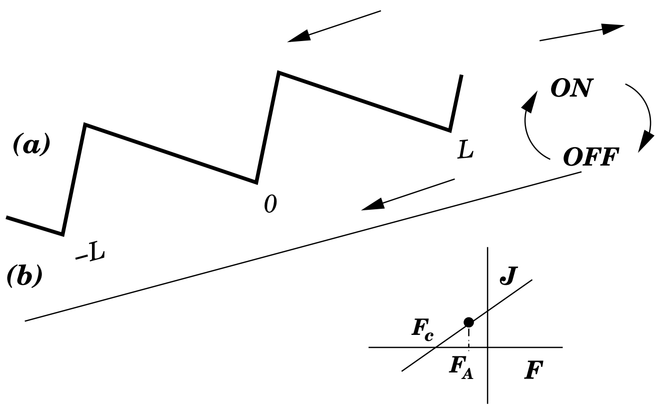

FIG. 3: Illustration of Parrando's paradox. (a) and (b) corresponds to the *on* and *off* potential states in the presence of bias field.

in the absence of any external force as discussed in Sec. III A with a current flow towards the right, Fig. 1, or in the positive direction when the potential is fluctuating between the *on* and *off* states. To this a constant tilting force F, as in Fig. 3, is applied opposite to the direction of current in the ratchet. Then, as expected, the current (J) in the positive direction decreases and beyond a particular value called the stopping force F_c the current crosses over to the negative direction. This is illustrated in the J vs F plot in Fig. 3. Let us consider a particular value of force say F_A in the J vs F plot. The corresponding flashing ratchet profile is as in the figure. In the presence of a bias force F_A, the current flows in the negative direction as expected in both the *on* and *off* states of the potential in the absence of flipping. But flipping between the *on* and *off* states randomly for a long time will result in a current in the positive direction. Thus at the point F_A one gets current in the positive direction though when considered separetely (i.e., when the potential is as shown either in Fig. 3a or in Fig. 3b) the flow of current is in the negative direction.

The current in the negative direction for the two different potentials is analogous to a losing game and thus alternating randomly between these two losing games one has a finite probability to win (current in the positive direction). The game reveals the fact that the outcome of the alternation of two stochastic dynamics can significantly be different from each separate one. Parrando's paradox, where one basically converts two losing strategies into a winning one, has numerous applications in the field of economics, sociology and many other interdisciplinary areas. An example is the formation of spatial patterns in spatially extended systems by alternation between two dynamics which in turn is absent in the presence of only a single dynamics.

B. Rocking ratchets

In the ratchet model we had considered before namely, flashing ratchet, the potential fluctuates between *on* and *off* states. Another type of ratchet corresponds to rocking ratchets where given an asymmetric potential one applies a random time varying force with mean zero. Due to the anisotropy of the potential (a special case is shown in Fig. 4a), when a force having same magnitude but different signs $+F$ and $-F$ are applied, the motion of the particle on the average will be along positive and negative direction respectively. However, particles will have to overcome only the smaller barriers along the direction of their average motion in the presence of a positive force as opposed to the case when the force is negative with the same magnitude. We consider a case where the average slope of the saw tooth potential, Fig. 4a, is changed in time either slowly or abruptly with a finite maximum value on either side of the zero slope line ensuring that the time average of the force acting on the particle due to rocking is zero (Fig. 4b and c). The rocking or changing of slopes can be done either periodically or randomly in time.

At very low temperature when the particles do not have enough energy to overcome the barrier the particles get trapped in the minima of the potential. A special case of rocking force, $+F$ and $-F$, imposed on the ratchet potential is shown in Fig. 4b and 4c respectively. For the case when force is negative, Fig. 4b, the particle remains trapped in one of the trenches whereas when the force is positive, Fig. 4c, the particle is capable of running down the potential hill. With such a geometrical construction one can notice that the current when

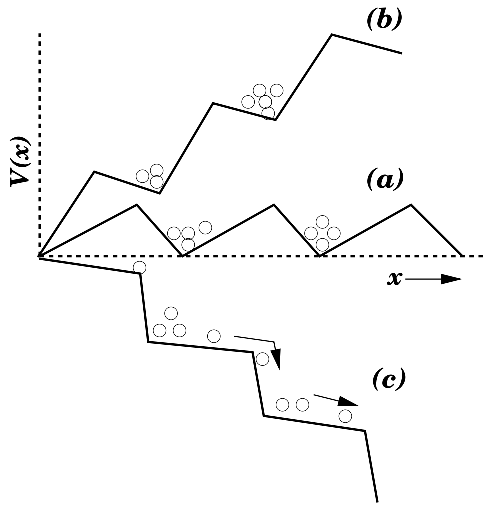

FIG. 4: Rocking ratchet model.

the force is $+|F|$ is not equal and opposite to the case when force is $-|F|$. In otherwords, $J(|F|) \neq -J(-|F|)$ remains valid even for finite temperatures. Thus system acts as a nonlinear rectifier in the presence of zero average periodic or random force.

Unlike in the case of flashing ratchets, the direction of current for the rocking ratchet is in the direction of the steeper slope and this mechanism of rocking is equivalent to generating dc current in semiconductor pn junctions under an applied ac bias.

C. Inhomogeneous ratchets

There is yet another type of ratchet, namely, frictional ratchets [7, 11] which unlike the ones discussed above gives unidirectional current even in the presence of spatially periodic symmetric potential, $V(x)$ but in the presence of space dependent diffusion coefficient $D(x) = k_B T(x)/\eta(x)$. The space dependence of diffusion coefficient could arise either due to space dependent temperature $T(x)$ or space dependent friction coefficient $\eta(x)$. Such inhomogeneous systems are common in nature. For example, particles diffusing close to surface have space dependent friction coefficient. The molecular motor proteins are believed to be moving close along the microtubules and therefore experience space dependent mobility. Semiconductor systems and superlattice structures also have space dependent friction coefficient. The peculiarity of this type of ratchets is that the system

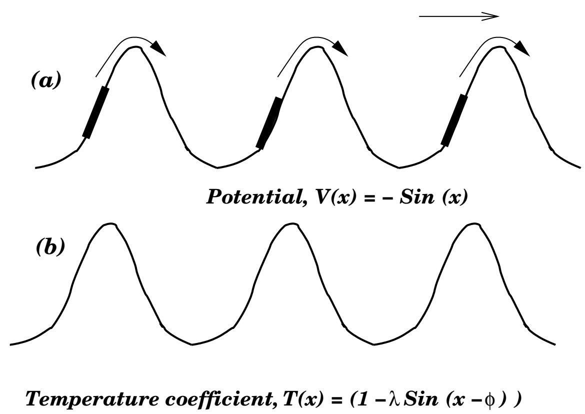

FIG. 5: Illustration of an inhomogeneous ratchet. The temperature and potential profiles are depicted in the figure.

dissipates energy during its time evolution differently at different places due to the space dependent diffusion coefficient arising from variation in temperature $T(x)$ which in turn implies that the system is out of equilibrium. The only criterion that has to be satisfied here is that both the potential $V(x)$ as well as temperture $T(x)$ has to be periodic and should be separated by a phase difference other than 0 and π with respect to the potential as shown in Fig. 5. A similar effect of variation in temperature can also be obtained if we have a medium with space dependent friction coefficient $\eta(x)$ in the presence of external noise. The noise being externally imposed the system always absorbs energy due to the absence of a concomitant loss factor. At the regions where the friction coefficient is high the overdamped particle stays for a longer time due to which the possibility of absorption of energy from the external noise in that region is correspondingly high. This leads to an increase in the local temperature in these regions. Thus system with space dependent friction at constant temperature in the presence of external parametric noise is equivalent to a system with a space dependent temperature field.

To illustrate the net unidirectional transport in these systems consider a periodic potential $V(x)$ as shown in Fig. 5a. Also consider a space dependent temperature profile with same periodicity as that of potential but shifted by a phase difference ϕ as shown in

Fig. 5b. In Fig. 5a the darkened regions specify regions of higher temperature corresponding to the regions where the temperature profile has peaks Fig. 5b. The particle in the darkened regions (high temperature regions) on the average gains more energy as compared to other regions. As a consequence the particle in any potential minima will find it easier to cross the peak of the potential and go over to the right side than to the left side. Hence current in the right side is assured. The magnitude as well as the direction of current depends on the phase difference ϕ.

It may be noted that unidirectional motion in inhomogeneous systems arise as a corrollary to the well known Landauer's blow torch principle [12]. This principle states that the behaviour of nonequilibrium systems will depend sensitively on the specific details of its kinetics, even on pathways that traverse infrequently occupied kinetic states far from the stable state. In otherwords, the stability criteria which examine only the immediate vicinity of a locally stable state are inadequate to assess the relative stability of states in nonequilibrium systems. In contrast, microscopically reversible systems can well be characterised by criteria that depend only on the local neighbourhood of the equilibrium state.

IV. ENERGETICS OF BROWNIAN MOTORS

As mentioned above, ratchets extract energy from random fluctuations and generate currents or ordered motion. In this sense they can be considered as information engines analogous to Maxwell's demon which extract work out of bath at the expense of an overall increase in entropy. The usefulness of any engine lie in the extent of work that can be efficiently extracted out of it. The molecular motors in living cells are found to be very efficient in their noisy environment. In all the ratchet models that we have discussed above no useful work has been performed. This is because particles moving in a periodic potential system ends up with the same potential energy even after crossing over the adjacent potential minimum. There is no extra energy stored in the particle which can be usefully expended when needed. To have an engine out of a ratchet it is necessary to use its systematic motion to store potential energy which inturn is achieved if a ratchet lifts a load. Thus for the ratchet to perform work a small force called load (Fig. 6) has to be applied opposite to the direction of current in the ratchet. Then the particles will keep on moving, on the average, against the force or load performing work. Part of the input energy, E_{in}, coming from the source of nonequilibrium is converted into mechanical energy related to the load.

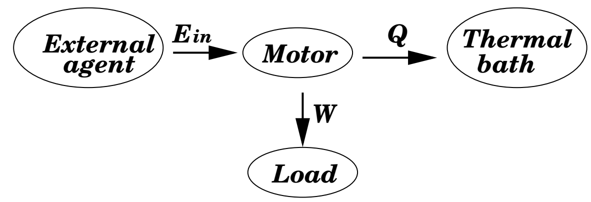

FIG. 6: Schematic figure of the energy flow in a Brownian motor.

A general framework has been developed wherein the compatibility between the Langevin or Fokker-Planck formalisms, used to discuss stochastic processes, and the laws of thermodynamics, which characterize the thermal and mechanical behaviour of macroscopic systems, have been established [13]. The concept of heat on mesoscopic scales has been defined in terms of Langevin dynamics and the essential point behind this formalism is that the heat transferred to the system is nothing but the microscopic work done by both the frictional and random force in the Langevin equation (i.e., work done by the bath on the system). This is also consistent with the fact that we cannot control all the details of energy transfer which inturn leads to the concept of heat (via stochastic dynamics) as a form of energy flow. The subject of the energetics of Brownian motors has developed into an entire subfield on its own right. Fig. 6 represents the energy flow between the isothermal Brownian motor and its surroundings. The first and the second law of thermodynamics is now given as

$$E_{in} = Q + W \tag{1}$$

$$\Delta S_{agent} + \frac{Q}{T} = S_{prod} \geq 0 \tag{2}$$

In the above equation E_{in} is the input energy into the system from the external agent, Q, the heat dissipated to the bath, and W the work done. All these quantities can be defined for each microscopic realization of the motion of Brownian particle or motor. Eqns. 1 and

2 correspond to the first and second law of thermodynamics. Again, ΔS_{agent} is the change in entropy of the external agent, Q/T is the entropy given to the bath and S_{prod} is the total entropy production of the universe. T corresponds to the absolute temperature. Magnitudes of all the physical quantities are taken over a cycle or per unit time in the stationary regime. In this regime the entropy and the internal energy of the motor (system) which are the state variables does not change. The formal expressions for all the above mentioned physical quantities are known in terms of the probability distribution of the particles.

Using this framework of stochastic energetics one can readily calculate various physical quantities like efficiency of energy conversion ($\eta = W/E_{in}$), energy dissipation (hysteresis loss), entropy production [14], input energy, work etc. The important point to be noted here is that an analysis of fluctuations, which is completely ignored in the working of heat engines at larger scale, is essential for the calculation of efficiency of ratchet systems at the molecular scale. The efficiency of the Brownian motors is sensitively dependent on system parameters.

The study of the efficiency of energy transduction by different types of ratchet models show the ratchets to have very low efficiency. The observed efficiency values of the several ratchet models like flashing ratchets, rocking ratchets etc., are found to fall in the subpercentage regime ($< .01$). This is due to the fact that every time the potential changes the particle distribution also changes and tries to adapt to the changing environmental conditions. This leads to an inevitable loss in the medium or in other words the mode of operation of the ratchets is intrinsically irreversible. As a consequence the unattainability of Carnot efficieny in Brownian heat engines has been emphasized in literature. Currently, the notion of reversible ratchets where the energy dissipation or entropy production is almost zero are being pursued. These reversible ratchets are sometimes termed as adiabatic pumps wherein transport of particles with zero entropy production is generated by cyclic adiabatic variations of atleast two parameters of the periodic potential (which are out of phase in time) in the absence of bias [13].

The energetic efficiency of a ratchet is not an intrinsic property of the device and it depends on the characteristics of the imposed external load. By a judicial choice of the external load one can improve the efficiency considerably. Recently [15] a flashing rachet model has been developed, wherein with two asymmetric double-well periodic-potential- states displaced by half a period a high efficiency has been achieved due to the blocking of particle motion in the opposite direction to that of the net average current. Such flashing ratchet models were found to be highly efficient with efficiency an order of magnitude higher than in the earlier models. The basic idea behind this enhanced efficiency is that even for diffusive Brownian motion the choice of appropriate potential profile ensures suppression of backward motion and hence a reduction in the accompanying dissipation.

We have studied the motion of a particle in a new class of rocking ratchets rocked purposefully as to favour current in one direction but to suppress motion in the opposite direction. This is accomplished by applying a temporally asymmetric but unbiased periodic forcings. It may be noted that in this type of ratchets a larger force field is applied for a short time interval of the period in the forward direction as compared to a smaller force for a longer time interval in the other direction. The intervals are so chosen that the net external force or bias acting on the particle over a period is zero. In these new class of temporally asymmetric driven ratchets one gets unidirectional current even in the presence of spatially symmetric potential [16].

At low temperatures when $k_B T$ is much less than V_0, the modulation amplitude of the periodic symmetric potential, significant current arises only when the bias field is greater than a critical field F_c, the value of which should be greater than V_0 [17]. If the bias field is less than V_0, the particle will feel the barriers and hence current flux in the negative direction is very small or there is blocking of current. A significant current flux in the positive direction arises only when the temporally asymmetric bias force field in that direction is greater than V_0. When this condition, of bias field being less than V_0 in the backward direction and being greater than V_0 in the forward direction, is satisfied the barriers for motion in the forward direction disappears and one gets unidirectional current. Interestingly, such choice of forcings help in obtaining rectified currents with high efficiency of the order of 50% without fine tuning of the physical parameters. This efficiency is several orders of magnitude larger than the obtained efficiency in several other ratchet models. Moreover, the range of parameters of operation of such ratchets is quite wide sustaining large loads.

A. Currents, Stochastic resonance and Coherence

The noise induced currents in most of the ratchet systems exhibits a peak with respect to noise strength and other physical parameters. Such peaking behaviour is expected when system exhibits a resonance phenomena. Infact, some recent studies have tried to reveal the relation between two unrelated phenomena, namely stochastic resonance and Brownian ratchets in a formal way through the consideration of Fokker-Planck equations. We have analysed this issue by using the method of stochastic energetics in several classes of adiabatically rocked ratchets.

The resonance behaviour can be well characterised by the behaviour of input energy. It is expected that at resonance the system extracts maximum energy from the external source and hence in the time periodic stationary state this energy is dissipated to the environment (hysteresis loss). Our studies show the input energy to have a monotonous behaviour as a function of noise strength. Thus the resonance like feature observed in the nature of currents as a function of noise strength is not related to the intrinsic resonance in the dynamics of the particle with the external ac drive. The above observations are valid only for a class of adiabatically rocked ratchets.

The presence of net currents (ordered motion) in the ratchets increases the amount of known information about the system than otherwise. This extra bit of information comes from the negentropy or the physical information supplied by the external nonequilibrium bath. The amount of information that is transferred by the nonequilibrium bath is quantified in terms of algorithmic complexity of the position of Brownian particle. It has been argued that the algorithmic complexity or Kolmogorov information entropy is maximum when the current is maximum [18]. Since the currents are generated at the expense of entropy we naively expect the maxima in current to be related to the maxima in the overall entropy production as a function of noise strength. However, we have shown that the total entropy production does not extremize at the same parameter value at which the current exhibits a maximum [14]. The fact

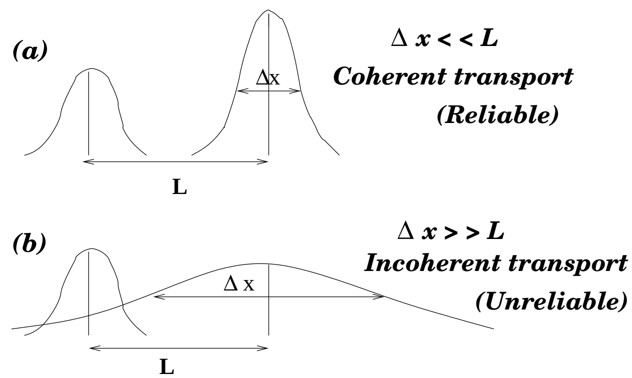

FIG. 7: Evolution of particle distribution for a given time interval is depicted for two separate cases of particle transport.

that noise strength at which the maxima seen in both current and entropy production do not coincide may be related to the quality of the current or the coherence in transport. Noise induced currents are always accompanied by a dispersion or diffusion. When the diffusion is large, $\Delta x >> L$ with Δx being the diffusive spread when the mean position of the particle is shifted by a distance L which is the period of the potential, then the quality of transport degrades and the coherence in the unidirectional motion is lost. The coherent transport (optimal transport) refers to the case of large mean velocity at fairly small diffusion (see Fig. 7). This is in turn quantified by a dimensionless Pećlet number which is the ratio of current to the diffusion constant [19]. From Fig. 7 we see that in both the cases there is a noise induced current of the same magnitude, but transport in Fig. 7a is more coherent due to the fact that particles are reliably transferred from one point to the other due to the less diffusive spread. In Fig. 7b though there is a shift in the peak (or there is current of same magnitude) due to the large diffusive spread the probability that the particle is delivered to the desired region is less. There is a finite probability that the particle may still be around the region where it has started.

For a given magnitude of current the transport may be coherent or incoherent. Thus analysis of the relation between current and the entropy production requires not only the magnitude of current but also the quality of transport. These studies are expected to reveal a deep connection between efficiency, quality of transport, entropy and information.

V. CONCLUSIONS

To summarize, we have given a qualitative picture of the constructive role of noise in nonequilibrium systems. This area has attracted great interest from diverse areas of science and technology. We have presented a brief description of the different ratchet models or Brownian motors and also the method of stochastic energetics that was developed in order to understand the energetics in such systems with special emphasis on our work and results.

In our discussion so far we had restricted only to the case of isolated Brownian motors. Infact, the cooperativity among Brownian motors has far reaching consequences [3]. The coupling among these motors can lead to a marked increase in the efficiency of energy transduction as well as the magnitude of macroscopic current. Cooperative motors also exhibit other fascinating phenomena such as phase transitions, normal to anomalous hysteretic behaviour, absolute negative mobility etc. The parallel development in mesoscopic systems has led to the discovery of quantum ratchets. These quantum ratchets make use of quantum effects such as tunneling and wave interference effects. Such electron ratchets can not only be used to generate particle current but also to pump heat in the reversible mode of operation.

To conclude, we have shown that molecular motors work as engines at molecular scale. These microscopic engines are not the microscopic equivalent of the efficient macroscopic engines that we come across in our daily life. Noise play an inherent constructive role in the mode of operation of these engines. Thus noise is not a nuisance but rather an inseparable part of the design of the engine operation. In some cases increase in noise strength is found to even enhance the efficiency of these engines. The operation of these engines are done by the engines themselves and they are out of equilibrium. As such there are no general principles that determines the mode of optimal efficiency of these engines. Further studies in this interdisciplinary area could lead to a better knowledge of the functioning of these biological motors in living cells and also in the creation of efficient man made nanoscale machines. Such studies could also bear important consequences in understanding the fundamental issues in nonequilibrium statistical mechanics.

---

[1] F. J*ü*licher, A. Adjari & J. Prost, (1997) Rev. Mod. Phys. **69** : 1269.

[2] R. D. Astumian, (2001) Scientific American, **285** : 56.

[3] P. Reimann, (2002) Phys. Rep. **361** : 57. and references therein.

[4] Special issue on "Ratchets and Brownian motors: basics, experiments and applications" ed. H. Linke, (2002) Appl. Phys. **A75(2)**.

[5] R. D. Astumian & P. Hanggi, (2002) Phys. Today, **55** : 33.

[6] M. C. Mahato & A. M. Jayannavar, Resonance (2003) **8**, Number 7 : 33, M. C. Mahato & A. M. Jayannavar, Resonance (2003) **8**, Number 9 : 4.

[7] A. M. Jayannavar, cond-mat 0107079; in Frontiers in Condensed Matter Physics, (A commemorative volume to mark the 75^{th} year of Indian Journal of Physics), ed. J. K. Bhattacharjee & B. K. Chakrabarti (in press).

[8] R. P. Feynman, R. B. Leighton & M. Sands, The Feynman Lectures on Physics (Addison-Wesley, Reading, MA, 1963), Vol. 1.

[9] D. Dan, M. C. Mahato & A. M. Jayannavar, (2000) Int. J. Mod. Phys. B **14** : 1585; (2001) Physica A **296** : 375; D. Dan & A. M. Jayannavar, (2002) Phys. Rev. **E65** : 37105.

[10] G. P. Harmer & D Abbott, (2002) Fluct. and Noise Lett. **2** : R71-R107; J. M. R. Parrondo & Luis Dinis, (2004) Contemp. Phys. **45** : 147.

[11] M. C. Mahato, T. P. Pareek & A. M. Jayannavar, (1996) Int. J. Mod. Phys. B **10** : 3857.

[12] R. Landauer, (1978) Phys. Today, **31** : 23.

[13] K. Sekimoto, (1997) J. Phys. Soc. Jpn. **66** : 1234; J. M. R. Parrondo & B. J. De Cisneros, (2002) Appl. Phys. **A75** : 179.

[14] Raishma Krishnan & A. M. Jayannavar, Physica A (in press), cond-mat 0310726, Debasis Dan & A. M. Jayannavar, Physica A (in press), cond-mat 0303417.

[15] Yu.A. Makhnovskii, V. M. Rozenbaum, D.-Y. Yang, S. H. Lin & Tsong, (2004) Phys. Rev. **E69** : 021102.

[16] M. Millonas, (1995) Phys. Rev. Lett. **74** : 10; M. C. Mahato & A. M. Jayannavar, (1995) Phys. Lett. A **209** :21.

[17] H. Risken, (1984) The Fokker-Planck Equation (Springer Verlag, Berlin).

[18] J. R. Sanchez, F. Family & C. M. Arizmendi, (1998) Phys. Lett. **A249** : 281.

[19] B. Linder & L Schimansky-Geier, (2002) Phys. Rev. Lett. **89** : 230602 ; P. Reimann et al., (2002) Phys. Rev. **E65** : 31104; D. Dan & A. M. Jayannavar, (2002) Phys. Rev. **E66** : 41106; Raishma Krishnan, D. Dan & A. M. Jayannavar, cond-mat 0309617.