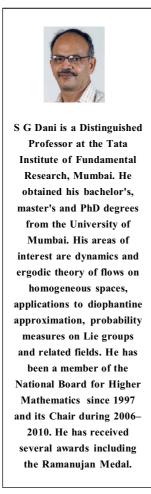
Kosambi, the Mathematician

S G Dani



¹ See *Resonance*, Vol.4, No.5, 1999.

Keywords

D D Kosambi, differential geometry, statistics. Damodar Dharmananda Kosambi is one of the rare individuals to have contributed to many diverse areas, both in the academic and public spheres, leaving an imprint of originality in each. Notwithstanding the fact that only a small part of his working time was devoted to mathematics, he stands out among the mathematicians of his generation for work of high quality and impact. There is an ample case for a full fledged appreciation of the mathematical legacy left by him. This may however take time to materialise, especially on account the paucity of expertise in the areas of his work from close quarters. Pending such an event, this is an attempt to put together a broad outline of Kosambi as a mathematician, with the aim to complement the writings on his achievements in other areas so that a bit more complete picture of his personality emerges. In producing this, though I did refer to the original papers of Kosambi to an extent, I have by and large based this article on other professional resources, and some material from the Tata Institute (TIFR) archives.

Let us begin with a sketch of his institutional affiliations, to set the context. Kosambi's career as a mathematician may be said to have begun at the Banaras Hindu University, though it is inextricably linked with his undergraduate studies at Harvard, following which he joined the University (for various biographical details the reader is referred to other articles in this issue). From Banaras he moved to Aligarh Muslim University, in 1931, at the invitation of André Weil¹. Weil, a young French mathematician, who would later join the ranks of celebrity mathematicians of the 20th century, had been invited by the University, in 1930, to head the mathematics department, and he was keenly looking out for truly



creative mathematical minds in the country. Weil had also enlisted T Vijayaraghavan, and together they made a good beginning towards a potentially strong mathematics department (Vijayaraghavan also went on to make important contributions in mathematics, and was the Director of the Ramanujan Institute for Mathematics, Madras, from 1949 until his early death in 1955). However Weil's relations with the University sourced and he returned to France in 1932. Vijayaraghavan had already left earlier and Kosambi no longer found it worthwhile to continue at Aligarh. He then found a position at Furgusson College, Pune where he worked from 1933 to 1945. On the whole this period seems to have been very fruitful for him, notwithstanding his reference to it as a 12-year "vanavas". Though he had to leave the College, since in the later years he did not get along well with the authorities, fortunately for him Dr. Homi Bhabha was just then setting up a new institution, the Tata Institute of Fundamental Research (TIFR) at Mumbai, and warmly welcomed Kosambi on the faculty. Kosambi joined the Institute in July 1946, to became the first mathematician on the roll of the Institute. He was acting Director, while Bhabha was abroad for several months in 1946, a role in which he seems to have been quite uncomfortable. Kosambi continued in the Institute until 1962, at which stage his appointment, which at the Institute used to be in terms of periodic contracts, was not renewed (he had attained the age of 55 years, the typical retirement-age in India those days, but that may not have been the whole reason for ending the affiliation). In the later years he stayed in Pune. From 1964 he had an Emeritus Professorship from CSIR, and was affiliated to the Maharashtra Vidyanavardhini (Maharashtra Association for the Cultivation of Science), Pune, until he passed away in 1966.

As is well known, Kosambi had travelled widely on various accounts, and some of the visits involved primarily

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During the winter session Kosambi was Visiting Professor of Geometry at the University of Chicago and gave a course of 36 lectures on "Tensor Analysis". mathematical activities. He visited US and UK during December 1948 to June 1949 on a UNESCO deputation. In the context of the efforts to secure him deputation, in a letter to the Ministry of Education, Government of India, on 1 April 1948 Bhabha wrote "In view of the circumstance that Prof. Kosambi, who is one of the most distinguished pure mathematicians in India, has shown special interest in calculating machines, having actually designed and constructed an electro-mechanical one with the help of an electrical engineer from RAF before he joined this institute in 1945, I suggest that Prof. Kosambi be deputed by the Government of India to study the latest types of machines, to work with mathematicians, like Prof. J von Neumann, who are designing one of them, ...". During the winter session Kosambi was Visiting Professor of Geometry at the University of Chicago and gave a course of 36 lectures on "Tensor Analysis". He visited also the Institute for Advanced Study, Princeton where he had the opportunity of discussions with Einstein. His extended visit helped considerably in networking with leading figures like Marshall Stone and Oppenheimer. (There is however no record of his involvement with designing calculating machines, on return to India.) During 1959–60 he spent several months in China, on deputation from the Tata Institute, as expert statistician. He is said to have provided consultancy to the Chinese government on statistical analysis of data in the industrial and agricultural sectors, a subject that was yet to develop in China at that time.

When the International Mathematical Union (IMU), which has been the representative body of the global mathematical community, was revived after the Second World War, Kosambi was appointed a member of the Interim Executive Committee, that served during 1950– 52. The Fields Medal, the coveted prize in mathematics, considered for long the equivalent of the Nobel Prize

(even though the prize amounts are scarcely comparable), was also reinstated after the war in 1950, after a gap of 14 years after its first award in 1936. The Fields Medals, upto 4, are awarded at the International Congresses of Mathematicians, organized every four years by the IMU. The selection committee for the award of the medals at the 1950 ICM, held at the Harvard University under the auspices of the American Mathematical Society, included Kosambi as one of the members; the other members of the committee were Harald Bohr (Chair), Ahlfors, Borsuk, Fréchet, Hodge, Kolmogorov and Morse.

Kosambi also enjoyed a significant standing in the Indian mathematical community. He was awarded in 1934 the Ramanujan Memorial Prize instituted by the University of Madras, along with S Chandrasekhar and S Chowla. He was a Foundation Fellow of the Indian Academy of Sciences, elected in 1935 (the Academy was founded in 1934, but Foundation Fellows were added also in 1935). He was also elected Fellow of the Indian National Science Academy. In 1947 he was awarded the Bhabha Prize, in honour of J H Bhabha. For the 34th session of Indian Science Congress held in Delhi in 1947 Kosambi was chosen President for the Mathematics Section. He was also involved with the Indian Mathematical Society (IMS). He was a member of the Editorial Committee of the Journal of IMS during 1940–49, when Vaidyanathaswamy was the Editor. According to the IMS records Kosambi was the librarian of the Society during the years from 1944–45 to 1949–50; the library had till then been located at the Furgusson College, and was shifted to Madras in 1950. Indeed during the first two years Kosambi must have been actively involved with the library. His biographers² mention that a tiff over some part of the space allocated to the library being taken away may have instigated his eventual parting with the college. During the later years when he moved

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² There are two biographies of Kosambi, both in Marathi: *Damodar Dharmananda Kosambi (Jeevan ani Karya)* by Chintamani Deshmukh, *Granthali*, 1993, and *Uttunga ani Ekaki Sanshodhak*: *Damodar Kosambi*, Sudhir Panse, Lok Vangmaya Griha, 2007.

Notwithstanding the imaginary authorname, the Ducray papers contain some serious mathematical results.

to TIFR his role as librarian was perhaps nominal.

In the course of his career Kosambi wrote over 60 mathematical papers. Interestingly, some of these were in French, one in German and one from later years in Chi-About 40% of the papers appeared in prestinese. gious international journals such as, Quarterly Journal of Mathematics, Oxford, Comptes Rendus of the Académie des Sciences, Paris and Proceedings of the National Academy of Sciences, USA. His publication folio is also unusual on account of some frivolity! At Aligarh, inspired by Weil's telling him of an incident of a talk at the École Normale at Paris featuring a theorem of a certain 'Bourbaki' who turned out to be quite fictitious, Kosambi published a paper in the Proceedings of the Academy of Sciences, UP, entitled 'On a generalisation of the second theorem of Bourbaki', in which he concocted also some details of Bourbaki's life, and death as well. It was only later that the Bourbaki myth evolved in France at the hands of Weil and other cohorts, and eventually acquired massive proportions; by the way, the latter was Nicolas Bourbaki, while Kosambi's was D Bourbaki. In the 1962–63 volume of the Journal of the University of Bombay, he published a paper under a pseudonym S Ducray, with the last name suggestive of the Marathi word for 'pig' (dukkar), containing an acknowledgement "This paper would not have been possible without the constant labour of Prof. D D Kosambi"! The figmental Mr. Ducray had the honour of later publishing two papers also in the Proceedings of the Indian Academy of Sciences, in 1964 and 1965. Notwithstanding the imaginary author-name, the Ducray papers contain some serious mathematical results.

As Kosambi published many books in his other areas of pursuit, one may wonder about his book-writing activity in mathematics. There are no mathematics books in print under his authorship. However, in a letter he wrote to Bhabha in November 1946, while Bhabha was abroad,



there is a mention, "The book on Path-Geometry will, according to a letter from Morse, appear in the Annals of Mathematics Studies, Princeton." He goes on to add "This means a planograph edition, not ordinary printing; however I am in quite distinguished company for Hermann Weyl has two books in that series and Morse himself has one." Indeed, Morse was one of the editors of the series and the mention of a letter from him conveys an "official" word, so it is unclear how the book failed to see light of day. Incidentally, only one of Weyl's books and none of Morse have appeared in that series. It has also been known that he had completed a manuscript on 'Prime Numbers' shortly before his death, which never got published and was eventually lost.

Work in Differential Geometry

It was remarkable for a young man of 22, not quite initiated into the art under the tutelage of any professional, to take to mathematical research on his own in the total isolation at Banaras. But we see that already in 1930, while he was still there, was his publication folio opened; this may be contrasted especially with the fact that many successful Cambridge-returned graduates of the time, including many who were part of the academic ethos in other respects, did not pursue research. It was however at Aligarh that young Kosambi is seen to have bloomed as a professional mathematician. While at Harvard he had been keenly interested in differential geometry, and studied the topic with Graustein; in his essay 'Mathematics at Harvard in the 1940s'³ Garret Birkhoff refers to Kosambi as "Graustein's student". It was a 'happening' area at that time, with several eminent mathematicians pursuing it actively, at Princeton and other centres of excellence. Interestingly, in his obituary note⁴ on G D Birkhoff, for whom he had a great regard, in 1944 Kosambi, apparently seeking a rationale as to why the former did not work in the area, wrote "If he did not follow up the enticing field of tensor analysis,

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³ In Proceedings of the American Philosophical Society, Vol.137. 1993.

³ In *Mathematics Student*, Vol.12 1945.

At Aligarh he took to a systematic study of "geometry of paths" in which he made over the years several contributions of lasting value. it must undoubtedly have been because the Princeton school (which he himself helped found with Veblen) had plunged into the heart of the subject with greater vigour and success than could have been expected at that time by a solitary research worker whose main interest lay in other branches of mathematics." At Aligarh he took to a systematic study of "geometry of paths" in which he made over the years several contributions of lasting value.

One of Kosambi's first papers on the theme was a Note, in French, appearing in the journal of the Accademia Nazionale dei Linceit, communicated by Levi-Civita in August 1932. A more detailed paper on the theme was submitted to Mathematische Zeitschrift within a few months and it appeared in the following year. This turned out to be a pioneering paper for what is now called 'KCC-theory', the name being derived from the first letters of the three authors, Kosambi, Elie Cartan and S S Chern; the paper of Cartan involved is a comment on Kosambi's paper, first received by Kosambi as a letter and published in the same issue of the journal, following Kosambi's paper; the paper of Chern in question appeared a few years later, in 1939, and mentions in the introduction "E. Cartan has made very important observations on the memoire (of Kosambi) in a Note published in the same journal. The objective in this Note is to place the problem in the general theory of generalised spaces of Cartan." (my translation from original French text). Incidentally, at Kosambi's initiative, Chern visited TIFR in 1946–47 and later a serious attempt was made to get him to join the Institute, when he left China in 1949–50 (Chern however opted to join the University of Chicago, which also made him an offer at that time).

The name 'KCC-theory' was introduced in a book of Antonelli, Ingarden and Matsumoto entitled *The Theory of Sprays and Finsler Spaces with Applications in Physics and Biology* published in 1993 and has since then been



extensively adopted in literature. As may be surmised from the source of its name, the theory has found interesting applications in physics and biology and continues to be cited in papers in these areas, apart from mathematical works.

The KCC theory concerns the following. Consider a second order differential equation $(d^2x_i/dt^2) + g_i(x, x', t) =$ 0, for i = 1, ..., n, where $x = (x_1, ..., x_n)$, t is the time parameter, x' denotes $((dx_1/dt), \ldots, (dx_n/dt))$, and g_i 's are smooth functions of (x, x', t) defined on a domain in the (2n+1)-dimensional Euclidean space. The aim is to understand what geometric properties of the system of integral curves – the paths associated with the system of differential equations – remain invariant under nonsingular transformation of the coordinates involved. The theory describes certain invariants, which are specific tensors depending on the q_i 's, which characterize the geometry of the system, in the sense that two such systems can be locally transformed into each other if and only if the corresponding invariants are equivalent tensors. In particular, a given system as above can be transformed into one for which the q_i 's are identically 0, so that the integral curves are all straight lines, if and only if the associated tensor invariants are all zero. The problem may be viewed also as that of realising the integral curves of a second order differential equation as geodesics for an associated linear connection on the tangent bundle. Kosambi introduced a method using calculus of variations, which involves realizing the paths as extremals of a variational principle; this is related to finding a 'metric' for the path space. In another path-breaking paper that appeared in the Quarterly Journal of Mathematics in 1935, Kosambi extends the work, giving further relations between geometrical concepts such as curvature, metric, isotropy, in terms of the integrands of regular problems of calculus of variation, in place of the Riemannian metric in classical

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Kosambi introduced a method using calculus of variations, which involves realizing the paths as extremals of a variational principle. In several papers during the later thirties Kosambi applied his ideas in differential geometry to the theory of relativity. Riemannian geometry. The overall theme was to engage Kosambi's mind over a long period, even after he got involved with many other areas, including within mathematics. Extensions to higher order path spaces involving more general differential equations, analogues for partial differential equations, developing a perspective in terms of general Finsler geometry, interplay with groups of transformations, are some of the directions the theme evolved into.

In several papers during the later thirties Kosambi applied his ideas in differential geometry to the theory of relativity. In a paper published in 1940 in the *Journal* of the London Mathematical Society, treating a fourdimensional path space of a second order differential equation as the universe with the world lines, Kosambi described the most general path-equations. Admitting the Lorentz group and, considering invariance of paths with respect to the Lorentz group, together with the similitude group and the translation group, he deduced Einstein's special relativity and Milne's expanding universe as special cases.

In the 1950's he wrote several papers exploring the relationship between Lie groups (continuous groups) and the differential geometry of path spaces. Intricate connections are established between projectively flat path spaces and Lie groups. One of the corollaries states that the paths of a symmetric affine connexion may be viewed as geodesics of a Riemannian space if they are unique for sufficiently near points.

Contributions in Statistics

Around 1940 Kosambi got interested in statistics. The topic is quite distant from his earlier work, and while it is not clear how the entry into the new arena was actuated in terms of training and equipping oneself, evidently it is linked to his newfound interest around that time in numismatics, and especially the analysis of hordes of old



Indian punch-marked coins. In the twin-areas of numismatics and statistics he wrote half a dozen papers in *Current Science* during 1940–42, two each year. Three of these are primarily in numismatics and, as is wellknown, proved to be path-breaking works in the subject. Of the two papers published in 1941, the first describes how Fisher's z distribution could be applied to study the ratio of two generalized sample variances in a bivariate situation, and the second proposes an alternative to correlation coefficient as a measure of the dependence of the variables in time series analysis, which seems to be inspired by his hands-on work on the punch-marked coins. One of the papers from 1942 proposes a new test procedure for distinguishing between multivariate normal populations.

In 1943 Kosambi published a long paper in the Journal of the Indian Mathematical Society, entitled 'Statistics in Function Space' which is considered his most important contribution in statistics. It discusses statistical problems concerning continuous stochastic processes whose representative functions have the form x(t) = $\sum_{j} x_{j} \varphi_{j}(t)$, where the φ_{j} determines an orthonormal set and x_1, x_2, \ldots are mutually independent Gaussian random variables with zero mean. A theory for second order random functions, describing a decomposition for them in terms of linear combinations of orthogonal random variables was developed by Loéve in his papers in 1945–46; similar results were also obtained by Karhunen during 1946–47. The decomposition, known as Karhunen–Loéve decomposition or proper orthogonal decomposition (POD), provides mathematical and conceptual tools to extend traditional techniques in the study of random processes, and has played a very important role, especially since the 1950's, in statistics as well as many applied areas including image processing, signal processing, data compression, pattern recognition, and a variety of problems in fluid mechanics. From a physical

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In 1943 Kosambi published a long paper in the *Journal of the Indian Mathematical Society*, entitled 'Statistics in Function Space' which is considered his most important contribution in statistics. In another subsequent short paper in the *Proceedings of the National Academy of Sciences*, USA, Kosambi gives an interesting characterisation of the Poisson distributions among a class of distributions defined through power series with positive terms. point of view orthogonal decompositions are important since orthogonal components can be isolated experimentally by means of suitable filters. While the analysis via POD is analogous to Fourier analysis, the approximants involved in the former are such that they are the best possible in the least square sense among those with the same number of terms.

According to a 1960 paper of Kelley, Reed and Root on 'The detection of radar echoes in noise', which itself involves the Karhunen–Loéve decomposition, and uses the name, the expansion was apparently first used in problems of statistical inference by Grenander in 1950. The paper of Grenander however does not adopt the name, and while reference is made to the work of Karhunen, papers of Loéve are not mentioned. During the last decade many authors using the Karhunen-Loéve decomposition, in a variety of the topics noted above, have also mentioned the paper of Kosambi, and sometimes the decomposition, or the associated transform, has been referred to with all the three names. The relevance of Kosambi's paper to the theory seems to have been first noted by J L Lumley in a paper in 1967, quoting a personal communication from A M Yaglom. The specific role or significance of Kosambi's paper in the theory however does not seem to have been discussed in literature, and perhaps calls for further exploration.

In another subsequent short paper in the *Proceedings* of the National Academy of Sciences, USA, Kosambi gives an interesting characterisation of the Poisson distributions among a class of distributions defined through power series with positive terms.

Kosambi also wrote expository articles on various topics in statistics. As is well-known he made some striking applications of statistics in various areas inlcuding eugenics, numismatics and linguistics; these are beyond the scope of this article.

The Riemann Hypothesis

His work in statistics led Kosambi to the idea of applying probabilistic arguments to prove the Riemann Hypothesis. For the non-mathematical reader I may mention that this is a long-standing and extremely important conjecture, proposed by Riemann in 1859, proving which will have enormous consequences in number theory and various allied areas of mathematics. As a gauge of its importance it may be recalled here that it is one of the seven mathematical problems, each carrying a prize of one million US dollars, instituted by the Clay Mathematics Institute, Cambridge, Massachusetts, USA, in 2000 to celebrate mathematics in the new millennium. It was also one of the 23 problems (the eighth) compiled by David Hilbert on the occasion of the International Congress of Mathematicians in 1900 that deeply influenced the mathematics of the twentieth century. There have been many attempts to prove the conjecture, including some by eminent mathematicians, but to-date only certain partial results are known. It was very daring on the part of Kosambi to hope to settle the conjecture. He published in 1959 a paper with a proof of the Riemann hypothesis. Unfortunately however, the 'proof' did not work, and the episode as a whole had some unsavoury consequences to his reputation in the mathematical community.

Actually, there *is* what may be considered a novel and intriguing strategy introduced in the paper for solving the problem, assuming strategies can be judged from a priori considerations, independently of whether or not they ultimately lead to success. In implementing the strategy he needed to prove a result, a lemma, on the distribution of prime numbers. Of the two proofs he offers for it, one goes through a chain of arguments which are described only in an intuitive fashion (what mathematicians call hand-waving) whose validity it is not possible to ascertain. The other proof is crisp, but is found to have

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His work in statistics led Kosambi to the idea of applying probabilistic arguments to prove the Riemann Hypothesis. Kosambi continued to pursue the theme of application of probabilistic methods in the study of distribution of the prime numbers, which is closely linked to the Riemann hypothesis. an error in terms of not taking note of the dependence of some constants involved on certain parameters. The paper was reviewed in *Mathematical Reviews* by a wellknown number theorist, LeVeque, who concluded that he "is unable either to accept this proof or to refute it conclusively."

Why was the paper published in the Journal of the Indian Society of Agricultural Statistics (JISAS)? It does seem strange. In this respect it may be recalled here that two of his earlier papers were also published in JISAS. One of them, published the previous year, describes a unified method for deriving the classical Tauberian theorems. It is clear from the introduction that the author was proud of it, and yet he chose to publish it in that journal. Evidently Kosambi was trying to support and strengthen the journal with his contributions, though the source for the solidarity involved would be difficult to determine at this stage. If only his proof of the Riemann hypothesis were to be right, JISAS would have got a big boost! Incidentally, notwithstanding what the name may suggest, JISAS was not devoted entirely to matters that would interest only agriculturists, and did publish many theoretical papers, largely in probability theory. In a way Kosambi was advocating a primacy for methods using probability theory, and may have considered JISAS a justifiable medium, though on balance the action may not seem prudent. It may also be borne in mind, that the versatile scholar that he was, he may have felt that it did not matter where he published, as good work would be read from wherever it is published.

Kosambi continued to pursue the theme of application of probabilistic methods in the study of distribution of the prime numbers, which is closely linked to the Riemann hypothesis, and as if to mock the critics who may have sniggered at his publishing the earlier paper in JISAS, published a note on the theme in the prestigious *Pro*ceedings of the National Academy of Sciences (USA).



The note was communicated by H S Vandiver, a number theorist member on the Editorial Board of the journal, and yet in the review of the paper in Mathematical *Reviews* one finds the reviewer, J B Kelley, complaining that he "could not follow the proof of the crucial Lemma 4.". Kosambi's last paper on the theme appeared in 1964, in which a proof of the Riemann hypothesis is again claimed, and this paper was also published in JISAS! It would be worthwhile to quote here in some detail from the review of this paper in Mathematical Reviews, that appeared after Kosambi's death, written by the well-known number theorist A Rényi: "The late author tried in the last 10 years of his life to prove the Riemann hypothesis by probabilistic methods. Though he did not succeed in this, he has formulated the following highly interesting conjecture on prime numbers." The reviewer goes on to describe the conjecture, comments that it would be even more difficult than the Riemann hypothesis to prove, and concludes with "Nevertheless, the conjecture is worthy of study in its own right, and the reviewer proposes to call it "the Kosambi hypothesis" in commemoration of the enthusiastic efforts of the late author." The review is indeed an apt tribute to Kosambi's work in the area.

Epilogue

Kosambi contributed novel ideas in many branches of mathematics. Apart from differential geometry and statistics which were the main areas of his pursuit, his work relates also to a variety of topics including differential equations, operational calculus, Fourier analysis, Lie groups, number theory, numerical analysis, probability theory and stochastic processes, sequences, series, summability theory. He was a fine example of a vigorous mathematical explorer and, though due to various circumstances he could not always get the best out of his strengths, he created a niche for himself in the edifice of mathematics of the mid-twentieth century. Kosambi contributed novel ideas in many branches of mathematics.

He was a fine example of a vigorous mathematical explorer.

Acknowledgement:

I am thankful to Somesh Bagchi, Pankaj Joshi, M S Raghunathan, Ram Ramaswamy and A Sitaram for their comments on a preliminary version of this article, and to the TIFR archives for making available some material on D D Kosambi.

I dedicate this article to the memory of an old friend, the late Chintamani (CD) Deshmukh, who was a professor of physics at VJTI, Mumbai, and authored many books, including the first biography of D D Kosambi, and to whom I owe my interest in Kosambi to a large extent.

A Note on Sources

A good deal of information about Kosambi is available online, and many print sources citing or related to his work could be located from the websites. It would not be worthwhile to include here the long list of such sources that were looked up, and I shall content myself mentioning some principal sites which an interested reader may like to visit.

An almost complete list of papers of D D Kosambi is available at the Wikipedia site http://en.wikipedia.org/ wiki/Damodar_Dharmananda_Kosambi/ A list with some more titles and a collection of his papers is available in the library of the Tata Institute of Fundamental Research, Mumbai. Some of his papers in numismatics, published in *Current Science*, that are seen to have a bearing to his work in statistics were brought to the author's notice by Ram Ramaswamy (a Kosambi enthusiast who has endeavoured to collect material relating to his work in various areas), but could be accessed online on the journal's site. Reviews of many papers of Kosambi and other related works could be accessed from http://www.ams.org/mathscinet/ (subscribed site of Mathematical Reviews). Considerable amount of information on citation of his work by various authors is available from the advanced googlescholar search. An article by Meera Kosambi at the site http://www.pragoti .org/node/1731 was helpful in respect of some details.

Address for Correspondence S G Dani School of Mathematics Tata Institute of Fundamental Research Homi Bhabha Road Mumbai 400 005, India. Email: dani@math.tifr.res.in

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