Quantum current enhancement effect in hybrid rings at equilibrium

Colin Benjamin† and A. M. Jayannavar⋆
Institute of Physics, Sachivalaya Marg, Bhubaneswar 751 005, Orissa, India
(June 21, 2011)

Current enhancement- a novel quantum phenomena is found to occur in a mesoscopic hybrid ring at equilibrium. The hybrid system is described by a ring with bubble which is in turn coupled to a reservoir. In the system the ring encloses a magnetic flux Φ while the bubble does not enclose any flux. The novelty of this work lies in the fact that while earlier current enhancement was observed in non-equilibrium systems (e.g., a ring coupled to two reservoirs at different chemical potentials μ_1 and μ_2), herein we prove that current enhancement can also arise in equilibrium. In addition, we show that the closed system analog of our chosen open hybrid ring system violates parity effects. Finally, we bring to focus the discrepancy between the equilibrium magnetic moment (obtained via energy eigenvalues) and that calculated from the currents in the system.

PACS Nos.: 73.23.-b, 05.60.Gg, 72.10.Bg, 72.25.-b

The physics of low dimensional systems particularly those whose system size is less than the electron phase coherence length has been quite vibrant in recent years thanks to technological advances in the field of nanoscience[1–5]. The study of such systems where the electron retains its wave nature over the entire sample is termed mesoscopic physics. In these systems experiments have revealed that several classical laws which hold for macroscopic systems breakdown[2]. This is attributed to the interference effects of electronic waves. One of the simple quantum mechanical phenomena which has been predicted in such systems is that of current enhancement or magnification[6–8]. Current enhancement can be defined as follows- In a metallic loop(see inset fig. 1) connected to two ideal leads transport current I flows through the system. Currents I_1 and I_2 flow in the upper and lower arms of the ring respectively. In general, I_1 is not equal to I_2 but $I = I_1 + I_2$, Kirchoff's law. In classical case both I_1 and I_2 are positive and flow in same direction as the applied bias. In quantum mechanics, for particular values of Fermi energy I_1 or I_2 can become much larger than I, this implies to obey Kirchoff's law the current in the other arm must be negative. The property that current in one of the arms is larger than the transport current is referred to as current enhancement effect. In this situation, we interpret the negative current flowing in one arm of the ring as a circulating current that flows continually in the loop. When the negative current flows in the upper arm the circulating current direction is taken to be anti-clockwise (or negative) and when it flows in the lower arm the circulating current direction is taken to be clockwise (or positive)[9].

The current enhancement effect leads to enhanced magnetic response (orbital magnetic moment) of a loop carrying current in the absence of magnetic flux which can lead to an experimental verification of this[4]. It is to be noted that these circulating currents arise in the absence of magnetic flux and in presence of transport currents (i.e., in a non-equilibrium system). In the present work our thrust is whether we can observe the aforesaid current enhancement effect and the resulting circulating currents in equilibrium. For this we consider the one dimensional hybrid ring system as depicted in figure 1 connected to a reservoir at chemical potential μ. The static localised flux piercing the loop is necessary to break the time reversal symmetry and induce a persistent current in the system. The reservoir acts as an inelastic scatterer and as a source of energy dissipation. All the scattering processes in the leads including the loop are assumed to be elastic. The loops J1J2aJ3J1 and J1J2bJ3J1 enclose the localised flux Φ. However, the bubble J2aJ3bJ2 does not enclose the flux Φ. This special situation we have considered, so as to answer the question of existence of circulating currents in equilibrium systems. We show that circulating currents (due to current enhancement) arise in a bubble which does not enclose a magnetic flux. We would like to mention here that the current enhancement effect and the associated circulating currents arise even when the magnetic field extends over the entire sample. However, for this the treatment is involved as one has to study separately persistent as well as circulating currents in the bubble as they have different symmetry properties. This has been studied in a simple loop in the presence of both transport currents and magnetic flux[9].

In the local coordinate system the wave-functions in the various regions of the ring in absence of magnetic flux are given as follows

$$\psi_0 = e^{ikx_0} + re^{-ikx_0},$$
$$\psi_j = a_j e^{i(k+\frac{\alpha_j}{l_j})x_j} + b_j e^{-ikx_j + i\frac{\alpha_j}{l_j}(x_j - l_j)}. \quad (1)$$

Here $x_j, j = 1,..4$ are coordinates along the the segments J1J2, J2bJ3, J2aJ4 and J3J1 respectively and x_0 is the coordinate along the connecting lead to the reservoir, while α_j's are the phases picked up by the electron as it traverses the various regions of the system with the restriction that $\alpha_1 + \alpha_2 + \alpha_4 = 2\pi\Phi/\Phi_0$, and $\alpha_1 + \alpha_3 + \alpha_4 = 2\pi\Phi/\Phi_0$ which implies $\alpha_2 = \alpha_3$. To solve for the unknown coefficients in eqn.(1) we use Griffith[10] boundary condition at the junctions $J1, J2$ and

$J3$. These boundary conditions are due to the continuity of wavefunctions and conservation of current (Kirchoff's law)[11].

In the lead connecting the reservoir to our circuit there is no current flow as $|r|^2 = 1$. The current densities (dimensionless form)[9,12] in the small interval dk around the Fermi energy k in the various segments of the circuit are given by - $I_j = |a_j|^2 - |b_j|^2$, The current densities are calculated from the usual formula of current density in presence of magnetic flux- $J_j = \frac{e\hbar}{2mi}(\psi_j^* \nabla \psi_j - \psi_j \nabla \psi_j^* - 2i\frac{\alpha_j}{l_j}\psi_j^* \psi_j)$, which implies $I_j = \frac{J_j}{e\hbar k/m}$.

The persistent current densities in various parts of the system show cyclic variation with flux and Φ_0 periodicity (reminiscent of Aharonov-Bohm oscillations), and oscillate between positive and negative values as a function of energy or the wave-vector k as expected. Since the analytical expressions for these currents are too lengthy we confine ourselves to a graphical interpretation of the results. It should be noted that in all these expressions for current densities, flux enters only through the combinations $\alpha_1 + \alpha_2 + \alpha_4$ and $\alpha_1 + \alpha_3 + \alpha_4$, the magnitude of these combinations is given by $2\pi\Phi/\Phi_0$ as expected. For us the current densities in the bubble, i.e., J2bJ3aJ2 are of special importance as in this region there is possibility of current enhancement which would be analysed below. The current density shows an extremum near the corresponding eigen-states of the system. We have calculated these eigen states for two different cases. For open system as depicted in figure 1, one can calculate the energies (or wave-vector) of these states by looking at the poles of the S-Matrix. In our case S-Matrix is simply a complex reflection amplitude r. We have also analysed the eigen states of a closed system (without coupling lead to reservoir).

We analyse the case of a bubble with unequal lengths, of its two arms i.e the length of $J2bJ3 \neq J2aJ3$. This asymmetry implies that currents in the two arms of the bubble are not equal. In figure 2 we plot the persistent current densities in various parts of the circuit. It should be noted that absolute value of the persistent current densities I_2 and I_3 are individually much larger than the input current density I_1 into the bubble and thus the current enhancement effect is evident (without violating the basic Kirchoff's law). The input current arises due to the presence of flux Φ as it breaks the time reversal symmetry. The system parameters are mentioned in the figure caption. In the interval, $5.2 < kL < 7.4$ current density I_1 changes from positive to negative and exhibits extremum around the real part of the poles of the S-Matrix. When I_1 is positive, negative current density of magnitude I_2 flows in the arm $J2bJ3$ of the bubble. Thus, when I_1 is positive circulating current flows in the anticlockwise direction in the bubble. In the range wherein I_1 is negative, i.e, input current into the bubble is in anti-clockwise direction, then positive current flows in arm $J2aJ3$. According, to our convention as mentioned earlier, circulating current flows in the anti-clockwise direction. In all the figures drawn, the length of the bubble is $l = l_2 + l_3$ which is taken as unity throughout our discussion. The current densities along with the Fermi wave-vectors are in their dimensionless form. Of course the phenomena of current enhancement is extremely sensitive to the arm lengths of the bubble. It should be noted that if we interchange the values of l_2 and l_3 keeping other parameters unchanged circulating current will flow in a clockwise direction. This is obvious from the geometry of the problem. Alongwith the current densities the persistent currents in various parts of the ring can also be plotted, to do that we integrate the current densities J_j in various regions of the circuit over the Fermi wave vector k_f. The persistent currents P_j at temperature $T = 0$ is given by

$$P_j = -\int_0^{k_f} dk J_j \tag{2}$$

In figure 3 we have plotted the persistent currents(in dimensionless units) for the system parameters as mentioned in the caption. An interesting point to note is that although in most cases current enhancement occurs around the eigenenergies of the closed system there are a few exceptions. In figure 4 we plot one of those exceptions. Herein, we show that current enhancement does not occur at a place which is an eigen'k' of the aforesaid system. Here the eigen wave-vector kL corresponds to 13.85. One can readily notice that the persistent current density (i.e, input current density I_1 into the bubble) shows extrema around this value. In this region both the currents in the bubble I_2 and I_3 are individually smaller than I_1 and they flow in the same direction as the input current. Hence we do not observe current enhancement.

The above discussion shows that current enhancement effect is not restricted to non-equilibrium systems only but is also expressed in mesoscopic systems at equilibrium. Our special system not only shows current enhancement but also in it parity effects are violated, of course in its closed system disguise. The study of parity effects or rather the lack of it especially in a metallic ring with a static localised flux at its center and a stub attached to it has attracted a lot of interest[13]. Parity effects are defined as follows- The persistent current carried by an electron in the eigen state E_n in case of an isolated single loop is defined as $I_n = -\frac{1}{c}\frac{\partial E_n}{\partial \Phi}$. In a closed single loop persistent current changes its sign as we go from one level to the next successive level, i.e., from diamagnetic to paramagnetic or vice-versa. The total persistent current is given by sum of currents carried by all electrons. Thus for spinless electrons at temperature ($T = 0$), a system with odd number of electrons behaves as a diamagnet while that with an even number of electrons behaves as a paramagnet. This is called parity effect. In figure 5 we have plotted the first few eigen energies $E = k_n^2$ of our isolated system (with the connecting lead to reservoir removed). The system parameters are mentioned in the figure caption. These eigen energies are calculated from the condition that the determinant of the coefficient matrix must vanish. The coefficient matrix is built from first principles using quantum waveguide theory with the second wavefunction of equation 1. The eigen energies are flux periodic with period Φ_0. For our system we immediately notice that certain number of successive energy levels from the bottom, have same direction of slope with respect to flux. Particularly the third and fourth energy levels from bottom carry diamagnetic current for small values of flux while levels five and six again from bottom carry a paramagnetic current. Thus breaking the well known parity effect. For details we refer to Ref.[13].

As a corollary to the discussion above, we consider the magnetic moments of our system. The orbital magnetic moment defined via currents in a loop, depends strongly on the topology of the system. If we consider our system as depicted in figure 1 to be planar and lying in the x-y plane then the magnetic moment density would be $\mu = \frac{1}{c}(I_1 A_r + I_3 A_b)$ wherein A_r and A_b are areas enclosed by the ring($J1J2aJ3J1$) and bubble($J2aJ3bJ2$) respectively, for details we refer the interested reader to Ref. [14]. Another orientation of the system in which the ring is not to the left but to the right of the bubble gives $\mu = \frac{1}{c}(-I_1 A'_r - I_2 A_b)$, where A'_r is the area of the ring for this changed configuration. Several other orientations also are possible, for example, if the bubble lies in x-y plane and the ring lies in x-z plane, then the magnetic moment density for the bubble is $\mu_z = \frac{1}{c}(I_3 A_b - I_2 A_b)/2$ and in case of the ring is $\mu_y = \frac{1}{c} I_1 A_r$. All the above examples buttress the fact that the orbital magnetic moment is inherently linked to the topology of the system. An important point to be noted is that the magnetic moment calculated herein is not same (qualitatively), as the magnetic moment calculated in case of the closed system from its eigenenergy spectra, which is same for all topological situations. A full discussion on this discrepancy alongwith a detailed analysis of different quantum effects exhibited by our chosen mesoscopic system will be presented elsewhere.

In conclusion, we have shown that the phenomenon of current enhancement is not restricted to non-equilibrium mesoscopic systems only but can also arise in equilibrium systems but ofcourse in the presence of magnetic flux. In addition to this quantum effect our hybrid ring geometry breaks parity effects in its closed system analog.

---

† Electronic address: colin@iopb.res.in
⋆ Electronic address: jayan@iopb.res.in
[1] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University, New York, 1997).
[2] S. Datta, *Electronic transport in mesoscopic systems* (Cambridge University Press, Cambridge, 1995).
[3] S. Washburn and R. A. Webb, *Adv. Phys.* **35**, 375.
[4] P. S. Deo and A. M. Jayannavar, Pramana J. Phys.**56**, 439 (2001).
[5] M. Büttiker, Y.Imry and R.Landauer , Phys. Lett. A **96**, 365 (1983).
[6] A. M. Jayannavar and P. S. Deo, Phys. Rev. **B49**, 13685 (1994).
[7] T. P. Pareek, P. S. Deo and A. M. Jayannavar, Phys. Rev. **B52**, 14 657 (1995).
[8] C. Benjamin, S. K. Joshi, D.Sahoo and A. M. Jayannavar, Mod. Phys. Lett. **B15**, 19 (2001).
[9] A. M. Jayannavar, P. Sinngha Deo, and T. P. Pareek, in Proceedings of International Workshop on Novel Physics in Low-Dimensional Electron Systems, Madras, India [Physica B**212**, 216 (1995)].
[10] S. Griffith, Trans. Faraday. Soc.**49**, 650 (1953)
[11] J-B. Xia, Phys. Rev. **B45**, 3593 (1992).
[12] M. Büttiker, Phys. Rev. **B32**, 1846 (1985).
[13] P. S. Deo, Phys. Rev. **B51**, 5441 (1995).
[14] P. Cedraschi and M. Büttiker, Phys. Rev. **B63**, 165312 (2001).

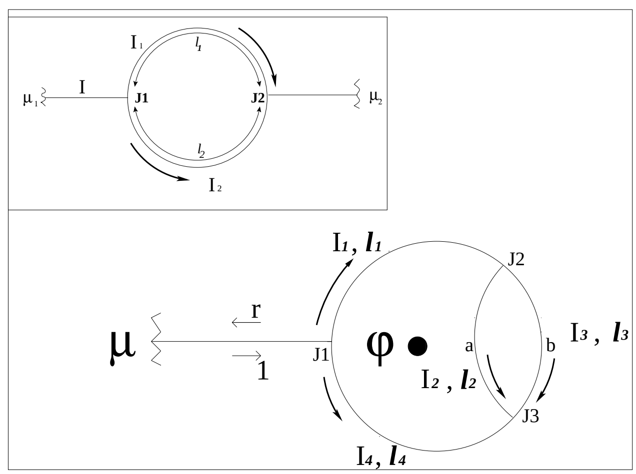

FIG. 1. The hybrid ring system connected to a reservoir at chemical potential μ. The bubble is denoted by the structure J2bJ3aJ2. The localised flux Φ penetrates the ring. The current densities in various parts of the structure are denoted by I's while the lengths of the various regions are denoted by l's. In the inset we have shown the non-equilibrium case, a one dimensional mesoscopic ring with leads is connected to two reservoirs at chemical potentials μ_1 and μ_2.

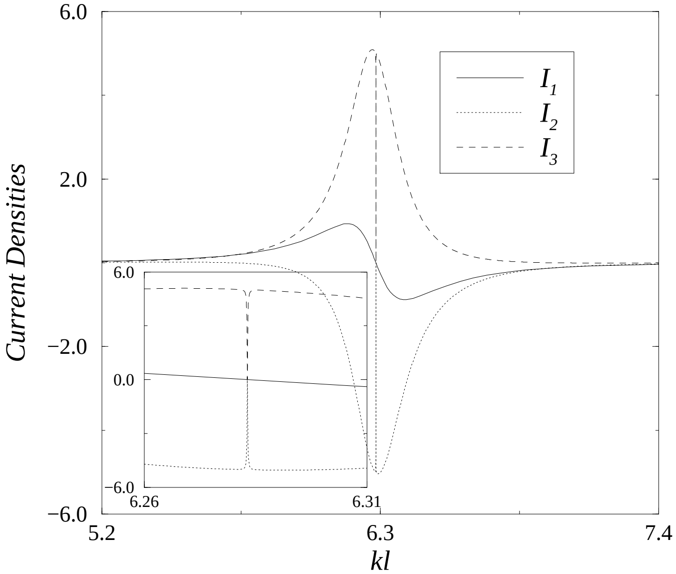

FIG. 2. Current enhancement shown with lengths $l_1/l = l_4/l = 0.75, l_2/l = 0.45, l_3/l = 0.55$. Herein the persistent current densities in the various parts of the circuit are plotted as function of the dimensionless Fermi wavevector kl. The persistent current density in $J1J2$ is denoted by the solid line while those in $J2bJ3$ and $J2aJ3$ are denoted by dotted and dashed line. Flux $\Phi = 0.1$.

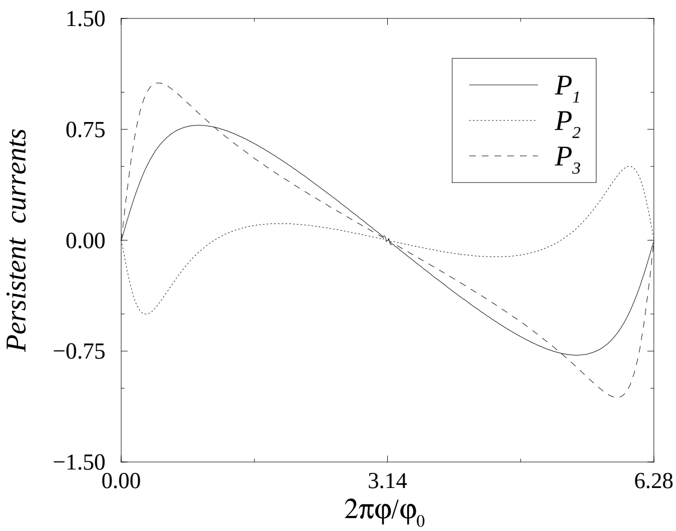

FIG. 3. Current enhancement shown with lengths $l_1/l = l_4/l = 0.25, l_2/l = 0.45, l_3/l = 0.55$. Herein the persistent currents in the various parts of the circuit are plotted as function of flux. The Fermi wavevector here is $k_f = 2\pi$.

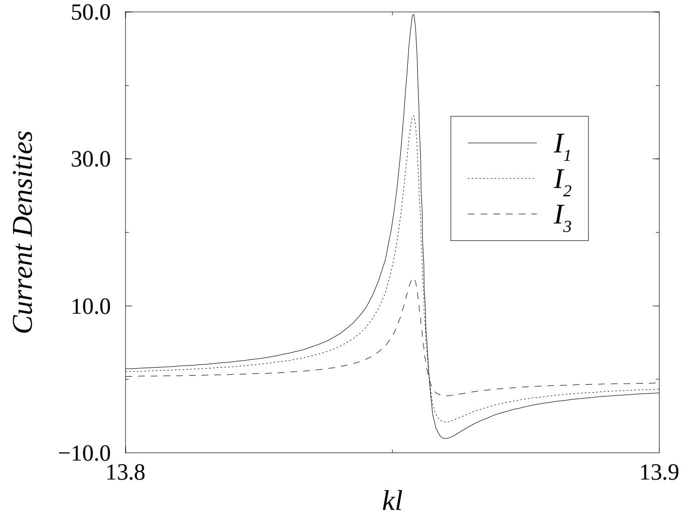

FIG. 4. Absence of current enhancement shown with lengths $l_1/l = l_4/l = 0.75, l_2/l = 0.25, l_3/l = 0.75$. Herein the persistent current densities in the various parts of the circuit are plotted. The persistent current density in $J1J2$ is denoted by the solid line while those in $J2aJ3$ and $J2bJ3$ are denoted by dotted and dashed line. The kl value 13.85 is an eigen wavevector of the closed system. Flux $\Phi = 0.1$.

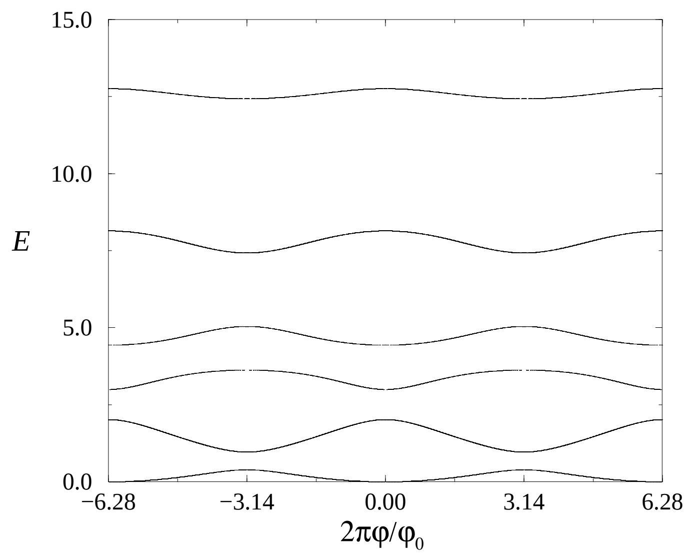

FIG. 5. Breakdown of parity effects in the closed form of our open hybrid ring system as depicted in figure 1. The length parameters are $l_1/l = 0.75, l_2/l = 0.35, l_3/l = 0.65$. The energies are normalised by π^2.