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Amplification Factor of Triodes with Elliptic Grids Having Support Rods

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ABSTRACT

A capacitance equivalence method to determine the amplification factor of triodes with irregular geometry was proposed by O'Neill. This method does not intrinsically take into its purview the presence of support rods of appreciable radius. In a previous communication, the author derived the equi-capacitance structure for elliptic grids using a method based on O'Neill's principle. In this paper the analysis given there at is supplemented with a criteria for obtaining an equivalent radius for the support rods, which would produce nearly the same screening fraction, if the position of these rods is reckoned at the circumference of the equivalent circular grid. The amplification factor formula is consequential.

INTRODUCTION

THE field analysis of vacuum tube triodes having elliptic grids and circular cathode have been reported[1,3]. Two of the methods currently available are O'Neill's approximation with the limiting geometry as the parallel planes[1] and an approximation due to the author having a cylindrical limiting configuration[3], the limit being decided by the applicability of the formula with maximum accuracy. But both these formulae do not take into consideration intrinsically the presence of support rods. Thus, their application is limited to structures wherein the effect of support rods can be neglected or structures wherein the support rods are not in the active region, e.g. sub-miniature tubes with a cantilever construction and tripod mounting. But in many of the conventional tubes the support rod effect figures prominently and the determination of the amplification factor of triodes necessitates the field analysis of triodes having support rod radius comparable to the dimensions of the grid. Several manufacturing concerns make use of sectionalization approximations but these are necessarily laborious and are ideally suited for a final design. But, the preliminary trial and error design requires more direct methods which are in the nature of formulae whose dimensions are not specified numerically. One such method which can be utilized conveniently for preliminary designs is O'Neill's equivalence capacitance method. This paper gives a criteria for calculating the screening fraction due to the support rods, when these are physically related to the equi-capacitance cylindrical grid.

PREVIOUS WORK

O'Neill has shown[1,2], to a first approximation, that the space-charge saturated currents in relation to the tube structure will be the same in any two tube forms wherein the electrostatic capacitance between the cathode and the plate per unit length is the same. This equivalence is seen to hold only if the cathode after final transformation retains its original shape. Using this proposition, the behaviour of a cylindrical cathode between a pair of parallel plane anodes has been analysed. It is shown that for a cylindrical cathode and an anode of elliptic cross-section, the formula for parallel planes is applicable with the proviso that the equivalent cylindrical anode radius will be somewhat less than 1·31 times the minor axis of the ellipse. This depends, to a large extent, on the actual curvature and that, in practical design problems, the appropriate factor is best determined by preliminary experiments supplemented, where necessary, by a reasonable guess. This approximation applies also to a cylindrical triode with an elliptic grid.

In the previous communication[3], expressions are derived for the equivalent radius of the equicapacitance grid structure by the method of conformal transformations supplemented by an electrolytic tank experiment. No provision for guess-work is incorporated in this analysis. The equivalent cylindrical radius of the elliptic grid was shown to be

$$R_g = R_c.\exp Q \qquad \ldots(1)$$

where for $0 < r_c/b < 0{\cdot}75$ and $1 < a/b < 2$

$$Q = \tfrac{1}{2}\cosh^{-1}\left(\frac{a^2b^2+r_{c_1}^2}{2a_1r_{c_1}}\right) \qquad \ldots(2)$$

and for $0 < r_c/b < 0{\cdot}75$ and $1 < a/b < 3$

$$Q = \frac{\cosh^{-1}\left(\dfrac{a^2b^2+r_{c_1}^2}{2a_1r_{c_1}}\right)\cosh^{-1}\left(\dfrac{a_1^2b_1^2+r_{c_2}^2}{2a_2r_{c_2}}\right)}{4\ln\left(\dfrac{a_1}{r_{c_1}}\right)} \qquad \ldots(3)$$

Here

a = semi-major axis of the ellipse
b = semi-minor axis of the ellipse
r_c = cathode radius
$a_1 = \frac{1}{2}(a^2+b^2)$
$b_1 = ab$
$r_{c_1} = r_c^2$
$a_2 = \frac{1}{2}(a_1^2+b_1^2)$ and
$r_{c_2} = r_{c_1}^2$

These expressions hold good for elliptic grids and the accuracy improves as the structure tends to circularity. This is complimentary to the parallel plane approximation given by O'Neill for elliptic grids. These two formulae may be supplemented to cover a wide range of ellipticities.

PRESENT ANALYSIS

The screening fraction of a triode is one of the main parameters governing the amplification factor. The total screening fraction has two components: one contributed by the grid and the other by the support rods. This proposition applies directly for regular structures, e.g. cylindrical triodes. The equicapacitance circular grid radius being determined, the screening fraction contributed by this in the absence of support rods result as a consequence. The support rods in the original grid being physically unconnected to the equicapacitance circular grid, a criteria to take this into effect at the latter situation has been examined. A linear relation in proportion to the distance from the cathode is found to give inconsistent and often erroneous results (as exemplified in Table 1). Since the electron trajectories are directly related to the electric lines of force for a specified geometry and since the former describes the amount of screening to the flow of electrons for a known obstruction, a knowledge of the electric lines of force is taken as the basis for computing the radius of the equivalent support rod which causes the same screening effect for various positions on the XX-axis (Fig. 1).

To a first approximation, we may consider that the force line which is tangential to the support rod in the initial position may be taken as the geodesic envelope expressing the non-linear relation for the radius in various possible positions of the support rod for the same screening, when physically related to regularized structures. Since there is a constraint for the radius of this regular structure, by virtue of the capacitance equivalence, only one such position need be considered. The support rod present anywhere along this envelope causes a field distortion and a corresponding trajectory distortion whose geodesics relate approximately one to one on any one of the force line present prior to the introduction of the support rods. Perforce, the converse follows, implying that the picture given above largely accounts for the effect of screening.

The necessary mathematical formalism for evaluating the radius of the equivalent support rod attached to the equicapacitance transformed grid is as follows:

A half section of the coaxial structure is transformed into a non-coaxial one by an analytic relation. The electric lines of force are mapped by resorting to

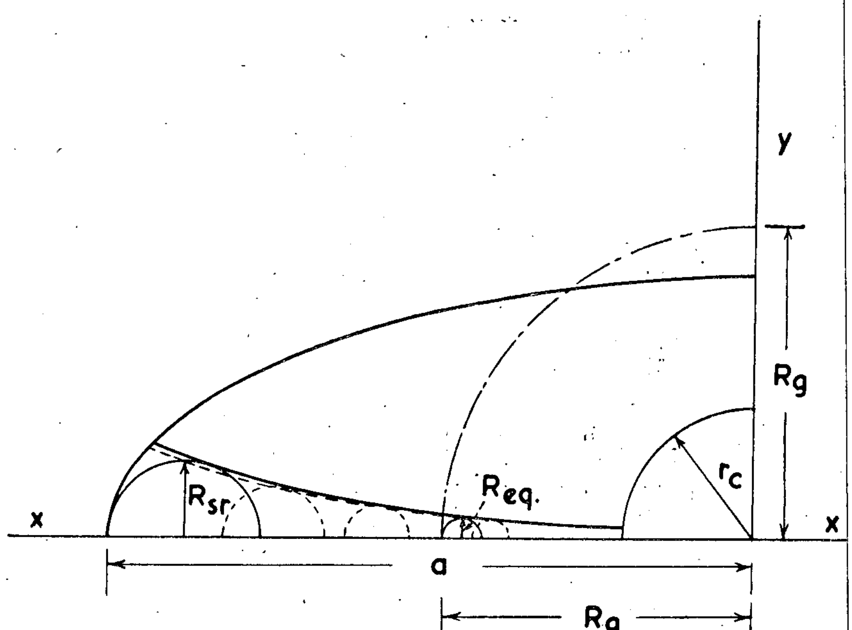

Fig. 1 — Sketch showing the original and the equivalent support rods

bicylindrical coordinate and a field resolution. A polar coordinate convertion with a subsequent boundary identification gives expressions for the effective field map. A line of force is identified by the condition imposed by the radius and the situation of the initial structure of the support rod. The new radius at the new situation of the support rod is computed in consequence.

It is necessary to point out that analysis of transmission lines[4,5], uniform and non-uniform, having the same cross-section as the one considered in this paper, have been reported.

RADIUS OF THE EQUIVALENT SUPPORT ROD

The total screening fraction for the equivalent grid structure is given by

$$S = \frac{2t_g}{d_p} + \frac{N_S R_{eq}}{\pi R_g} \qquad \ldots(4)$$

where

t_g = grid wire thickness
d_p = pitch of the grid
N_s = number of support rods
R_g = radius of the equicapacitance grid
R_{eq} = radius of the equivalent support rods.

Fig. 2 shows a non-linear transformation of a half-plane structure in the Z-plane consisting of a half-ellipse and a semicircle by an analytic relation

$$W = Z^2 \qquad \ldots(5)$$

The transformed structure in the W-plane is an ellipse of lesser eccentricity than that of the original structure. From the transformation we get

$$\left.\begin{aligned} a_1 &= \tfrac{1}{2}(a^2+b^2) \\ e_1 &= \tfrac{1}{2}(a^2-b^2) \\ r_1 &= r_c^2 \end{aligned}\right\} \qquad \ldots(6)$$

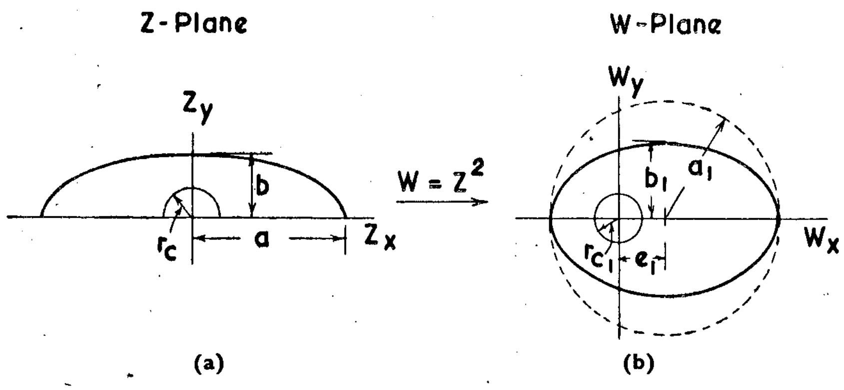

Fig. 2 — Transformation of the half-structure of the elliptic circular coaxial tube into a non-coaxial configuration

This transformed structure can be analysed using the bicylindrical coordinate (u, v) (Fig. 3).

Two families of orthogonal circles in the xy-plane are drawn. $x = a$ and $x = -a$ are two poles about which constant u are drawn, while the constant v are their orthogonal counterparts.

$$\left.\begin{aligned} x &= \frac{a \sinh u}{\cosh u - \cos v} \\ y &= \frac{a \sin v}{\cosh u - \cos v} \end{aligned}\right\} \qquad \ldots(7)$$

The metric coefficients are

$$g_{11} = g_{22} = \frac{a_r^2}{(\cosh u - \cos v)^2} \qquad \ldots(8)$$

where
$$a_r = \tfrac{1}{2}e_1 \left[e_1^4 - 2e_1^2(a_1^2+r_1^2)+(a_1^2-r_1^2)^2\right]^{1/2} \qquad \ldots(9)$$

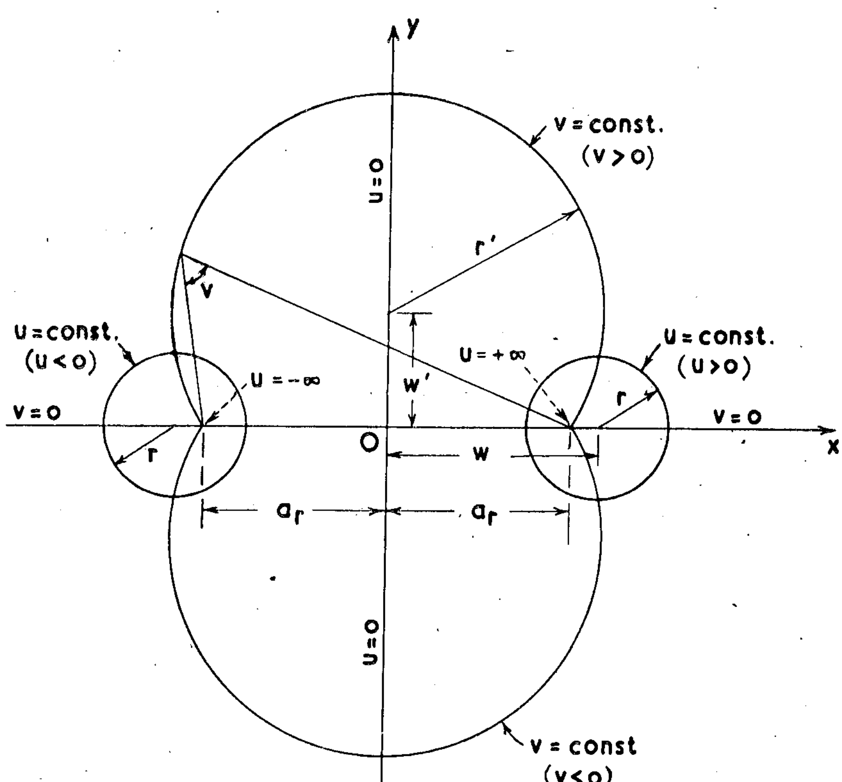

Fig. 3 — The bicylindrical coordinate system

With this as the basis, the information concerning the electric lines of force can be derived. It has been shown in a previous paper[4] that, if (r, θ) represents the polar coordinate with the origin at the centre of the transformed inner conductor (Fig. 2b),

$$v(r, \theta) = \sin^{-1}\left[\frac{2a_r r \sin\theta}{\sqrt{D(r, \theta)}}\right] \quad \ldots(10)$$

where
$$D(r, \theta) = (r^2+w_1^2+2rw_1\cos\theta)^2+a_r^4-2a_r^2(r^2\cos 2\theta+w_1^2+2rw_1\cos\theta) \quad \ldots(10a)$$

$$w_1 = \sqrt{a_r^2+r_1^2} \quad \ldots(10b)$$

The above relations depict the field map for the auxiliary circle only. To take into consideration the small ellipticity in the transformed figure, boundary conditions are examined. At the inner boundary, namely the cathode surface, there should be no change in the field configuration, while at the other boundary the auxiliary circle should be identified with the ellipse. Due to the near circularity of this ellipse, for $0<a/b<3$, a linear radial coordinate shrinking may be assumed.

This is given by a ratio

$$R = \frac{r_e-r_1}{r_c-r_1} \quad \ldots(11)$$

where
$$r_c = e_1\cos\theta+(a_1^2-e_1^2\sin^2\theta)^{1/2} \quad \ldots(11a)$$

$$r_e = \frac{b_1^2 e_1\cos\theta+a_1 b_1\,[(a_1^2-e_1^2)\sin^2\theta+b_1^2\cos^2\theta]^{1/2}}{b_1^2\cos^2\theta+a_1^2\sin^2\theta} \quad \ldots(11b)$$

Further, if (η, Θ) refer to the original Z-plane structure Fig. 2(a) then, if

$$\left.\begin{aligned} v, D &= F_1, F_2\,(r, \theta) \\ R &= F_3\,(\theta) \end{aligned}\right\} \quad \ldots(12)$$

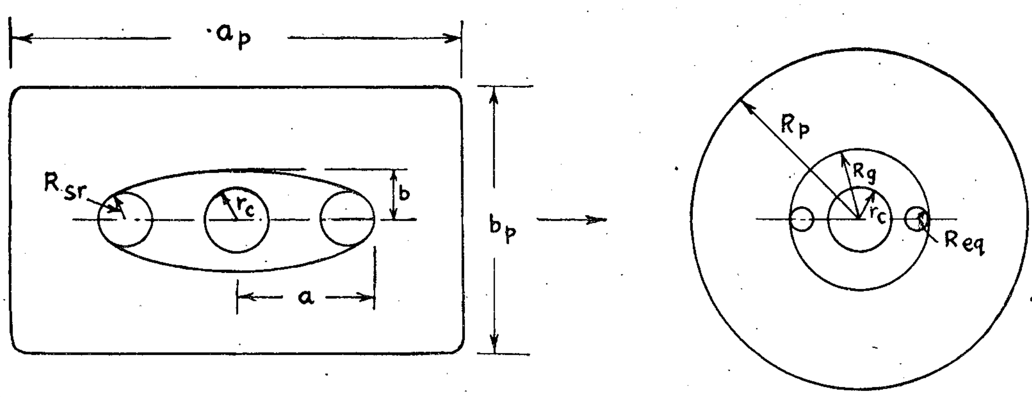

Fig. 4 — The original and the equicapacitance structures for a tube of the 12AX7 or ECC83 type [a_p=6·5 mm., a=1·83 mm., R_p=2·09 mm. (from O'Neill's formula), b_p=3·3 mm., b=0·67 mm., R_g=0·79 mm. (from Eq. 3), r_c=0·46 mm., R_{SR}=0·37 mm., R_{eq}=0·04 mm. (from Eq. 14)]

TABLE 1

Equations and conditions specified	πS	μ
Formula (1) and without taking support rods into consideration	0·83	73
Formula (1) and taking a linear relation for the effect of support rods	1·18	190
Formula (1) and taking a non-linear relation for the effect of support rods	0·93	98
Actual value	—	100

Then
$$v',\ D' = F_1,\ F_2\left(\frac{\eta^2+R_{eq}r_1-r_1}{R_{eq}},\ 2\Theta\right) \text{ and } R_{eq} = F_3(2\Theta) \quad \ldots(13)$$

But, in practical design, a simplification assumption may be useful. That is, the force line tangential to the original support rod is not much affected by slight ellipticity in Fig. 2(b). Hence, Eq. (10) itself may be used taking $r = \eta^2$ and $\theta = 2\Theta$. The resulting expression may be written as

$$v(\eta^2,\ 2\Theta) = \sin^{-1}\left[\frac{2a_r\eta^2 \sin 2\Theta}{\sqrt{D(\eta^2,\ 2\Theta}}\right] \quad \ldots(14)$$

From Fig. 1 and Eq. (14)

$$v(\eta_1^2,\ 2\Theta_1) = v(\eta_2^2,\ 2\Theta_2) \quad \ldots(15)$$

where, if R_{SR} is the radius of the original support rod,

$$\left.\begin{aligned} \eta_1 &= (a^2+2aR_{SR})^{1/2} \\ \Theta_1 &= \tan^{-1}\left(\frac{R_{SR}}{a-R_{SR}}\right) \\ \eta_2 &= (R_g^2+2R_gR_{eq})^{1/2} \\ \Theta_2 &= \tan^{-1}\left(\frac{R_{eq}}{R_g-R_{eq}}\right) \end{aligned}\right\} \quad \ldots(16)$$

Thus, if

$$\left.\begin{aligned} \frac{D(\eta_1^2,\ 2\Theta_1)}{(a-R_{SR})^2\, R_{SR}^2} &= K \\ \text{and} \quad \frac{D(\eta_2^2,\ 2\Theta_2)}{(R_g-R_{eq})^2\, R_{eq}^2} &= F(R_{eq}) \end{aligned}\right\} \quad \ldots(17)$$

then, $F(R_{eq}) = K$, gives the value of R_{eq} by iteration to any desired accuracy.

As an illustration, the amplification factor of a triode with an elliptic grid, a rectangular anode and a circular cathode is worked out (Fig. 4).

Table 1 gives the computed values of μ for three different cases, viz. neglecting the support rods, a linear relation for including the effect of support rods, and a non-linear relation for the same. The non-linear relation gives a value of μ nearer the actual.

CONCLUSION

This paper gives one possible method of including the effect of support rods for methods making use of O'Neill's equicapacitance principle. The sequence of analysis adopted for the elliptic configuration can be used for any other structure if the approximate field map can be obtained. Based on this method simpler design guides than the one given in this paper may be evolved to suit the tube designer's necessities, the purpose of the present paper being principally to being to note the equivalent support rod concept.

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