Transport Coherence in Frictional Ratchets

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Abstract: We study the phenomena of noise induced transport in frictional ratchet systems. For this we consider a Brownian particle moving in a space dependent frictional medium in the presence of external white noise fluctuations. To get the directed transport, unlike in other ratchet models like flashing or rocking ratchets etc., we do not require the potential experienced by the particle to be asymmetric nor do we require the external fluctuations to be correlated. We have obtained analytical expressions for current and the diffusion coefficient. We show that the frictional ratchets do not exhibit a pronounced coherence in the transport in that the diffusion spread overshadows the accompanying directed transport in system with finite spatial extensions.

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I. INTRODUCTION

Ratchets, also termed as Brownian motors, are systems that exploit the nonequilibrium fluctuations that are present in the medium to generate directed flow of Brownian particles in the absence of any net external force or bias [1, 2]. Recently, such Brownian motors have been studied extensively because they are believed to share common features with biological motors. Biological protien motors convert the chemical energy into macroscopic work i.e., they transport cargo efficiently with high reliablility at room temperature in the presence of a very noisy environment [3, 4].

In thermal equilibrium, Onsager's principle of detailed balance, where the forward transition in any pathway is on average balanced by an identical transition in the backward direction, holds due to which there cannot be any particle current in the system. Also, an understanding of the second law of thermodynamics [5] tells us that a system when left to itself in thermal equilibrium would not cause any preferential motion even in the presence of spatial anisotropy. Thus external nonequilibrium fluctuations are ubiquitous to drive a net directed flow [1, 2].

Much has been studied from the viewpoint of statistical physics in ratchet models to understand how unidirectional motion emerges from nonequilibrium fluctuations [1]. Several physical models like flashing ratchets, rocking ratchets, time asymmetric ratchets etc., where potential has been taken to be asymmetric in space, have been developed. In these models to generate noise induced directional transport the nonequilibrium fluctuations have to be time correlated. In an earlier work it was shown that in the case of inhomogeneous systems, where the friction coefficient and/or temperature varies in space [6], it is possible to get unidirectional currents even in a symmetric potential in the absence of a net bias. Moreover, external fluctuations need not be correlated in time [7, 8]. Such ratchets are termed as frictional ratchets. In the presence of external parametric noise the particle on an average absorbs energy from the noise source. The particle spends larger time in the region of space where the friction is higher and hence the energy absorption from the noise source is higher in these regions. Therefore, the particle in the higher friction regions feel effectively higher temperatures. Thus, the problem of particle motion in an inhomogeneous medium in presence of an external noise becomes equivalent to the problem in a space dependent temperature. Such systems are known to generate unidirectional currents [2, 6, 8, 9]. This follows as a corollary to Landauer's blow torch theorem that the notion of stability changes dramatically in the presence of temperature inhomogenieties. In such cases the notion of local stability, valid in equilibrium systems, does not hold. Frictional inhomogenieties are common

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in superlattice structures, semiconductors or motion in porous media. It is believed that molecular motor protiens moving close along the periodic structures of microtubules experience a space dependent friction. Frictional inhomogenity changes the dynamics of the particle nontrivially as compared to the homogeneous case. This in turn has been shown to give rise to many counter intuitive phenomena in driven non-equilibrium systems [10].

Considerable amount of work has been done on the nature of current, current reversals as well as thermodynamic efficiency in these systems. However, the question of reliability or the coherence of transport has recieved only little attention [9, 11, 12]. Transport of Brownian particles are always accompanied by a diffusive spread. The coherence of transport means a large particle current accompanied by a minimal diffusive spread. When a particle on an average moves a distance L due to its velocity, there will always be an accompanying diffusive spread. If this diffusive spread is much smaller than the distance travelled, then the particle motion is considered to be coherent or optimal or reliable. This is in turn quantified by a dimensionless quantity, Pećlet number Pe, which is the ratio of current to the diffusion constant. Higher the Pećlet number, more coherent is the transport. The Pećlet numbers for some of the models studied show low coherence of transport ($Pe = 0.2$) [11]. But experimental studies on molecular motors showed more reliable transport with Pećlet number ranging from 2 to 6 [13]. Thus study of transport coherence is of importance in identifying a proper model for the biological motors and also in nanoscale particle separation devices based on current reversals.

In this work we study the coherence of transport of a Brownian particle in a medium with frictional inhomogeneity and an external parametric white noise. We show that this system does not exhibit pronounced coherence.

II. THE MODEL:

We consider the overdamped dynamics of a Brownian particle moving in a medium with spatially varying frictional coefficient $\eta(q)$ at temperature T. Using a microscopic treatment the Langevin equations for the Brownian particle in a space dependent frictional medium has been obtained earlier [7, 8]. The corresponding overdamped Langevin equation of motion is given by

$$\dot{q} = -\frac{V'}{\eta(q)} - \frac{k_B T \eta'(q)}{2[\eta(q)]^2} + \sqrt{\frac{k_B T}{\eta(q)}} f(t) \qquad (1)$$

with $< f(t) >= 0$, and $< f(t) f(t') >= 2\delta(t - t')$ where $< ... >$ denotes the ensemble average and q the coordinate of the particle.

The system is then subjected to an external parametric additive white noise fluctuating force $\xi(t)$, so that the equation of motion becomes

$$\dot{q} = -\frac{V'}{\eta(q)} - \frac{k_B T \eta'(q)}{2[\eta(q)]^2} + \sqrt{\frac{k_B T}{\eta(q)}} f(t) + \xi(t) \qquad (2)$$

with $< \xi(t) >= 0$ and $< \xi(t)\xi(t') >= 2\Gamma\delta(t - t')$, where Γ is the strength of the external white noise $\xi(t)$. The corresponding Fokker-Planck equation is given by [14]

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial q}\left[\left\{\frac{V'(q)}{\eta(q)}\right\} P + \left\{\frac{k_B T}{\eta(q)} + \Gamma\right\} \frac{\partial P}{\partial q}\right] \qquad (3)$$

For periodic functions $V(q)$ and $\eta(q)$ with periodicity 2π, one can readily obtain analytical expression for current and is given by [7, 8]

$$J = \frac{1 - exp\,[-2\,\pi\,\delta]}{\int_0^{2\pi} dy exp\,[-\psi(y)] \int_y^{y+2\pi} dx \frac{exp\,[\psi(x)]}{A(x)}} \qquad (4)$$

with the generalized potential $\psi(q)$ as

$$\psi(q) = \int^q dx \frac{V'(x)}{k_B T + \Gamma\eta(x)} \qquad (5)$$

and $A(q)$ as

$$A(q) = \frac{k_B T + \Gamma\eta(q)}{\eta(q)} \qquad (6)$$

with

$$\delta = \psi(q) - \psi(q + 2\pi)$$

which inturn determines the effective slope of the generalized potential $\psi(q)$. Hence the sign of δ gives the direction of current which follows from Eqn(4).

In our present work we have taken the potential $V(q) = V_0[1 - cos(q)]$ and $\eta(q) = \eta_0[1 - \alpha cos(q - \phi)]^{-1}$, $0 < \alpha < 1$. For simplicity we have restricted to the case where $T = 0$. For this case the effective potential $\psi(q)$, δ and $A(q)$ are given by

$$\psi(q) = \frac{V_0}{\Gamma\eta_0}[1 - cos(q) + \frac{\alpha}{4}[cos(2q - \phi) - cos(\phi)] - \frac{\alpha}{2} q sin(\phi)] \quad (7)$$

$$\delta = \frac{V_0 \pi \alpha sin(\phi)}{\Gamma\eta_0} \quad (8)$$

$$A(q) = \Gamma \quad (9)$$

Following references [15], one can obtain exact analytical expressions for the diffusion coefficient D and the average current J as

$$D = \frac{\int_{q_0}^{q_0+L} \frac{dx}{L} A(x) \left[I_+(x)\right]^2 I_-(x)}{\left[\int_{q_0}^{q_0+L} \frac{dx}{L} I_+(x)\right]^3} \quad (10)$$

$$J = L \frac{1 - exp\left[-L\delta\right]}{\int_{q_0}^{q_0+L} \frac{dx}{L} I_+(x)} \quad (11)$$

where $I_+(x)$ and $I_-(x)$ are as given below

$$I_+(x) = \frac{1}{A(x)}\, exp\left[\psi(x)\right] \int_{x-L}^{x} dy\; exp\left[-\psi(y)\right] \quad (12)$$

$$I_-(x) = exp\left[-\psi(x)\right] \int_{x}^{x+L} dy\; \frac{1}{A(y)}\, exp\left[\psi(y)\right] \quad (13)$$

L here represents the period of the potential ($= 2\pi$ in our case). Now, the time taken for a Brownian particle to travel a distance L is given as $\tau = L/v$ and the spread of the particle in the same time is given as $< (\Delta q)^2 >= 2D\tau$. For a reliable transport we require $< (\Delta q)^2 >= 2D\tau < L^2$. This in turn implies that $Pe = Lv/D > 2$ for coherent transport.

III. RESULTS AND DISCUSSIONS

We study the current (J), diffusion constant (D) and the Peclet number (Pe) as a function of the phase difference ϕ between the periodic functions $V(q)$ and $\eta(q)$, noise strength, Γ and α, the amplitude of the periodic modulation of $\eta(q)$. All the physical quantities are taken in dimensionless form.

In Fig. (1) we plotted J, D and Pe as a function of phase ϕ by numerically integrating Eqn 9 & 10 using quadrature methods. All other physical parameters are mentioned in the figure caption. As expected, all the quantities are periodic functions of phase ϕ with the current J being zero at $\phi = 0 \,\&\, 2\pi$ [8, 16]. The Peclet number Pe exhibits a maximum with the maximum value being 1. Thus in the parameters considered in Fig. 1 the transport is not coherent.

In figures 2 and 3 we have plotted J, D and Pe as a function of α and Γ respectively for some typical values of physical parameters given in the figure captions. It is clear from these figures that in the parameter range we have considered the transport is not very coherent. However, the transport coherence in our model is found to be much larger than those in the earlier nonfrictional ratchet models where the Peclet number is always less than 0.2 [11].

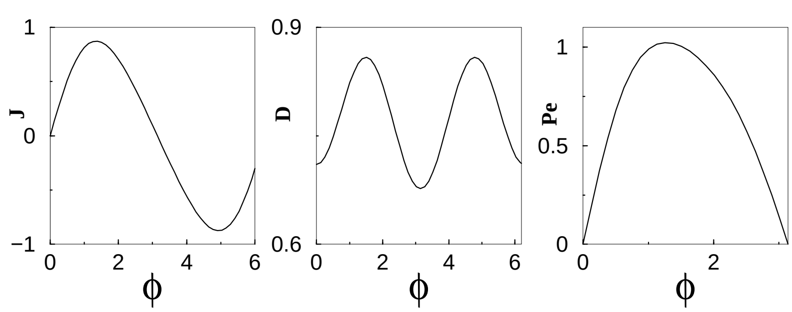

FIG. 1: Plot J, D and Pe for $\Gamma = 1$, $\alpha = 0.5$

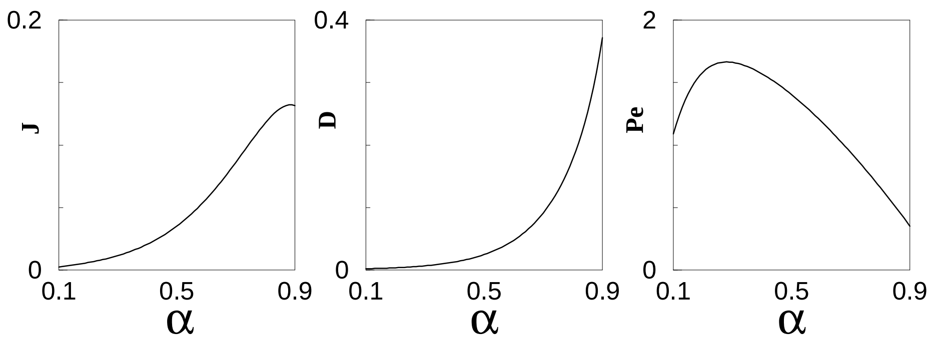

FIG. 2: Plot of J, D, and Pe for $\phi = \pi/2$, $\Gamma = 1$

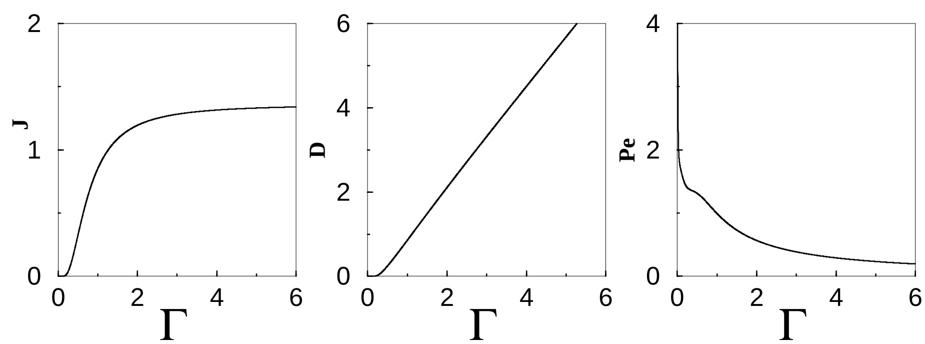

FIG. 3: Plot of J, D and Pe for $\phi = \pi/2$, $\alpha = 0.5$

IV. CONCLUSIONS

We have studied the reliability of transport in frictional ratchet systems in the presence of external white noise fluctuations. We restrict our study to the simple case where the temperature of the heat bath is zero (absence of thermal noise). The transport coefficients, both the current as well as diffusion, arises solely due to the presence of external noise. Our analysis indicates that for this special case, the transport is not very coherent. However, the obtained Pe values are much larger than those obtained from the nonfrictional ratchet models that were studied before. In the small parameter range of $\Gamma \sim 0.05$, $\phi \sim 0.2$, $\alpha \sim 0.9$ we find larger coherence with $Pe \sim 3.5$. However, the transport coefficients are found to be very small. We have not considered here the mutual interplay between the thermal noise and the external noise which may lead to an increase in coherence in appropriate parameter range. Work along this line as well as on different frictional ratchet models is under progress.

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