

# AC driven thermal ratchets.

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## Abstract :

We consider the motion of an overdamped Brownian particle in a periodic asymmetric potential with space-dependent friction coefficient. In the presence of external time-periodic forcing, the system shows multiple current reversals as a function of noise strength, which can only be accounted for by the presence of space-dependent friction coefficient. For very large forcing, the current does not go to zero; instead, it asymptotically tends to a limiting value depending on the phase shift between the potential and the friction. This fact plays an important role in obtaining multiple current reversals.

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## 1. Introduction

Rectification of noise leading to unidirectional motion in ratchet systems has been an active field of research over the last decade. In these systems, directed Brownian motion of particles is induced by nonequilibrium noise in the absence of any net macroscopic forces and potential gradients. Several models have been proposed to explain this transport mechanism under various nonequilibrium situations, like (a) flashing ratchets [1,2], wherein the particles experience a fluctuating energy profile, (b) rocking ratchets [3,4], where the Brownian particles experience a spatially uniform, time-periodic uniform force,  $F(t + \tau) = F(t)$ , (c) diffusion ratchets [5], wherein particles are driven by a time-periodic diffusion coefficient  $D(t) = D(t + \tau)$  and (d) correlation ratchets [6], where particles are driven by a spatially uniform but temporally correlated force  $\zeta(t)$  of zero average. In these systems, to get a unidirectional current, either spatially asymmetric periodic potentials or time-asymmetric external forces are necessary. The recent burst of work on ratchet systems is motivated in part to explain the unidirectional transport of molecular motors in biological systems and a new prospect of novel techniques for separation or segregation of particles of macro-meter size. This technique for particle separation is based on the property of current reversal in a ratchet by varying the strength of thermal noise, amplitude of the forcing, size of the particle, or any other relevant variable in the problem.

In the area of Brownian rectifiers or ratchets, the study

of current reversal has become a subject by itself. Bar-tussek *et. al.* [4] showed the occurrence of current reversals in a rocked thermal ratchet as a function of amplitude of rocking force as well as the temperature of the thermal bath. Multiple current reversals have also been shown in the deterministic limit of these ratchets when the inertial term is taken into account. However, these multiple current reversals in inertial ratchets are not robust in the presence of noise. Besides rocking ratchets, current reversals have also been observed in flashing ratchets [7,2]. We in this work report that *multiple reversals* can be achieved even in a rocked *overdamped* ratchet in the presence of space-dependent mobility, as a function of the noise strength and amplitude of the rocking force. In the deterministic overdamped case, we get current reversal as a function of the amplitude of rocking force. It is to be noted that systems with space-dependent friction are not uncommon. Brownian motion in confined geometries shows space-dependent friction [8]. It is believed that molecular motor proteins move close along the periodic structure of microtubules and will therefore experience a position-dependent mobility [9]. Frictional inhomogeneities are common in superlattice structures and Josephson junctions, etc.

## 2. Model

We consider an overdamped Brownian particle moving in an asymmetric potential  $V(x)$  with space-dependent

dent friction coefficient  $\eta(x)$  under the influence of external force field  $F(t)$  at temperature  $T$ . Through our analysis we take the ratchet potential  $V(x) = -\frac{1}{2\pi}(\sin(2\pi x) + \frac{\mu}{4}\sin(4\pi x))$ . Here  $\mu$  is the asymmetry parameter with values taken in the range  $0 < \mu < 1$ , friction coefficient  $\eta(x) = \eta_0(1 - \lambda \sin(2\pi x + \phi))$ ,  $|\lambda| < 1$ .  $\phi$  determines the relative phase shift between friction coefficient and potential. The forcing term is taken to be  $F(t) = A \sin(\omega t + \theta)$ , ( $\omega = \frac{2\pi}{\tau}$ , where  $\tau$  is period of force). Without any loss of generality  $\theta$  is taken to be zero. The Langevin equation for this system in the overdamped limit is given by [11,12]

$$\dot{x} = -\frac{(V'(x) - F(t))}{\eta(x)} - k_B T \frac{\eta'(x)}{(\eta(x))^2} + \sqrt{\frac{k_B T}{\eta(x)}} \xi(t), \quad (1)$$

where  $\xi(t)$  is Gaussian thermal noise with correlation  $\langle \xi(t)\xi(t') \rangle = 2\delta(t - t')$ . The Fokker Planck Equation (FPE) [11] corresponding to Eqn. (1) is given by

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x} \frac{1}{\eta(x)} [k_B T \frac{\partial}{\partial x} + (V'(x) - F(t))] P(x,t). \quad (2)$$

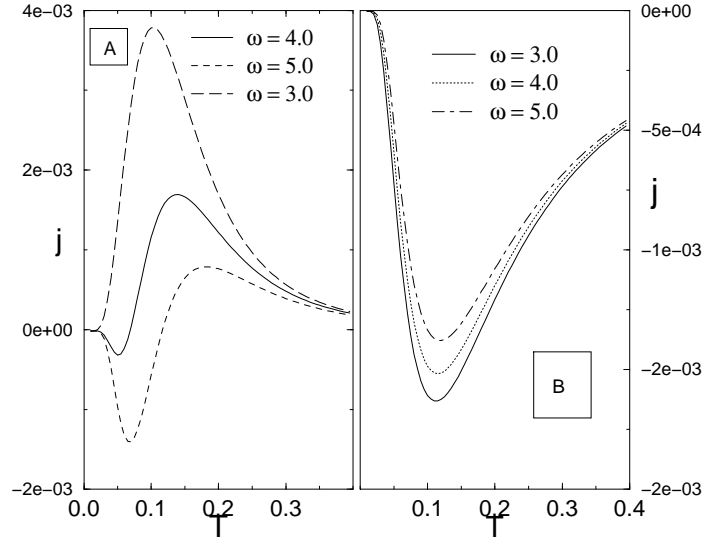
Here  $P(x,t)$  is the probability density at position  $x$  at time  $t$ . Equation (2) can be recasted to a continuity equation  $\frac{\partial P(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}$ , where

$$J(x,t) = -\frac{1}{\eta(x)} [(V'(x) - F(t)) + k_B T \frac{\partial}{\partial x}] P(x,t), \quad (3)$$

is the probability current density. Since the potential and the driving force have spatial and temporal periodicity respectively, therefore  $J(x,t) = J(x+1, t+\tau)$ , [4,13]. The net current  $j$  in the system is given by  $j = \lim_{t \rightarrow \infty} \frac{1}{\tau} \int_t^{t+\tau} dt \int_0^1 J(x,t) dx$ . It should be noted that for symmetric potential and  $\lambda = 0$ ,  $j = 0$ . Rectification of current is possible if the potential is either asymmetric or  $\lambda \neq 0$  with  $\phi \neq 0, \pi$  [14]. It is important to note that in these systems rectification is due to a symmetry breaking as the potential is asymmetric and  $\lambda = 0$ . For the case when the potential is symmetric and  $\lambda \neq 0$ , the symmetry breaking results from the dynamics of the system. It is the space dependent mobility that breaks the symmetry in this nonequilibrium problem and the magnitude of the symmetry breaking is related to the phase shift between the friction and potential profile.  $j$  is independent of the initial phase  $\theta$  of the driving force. We solve the FPE numerically by method of finite difference and calculate the current  $j$ . All the physical quantities such as  $j, T, A, \omega$  are in dimensionless units [13–15].

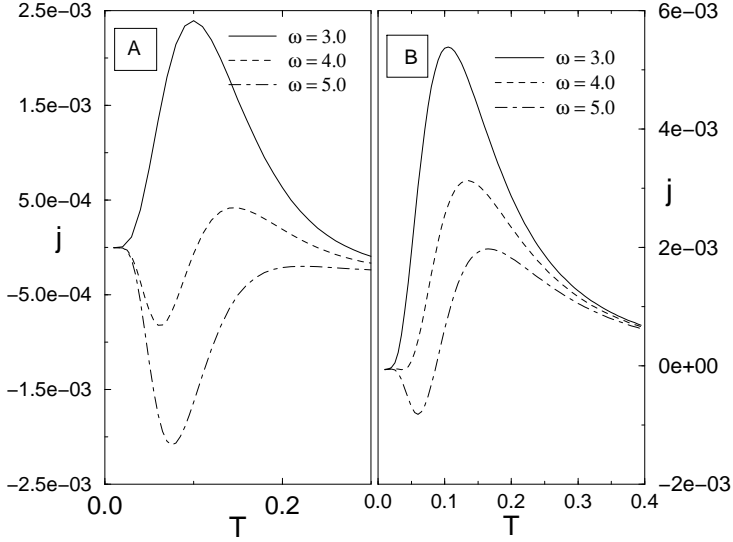
### 3. Results and Discussions

In the Fig. (1A), the average current  $j$  is plotted as a function of temperature  $T$  for different values of  $\omega$ . Here the asymmetry parameter  $\mu = 1.0$  and  $\lambda = 0.0$ .



**Figure 1.** Mean current  $j$  vs temperature  $T$  for (a)  $\lambda = 0, \mu = 1.0$  and (b)  $\lambda = 0.1, \mu = 0$  and  $\phi = 0.2\pi$ . Note there is no current reversal when the potential is symmetric.

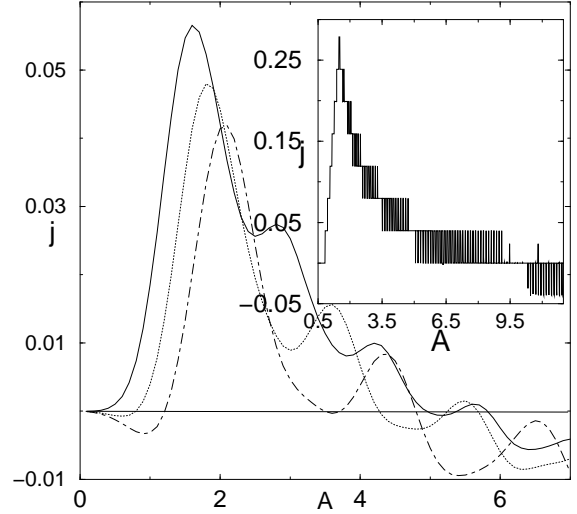
In this case the current reverses its sign (only *once*) for frequencies sufficiently large as shown in the  $\omega = 4.0, \omega = 5.0$  case. In the absence of asymmetric potential and presence of space dependent friction ( $\lambda = 1.0$ ), there is no current reversal irrespective of  $\omega$  and phase shift  $\phi$  as shown in Fig (1B). Hence asymmetric potential is essential for current reversal. However the direction of the current depends on the phase lag  $\phi$ . Separately in both these cases absolute value of current exhibits a maxima as a function of  $T$ , reminiscent of stochastic resonance phenomena. In a purely asymmetric case ( $\lambda = 0$ ) current vanishes rapidly when  $T$  exceeds the temperature associated with the barrier height. Whereas, in the symmetric case due to space dependent friction absolute values of currents are significantly higher and decay slowly to zero in the large temperature regime. Naturally in the presence of both asymmetry and space dependent friction for the case under study the low temperature regime is dominated by the effect of asymmetry while the high temperature regime is dominated by space dependent friction. From this, one can qualitatively explain the current reversals from positive to negative side as a function of temperature even in the adiabatic limit. Hence in the presence of both space dependent friction and asymmetric potential we can have current reversal with  $T$  even in the *adiabatic regime* for a suitably chosen value of  $\phi$  as shown in Fig (2A), where we have plotted  $j$  vs  $T$  with  $\mu = 1.0, \lambda = 1.0$  and  $\phi = 0.2\pi$  for different values of  $\omega$ .



**Figure 2.** The mean current  $j$  vs temperature  $T$  for  $\phi = 0.2\pi$ ,  $A = 0.5$  and  $\lambda = 0.1$ . The driving frequencies are  $\omega = 3.0, 4.0$  and  $5.0$ . The right hand side figure shows current  $j$  vs  $T$  for  $\phi = 1.2\pi$ .

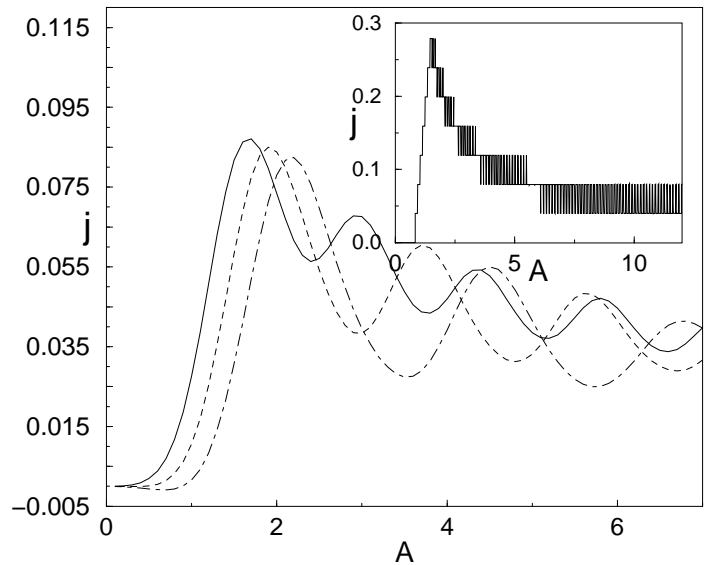
In the presence of finite frequency drive there are twice current reversals as shown in the figure. This phenomena of twice current reversal with temperature  $T$  is the foremost feature of our system, previously unseen in any overdamped system. When the phase difference  $\phi$  is such that the current due to space dependent friction alone is in the same direction as that of current due to potential asymmetry only, then we do not have current reversal in the adiabatic case as shown in Fig (2B) where  $\phi = 1.2\pi$ , though a *single* current reversal due to finite frequency drive may be present. In all the cases studied so far we have observed that the current reversal don not take place above a critical frequency  $\omega_c$  of driving, which in turn depends on  $\phi$  and other parameters in the problem.

Multiple current reversals can also be seen when the amplitude ( $A$ ) of the forcing term is varied in a suitable parameter regime of our system.



**Figure 3.** The mean current  $j$  with amplitude  $A$  of the forcing term for  $\phi = 0.88\pi$ ,  $T = 0.05$  and  $\lambda = 0.1$  with  $\omega = 3.0, 4.0, 5.0$ . The inset shows the reversal of deterministic current vs the amplitude of the forcing.

In Fig. 3, the plot of  $j$  versus  $A$  is shown for different values of  $\omega$ , keeping  $\lambda$ ,  $\phi$  and  $T$  fixed at  $0.1$ ,  $0.88\pi$  and  $0.05$  respectively. For  $\omega = 4.0$  curve, we can see as many as four current reversals. For very large value of  $A$ , the current asymptotically goes to a constant value depending on the value of  $\phi$ , as was previously shown for the adiabatic case[14]. The inset in Fig. 3 shows current reversal even for the deterministic case also. In addition it exhibits an interesting phenomena of current quantization and phase locking. These features are washed out with increasing noise strength.



**Figure 4.** Mean current  $j$  vs temperature  $T$  for (a)  $\lambda = 0, \mu = 1.0$  and (b)  $\lambda = 0.1, \mu = 0$  and  $\phi = 0.2\pi$ . Note there is no current reversal when the potential is symmetric.

In Fig (4) we have plotted  $j$  versus  $A$  for  $\phi = 1.2\pi$ . There is no current reversal in the adiabatic regime for both deterministic as well as finite temperature case. The observation of multiple current reversals can be attributed to a cooperative interplay between the spatial asymmetry of the potential, the friction inhomogeneity and the finite frequency drive. Depending on the system parameters we may have multiple current reversal or no current reversal at all ( see Fig.( 3 and 4)).

In conclusion, we have studied the transport properties of overdamped Brownian particles moving in an asymmetric potential with space dependent friction coefficient and rocked by periodic force. We observe multiple current reversal with both temperature  $T$  and  $A$  the amplitude of the rocking force in the presence of finite frequency driving. Current reversal also occurs in the deterministic adiabatic regime for suitable values of phase difference  $\phi$ . All the above results can be understood qualitatively. The space dependent friction plays an active role in these systems in determining several counter intuitive phenomena<sup>[14–16]</sup> observed in the transport processes.

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