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# Axiomatic Derivation of Maxwell's Equations*

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**ABSTRACT**

**The derivation of Maxwell's equations commencing with Coulomb's law can be carried out in several ways by extending the work of Page. However, their derivation without assuming any experimental results of electromagnetics like Coulomb's law has not so far been carried out. This communication gives an axiomatic derivation of Maxwell's equations assuming only the conservation of charge, special relativity with the associated structure of space and a relation connecting electric and magnetic field intensities.**

## INTRODUCTION

SEVEN years after the publication of the special theory of relativity by Albert Einstein[1], Leigh Page[2,3] set the norms for its use in developing Maxwell's equations starting from Coulomb's law. Since then, several improvements were made culminating in a generalization of his method by Elliott[4]. All these methods, however, centre around at least one experimental result like Coulomb's law or Biot-Savart law apart from the basic assumption of conservation of charge. If Maxwell's equations are required to be derived axiomatically, only the following three axioms and the associated definitions may be assumed, viz. (1) impossibility of action at a distance (special relativity); (2) impossibility of creating or destroying charge without creating or destroying the mass carrying them; and (3) a relation connecting $\boldsymbol{B}$ with $\boldsymbol{E}$.

It has been implied in current literature on electromagnetic theory that no derivation exists up to now which can purport to be axiomatic. The purpose of this communication is to give a method of derivation which extends and parallels the works of Page and Elliott but utilizes only the above two assumptions and the associated definitions.

The first assumption is covered essentially by special relativity and the associated concepts like the retarded potential. The second assumption is equivalent to the principle of conservation of charge. Along with these the properties of charge as defined implicitly in the current theories of the structure of elementary particles are also assumed. One such property with which we commence the derivation of Maxwell's equation is that the force between two charges in Euclidean space decreases as the distance between them increases.

The subject matter to follow is treated in the following order: In the next section a general expression for the electric field intensity is derived. This is followed by the relativistic transformations of the expression resulting in a generalized form of Lorentz's force law. In sections 4, 5, 6 and 7, the general form of the gradient and curl of the electric field intensity and the magnetic field intensity are derived. In all these sections

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only the first assumption is present without implying the second assumption. However, when the principle of conservation of charge is invoked, as in section 8, the generalized equations so derived reduce exactly to Maxwell's equations. The last section gives a discussion of some of the implications of the approach.

## 2. GENERAL EXPRESSION FOR THE ELECTRIC FIELD INTENSITY

Since Coulomb's inverse square law or any other experimental postulate of electromagnetics is sought to be avoided, the logical starting point of the derivation is a general expression for electric field intensity covering all admissible laws of variation of the field intensity as a function of distance. The basic property of charge which prohibits the force between two charges from increasing as the distance between them increases, gives rise to the condition that the general force law should monotonically decrease or at least be constant with the increase in distance. Hence, if a test charge $Q_X$ is placed at $(x_1, x_2, x_3)$ in the field created by a static volumetric charge distribution $\rho_X(\bar{x}_1, \bar{x}_2, \bar{x}_3)$, the total force exerted on $Q_X$ is given by

$$\boldsymbol{F}_X, (x_1, x_2, x_3) = \frac{Q_x}{4\pi\epsilon_0}\iiint_{\bar{V}} \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \frac{\boldsymbol{r}-\bar{\boldsymbol{r}}}{r(r+a_i)^2}\rho_X(\bar{x}_1, \bar{x}_2, \bar{x}_3)\, d\bar{V} \qquad \ldots(1)$$

where, if $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ are unit vectors along the three principal axes,

$\boldsymbol{r} = \boldsymbol{a}_1 x_1 + \boldsymbol{a}_2 x_2 + \boldsymbol{a}_3 x_3$ in the coordinates $X_1X_2X_3$

$\bar{\boldsymbol{r}} = \boldsymbol{a}_1 \bar{x}_1 + \boldsymbol{a}_2 \bar{x}_2 + \boldsymbol{a}_3 \bar{x}_3$ in the coordinates $X_1X_2X_3$

$\epsilon_0$ = dielectric constant of free space (in general, units adjusting parameter)

$\bar{V}$ = volume containing the charges

$a_i$ = non-negative undetermined coefficients.

The undetermined coefficients enable Eq. (1) to represent any conceivable monotonically non-increasing force law.

If the electric field intensity $E_X$ is defined as the force per unit charge, then

$$\boldsymbol{E}_X(x_1, x_2, x_3) = \frac{1}{Q_X}\boldsymbol{F}_X(x_1, x_2, x_3) \qquad \ldots(2)$$

When the charges are not static, the force and field intensity expressions should be derived from the principles of special relativity. Thus, if a static observer $O_X$ with his coordinates $X_1X_2X_3$ interprets the force and field intensity created by a charge distribution $\rho_X(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ with respect to a test charge $Q_X$ as $\boldsymbol{F}_X$ $(x_1, x_2, x_3)$ and $\boldsymbol{E}_X$ $(x_1, x_2, x_3)$ respectively and relates these according to Eq. (2), then an observer $O_Y$ in another coordinate $Y_1Y_2Y_3$, one of whose coordinate axis, say $Y_3Y_3$, is moving along $X_3X_3$ axis with a relative velocity $u_3$, interprets the force and field intensity created by the same charge distribution with respect to the same test charge, differently estimated as $\rho_Y(\bar{y}_1, \bar{y}_2, \bar{y}_3, t)$ and $Q_Y$, as $\boldsymbol{F}_Y(y_1, y_2, y_3, t)$ and $\boldsymbol{E}_Y(y_1, y_2, y_3, t)$ respectively and relates them according to the equation

$$\boldsymbol{E}_Y(y_1, y_2, y_3, t) = \frac{1}{Q_Y}\boldsymbol{F}_Y(y_1, y_2, y_3, t) \qquad \ldots(3)$$

When the test charge is at rest with respect to $O_Y$, the force transformation law[5] in special relativity will be

$$\boldsymbol{F}_Y = \lambda(\boldsymbol{a}_1 F_{X1} + \boldsymbol{a}_2\, F_{X2}) + \boldsymbol{a}_3 F_{X3} \qquad \ldots(4)$$

where, $F_{X1}$, $F_{X2}$, $F_{X3}$ are the components of the force $\boldsymbol{F}_X$ and $\lambda$ is the Lorentz factor $(1-v_3^2/c^2)^{-1/2}$. Combining Eqs. (3) and (4) and separating the components of the vectors, one obtains the relations

$$\left.\begin{aligned} E_{Y1}(y_1, y_2, y_3, t) &= \lambda \frac{Q_X}{Q_Y} E_{X1}(x_1, x_2, x_3) \\ E_{Y2}(y_1, y_2, y_3, t) &= \lambda \frac{Q_X}{Q_Y} E_{X2}(x_1, x_2, x_3) \\ E_{Y3}(y_1, y_2, y_3, t) &= \frac{Q_X}{Q_Y} E_{X3}(x_1, x_2, x_3) \end{aligned}\right\} \quad \ldots(5)$$

where

$$\boldsymbol{E}_X = \boldsymbol{a}_1 E_{X1} + \boldsymbol{a}_2 E_{X2} + \boldsymbol{a}_3 E_{X3}$$
$$\boldsymbol{E}_Y = \boldsymbol{a}_1 E_{Y1} + \boldsymbol{a}_2 E_{Y2} + \boldsymbol{a}_3 E_{Y3}$$

### 3. RELATIVISTIC TRANSFORMATION OF THE GENERAL FORCE LAW

When a velocity $\boldsymbol{V}(t)$ is assigned to the test charge with respect to the coordinates $Y_1 Y_2 Y_3$, the relativistic transformation law for the force $\boldsymbol{F}_Y$ will be, after combining it with Eqs. (3) and (4),

$$\boldsymbol{F}_Y = Q_Y \boldsymbol{E}_Y + Q_Y \boldsymbol{V} \times \left( \boldsymbol{a}_3 \frac{\lambda v_3}{c^2} \times \boldsymbol{E}_X \frac{Q_X}{Q_Y} \right) \quad \ldots(6)$$

Abbreviating the term in the bracket of Eq. (6) as $\boldsymbol{B}_Y$, the Magnetic field intensity, which has as its components

$$\left.\begin{aligned} B_{Y1}(y_1, y_2, y_3, t) &= -\frac{\lambda v_3}{c^2} E_{X2}(x_1, x_2, x_3) \frac{Q_X}{Q_Y} \\ B_{Y2}(y_1, y_2, y_3, t) &= +\frac{\lambda v_3}{c^2} E_{X1}(x_1, c_2, x_3) \\ B_{Y3}(y_1, y_2, y_3, t) &= 0 \end{aligned}\right\} \quad \ldots(7)$$

Eq. (6) can be written analogous to Lorentz's force law as

$$\boldsymbol{F}_Y = Q_Y \boldsymbol{E}_Y + Q_Y \boldsymbol{V} \times \boldsymbol{B}_Y \quad \ldots(8)$$

The relativistic transformations (5) and (7) can be inverted to realize the transformations

$$\left.\begin{aligned} E_{X1} &= \lambda \frac{Q_Y}{Q_X} (E_{Y1} - v_3 B_{Y2}) \\ E_{X2} &= \lambda \frac{Q_Y}{Q_X} (E_{Y2} + v_3 B_{Y1}) \\ E_{X3} &= \frac{Q_Y}{Q_X} E_{Y3} \end{aligned}\right\} \quad \ldots(9)$$

### 4. DIVERGENCE RELATION FOR ELECTRIC FIELD INTENSITY

Let $S$ be an imaginary closed surface which does not intersect the charges in the region. Since the divergence theorem cannot be applied to the force law of Eq. (1), we have to consider an arbitrarily small auxiliary sphere $\Sigma$ of radius $\alpha$ surrounding $r = 0$.

For the connected region enclosing the volume $V'$, divergence theorem can be applied[6] resulting in the equation

$$\iiint_{V'} \nabla \cdot \left\{ \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\rho_X(\boldsymbol{r}-\bar{\boldsymbol{r}})}{4\pi\epsilon_0 r(r+a)^2} \right\} d\tau = \iint_S \lim_{N\to\infty} \frac{-1}{N} \sum_{i=1}^{N} \frac{\rho_X(\boldsymbol{r}-\bar{\boldsymbol{r}}).d\boldsymbol{\sigma}}{4\pi\epsilon_0 r(r+a_i)^2}$$

$$+ \iint_\Sigma \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\rho_X(\boldsymbol{r}-\bar{\boldsymbol{r}}).d\boldsymbol{\sigma}}{4\pi\epsilon_0 r(r+a_i)^2} \qquad \ldots(10)$$

utilizing the identities,

$$\nabla \cdot \left\{ \lim_{N\to} \frac{1}{N} \sum_{i=1}^{N} \frac{\rho_X(\boldsymbol{r}-\bar{\boldsymbol{r}})}{4\pi\epsilon_0 r(r+a_i)^2} \right\} = \frac{\rho_X}{4\pi\epsilon_0} \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ \nabla^2 \frac{1}{r+a_i} \right\}$$

and

$$\iint_\Sigma \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\rho_X(\boldsymbol{r}-\bar{\boldsymbol{r}}).d\sigma}{4\pi\epsilon_0 r(r+a_i)^2} = -\frac{\rho_X}{\epsilon_0} \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\alpha^2}{(\alpha+a)^2}$$

Eq. (10) can be written in the infinitesimal form as

$$\nabla . E_X = \frac{\rho_X}{\epsilon_0} \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{a_i b}{2\pi r(r+a_i)^3} + \frac{\alpha^2}{(\alpha+a_i)^2} \right\} \qquad \ldots(11)$$

where $b$ is a units adjusting constant with the dimensions of volume. It is important to consider the point why $\alpha$ should not be allowed to tend to zero. The reason is as follows: In the general force law of Eq. (1) the possibility of some of the $a_i$ taking a value of zero is not ruled out. The only constraint is that $a_i$ should be non-negative. Therefore, if a particular $a_i = 0$, then

$$\lim_{\alpha\to 0} \frac{\alpha^2}{(\alpha+a_i)^2} = 1$$

whereas if $a_i \neq 0$,

$$\lim_{\alpha\to 0} \frac{\alpha^2}{(\alpha+a_i)^2} = 0$$

Hence, the limit for $\alpha \to 0$ should be determined only after the values of the undetermined coefficients $a_i$ are determined.

The divergence relation can be determined for the dynamic case as a relativistic transformation of Eq. (11). By definition

$$\nabla . \boldsymbol{F}_Y = \frac{\partial \boldsymbol{E}_{Y1}}{\partial y_1} + \frac{\partial E_{Y2}}{\partial y_2} + \frac{\partial E_{Y3}}{\partial y_3} \qquad \ldots(12)$$

utilizing Eq. (5) and the following relations derived for any function $F$ from the Lorentz transformations[4], viz.

$$\frac{\partial \boldsymbol{F}}{\partial x_1} = \frac{\partial F}{\partial y_1} \quad \frac{\partial F}{\partial x_2} = \frac{\partial F}{\partial y_2}$$

$$\frac{\partial F}{\partial x_3} = \lambda \left[ \frac{\partial F}{\partial y_3} + \frac{v_3}{c^2} \frac{\partial F}{\partial t} \right] \qquad \ldots(13)$$

Eq. (12) reduces to

$$\nabla . \boldsymbol{E}_Y = \lambda \frac{Q_X}{Q_Y} (\nabla . \boldsymbol{E}_X) \quad \ldots (14)$$

Combining Eq. (14) with Eq. (11),

$$\Delta . \boldsymbol{E}_Y = \lambda \frac{Q_X}{Q_Y} \frac{\rho_X}{\epsilon_0} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{a_i b}{2\pi r (r+a_i)^3} + \frac{\alpha^2}{(\alpha + a_i)^2} \right\} \quad \ldots (15)$$

Eq. (15) gives the divergence relation for dynamic case.

## 5. DIVERGENCE RELATION FOR MAGNETIC FIELD INTENSITY

The divergence relation for magnetic field intensity for the dynamic case can be obtained straightforwardly as follows:

By definition,

$$\nabla . \boldsymbol{B}_Y = \frac{\partial B_{Y1}}{\partial y_1} + \frac{\partial B_{Y2}}{\partial y_2} + \frac{\partial B_{Y3}}{\partial y_3}$$

Applying Eqs. (7) and (13), it transforms to

$$\nabla . \boldsymbol{B}_Y = \frac{\lambda v_3}{c^2} \left( \frac{\partial E_{X1}}{\partial x_2} - \frac{\partial E_{X2}}{\partial x_1} \right) \frac{Q_X}{Q_Y}.$$

The right-hand side is identically zero. Therefore,

$$\nabla . \boldsymbol{B}_Y = 0 \quad \ldots (16)$$

Eq. (16) implies the non-existence of the magnetic charge irrespective of the validity of the law of conservation of charge and for any conceivable monotonically non-increasing force law. This conclusion appears to point out, without ambiguity, the futility of the efforts to discover the magnetic charge by any experiment.

## 6. CURL RELATION FOR ELECTRIC FIELD INTENSITY

For the electrostatic case, curl $\boldsymbol{E}_X$ can be obtained by methods similar to those employed earlier[7]. Because of the identity,

$$\nabla \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{(r+a_i)} = - \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{\boldsymbol{r} - \bar{\boldsymbol{r}}}{r (r+a_i)^2}$$

Eq. (2) can be rewritten after substituting Eq. (1), into

$$E_X = - \frac{1}{4\pi\epsilon_0} \iiint_{\bar{V}} \nabla \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{(r+a_i)} \rho_X d\bar{V} \quad \ldots (17)$$

If a scalar potential

$$\phi (x_1, x_2, x_3) = \iiint_{\bar{V}} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{(r+a_i)} \rho_X d\bar{V} \quad \ldots (18)$$

is defined, then Eq. (17) reduces to

$$\boldsymbol{E}_X = -\nabla\phi$$

Since

$$\nabla \times \nabla \phi = 0; \quad \nabla \times \boldsymbol{E}_X = 0 \quad \ldots (19)$$

For the dynamic case, by definition

$$\nabla\times \boldsymbol{E}_Y = \boldsymbol{a}_1\left(\frac{\partial E_{Y3}}{\partial y_2}-\frac{\partial E_{Y2}}{\partial y_3}\right)+\boldsymbol{a}_2\left(\frac{\partial E_{Y1}}{\partial y_3}-\frac{\partial E_{Y3}}{\partial y_1}\right)+\boldsymbol{a}_3\frac{\partial E_{Y2}}{\partial y_1}-\frac{\partial E_{Y1}}{\partial y_2} \quad \ldots(20)$$

From Eqs. (9), (13), (16) and (19), it follows that

$$\frac{\partial E_{X2}}{\partial x_1}-\frac{\partial E_{X1}}{\partial x_2}=0=\lambda\frac{Q_X}{Q_X}\left[\frac{\partial E_{Y2}}{\partial y_1}-\frac{\partial E_{Y1}}{\partial y_2}\right] \quad \ldots(21)$$

If in addition to Eqs. (9), (13), (16) and (19) an additional relativistic transformation, viz.

$$\frac{\partial F}{\partial t}=-v_3\frac{\partial F}{\partial y_3}$$

is utilized, it can be shown that

$$\frac{\partial E_{X3}}{\partial x_2}-\frac{\partial E_{X2}}{\partial x_3}=0=\frac{Q_Y}{Q_X}\left(\frac{\partial E_{Y3}}{\partial y_2}-\frac{\partial E_{Y2}}{\partial y_3}\right)+\left(\frac{\partial B_{Y1}}{\partial t}+\frac{Q_X}{Q_Y}\right) \quad \ldots(22)$$

and

$$\frac{\partial E_{X1}}{\partial x_3}-\frac{\partial E_{X3}}{\partial x_1}=0=\frac{Q_Y}{Q_X}\left(\frac{\partial E_{Y1}}{\partial y_3}-\frac{\partial E_{Y3}}{\partial y_1}\right)+\left(\frac{\partial B_{Y2}}{\partial t}+\frac{Q_X}{Q_Y}\right) \quad \ldots(23)$$

From Eqs. (21), (22), (23) and (7), the curl relation for the dynamic $C$ as results and is given by

$$\nabla\times \boldsymbol{E}_Y=-\frac{Q_X}{Q_Y}\boldsymbol{B}_Y-\frac{Q_X^2}{Q_Y^2}\lambda^2\left(B_Y\frac{\partial}{\partial t}\frac{Q_Y}{Q_X}\right) \quad \ldots(24)$$

## 7. CURL RELATION FOR MAGNETIC FIELD INTENSITY

The curl relation for magnetic field intensity can be determined directly for the dynamic case, starting from the static divergence relation for electric field intensity. By definition,

$$\nabla.\boldsymbol{E}_X=\frac{\partial E_{X1}}{\partial x_1}+\frac{\partial E_{X2}}{\partial x_2}+\frac{\partial E_{X3}}{\partial x_3}$$

which can be transformed relativistically using Eqs. (13), (9) and (7) into

$$\nabla.\boldsymbol{E}_X=\lambda\frac{Q_Y}{Q_X}(\nabla.\boldsymbol{E}_Y)-\lambda v_3\left(\frac{\partial B_{Y2}}{\partial y_1}-\frac{\partial B_{Y1}}{\partial y_2}\right)\frac{Q_Y}{Q_X}+\frac{\lambda v_3}{c^2}\frac{Q_Y}{Q_X}\frac{\partial E_{X3}}{\partial t} \quad \ldots(25)$$

Eq. (25) can be rewritten using Eqs. (14) and (15) into

$$\frac{\partial B_{Y2}}{\partial y_1}-\frac{\partial B_{\ 1}}{\partial y_2}=\frac{v_3\lambda}{c^2}\left[\frac{\rho_X}{\epsilon_0}\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\left(\frac{a_i b}{2\pi r(r+a_i)^3}+\frac{\alpha^2}{(\alpha+a_i)^2}\right)\right]\frac{Q_X}{Q_Y}+\frac{1}{c^2}\frac{\partial E_{Y3}}{\partial t} \quad \ldots(26)$$

Eq. (26) is for the assumption that the axis $Y_3Y_3$ is sliding along the axis $X_3X_3$. Similar equations can be obtained for assumed relative velocities of the other axis pairs. This enables us to generalize Eq. (26) and write the combination of the three components as

$$\boldsymbol{B}_Y=\frac{\boldsymbol{I}_D}{c^2\epsilon_0}\frac{Q_X}{Q_Y}+\frac{1}{c^2}E_Y \quad \ldots(27)$$

where $\boldsymbol{I}_D$ is the general form of the current density and is a vector formed out of the three components corresponding to the first term in the right-hand side of Eq. (26).

## 8. REDUCTION TO MAXWELL'S EQUATIONS

In the foregoing sections a complete set of four equations, viz. Eqs. (15), (16), (24) and (27) were derived axiomatically without even assuming the conservation of charge. Thus they constitute a set of general electromagnetic equations in which charge conservation is immaterial. However, our present purpose being the derivation of Maxwell's equations, the second axiom or assumption mentioned in the introductory section will now be invoked.

If charge is conserved the first condition to be satisfied is

$$Q_X = Q_Y \qquad \ldots(28)$$

and the second condition consequential to Eq. (28) is

$$\rho_X dV_X = \rho_Y dV_Y \qquad \ldots(29)$$

Since volume transforms like distance in the present case,

$$\rho_Y = \lambda \rho_X \qquad \ldots(30)$$

To derive the condition that must be satisfied by Eqs. (15), (16), (24) and (27), as a result of the restrictions imposed by Eqs. (28) and (30), the expression for $\nabla . \boldsymbol{E}_Y$ must be examined. Though several derivations lead to the same condition, the simplest one out of these methods is given in the following. Eq. (15) can be rewritten, after applying Eqs. (28) and (30) into

$$\nabla . \boldsymbol{E}_Y = \frac{\rho_Y}{\epsilon_0} \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{a_i b}{2\pi r(r+a)^3} + \frac{\alpha^2}{(\alpha+a)^2} \right\} \qquad \ldots(31)$$

comparing Eq. (31) with Eq. (11), it can be inferred that whereas $\rho_X$ transformed to $\rho_Y$ while $\nabla . \boldsymbol{E}_X$ transforms to $\nabla . \boldsymbol{E}_Y$, the terms inside the bracket of Eq. (11) have remained an invariant. Since $b$ is a units adjusting constant and is invariant under the transformation analogous to $\epsilon_0$, and since the quantities $a$, $r$ and $\alpha$ suffer Lorentz-Fitzerald contraction in the direction of motion, the condition that the terms in the bracket of Eq. (11) is invariant under the transformation is

$$(\lambda^3 - 1) \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{a_i b}{2\pi r(r+a_i)^3} \right\} = 0 \qquad \ldots(32)$$

Since the velocity in general need not be zero, $\lambda \neq 1$, and since $a_i$, $b$ and $r$ are non-negative, Eq. (32) is satisfied only if all $a_i$ assume zero values. Imposing this condition Eqs. (15), (16), (24) and (27) reduced to

$$\left. \begin{aligned} \nabla . \boldsymbol{E}_Y &= \frac{\rho_Y}{\epsilon_0} \\ \nabla . \boldsymbol{B}_Y &= 0 \\ \nabla . \boldsymbol{E}_Y &= - \dot{B}_Y \\ \nabla . \boldsymbol{B}_Y &= \frac{\boldsymbol{\sigma}}{c^2 \epsilon_0} + \frac{\dot{E}_Y}{c} \end{aligned} \right\} \qquad \ldots(33)$$

which are the well-known equations of Maxwell.

## DISCUSSION

In the foregoing sections Maxwell's equations were derived axiomatically without assuming any experimental result. It may be pointed out that the velocity of light, $C$, is the result of Michaelson's experiment. However, it should be noted that no where in the derivation a value for $C$ has been assumed, nor the fact that it is the velocity of light. On the contrary $C$ comes into picture only through the Lorentz factor $[1-(v^2/c^2)^{-1/2}]$ so that throughout the derivation $C$ stands for the upper limit for the velocity of travel of any effect from its cause since action at a distance is assumed to be impossible.

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