Hybrid Methods for Multi-Objective Evolutionary Algorithms

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Abstract- Hybrid methods of using evolutionary algorithms with a local search method are often used in the context of single-objective real-world optimization. In this paper, we discuss a couple of hybrid methods for multi-objective real-world optimization. In the *posteriori* approach, the obtained non-dominated solutions of a multi-objective evolutionary algorithm (MOEA) run are modified using a local search method. In the *online* approach, a local search method is applied to each solution obtained by genetic operations in a MOEA run. Both these approaches are compared on three engineering shape optimization problems for a fixed number of overall function evaluations. Simulation results suggest important insights about the extent of local search and the extent of an MOEA needed to achieve an overall efficient hybrid approach.

1 Introduction

In the multi-objective optimization problem there exist more than one objective. If the objectives are of conflicting in nature, one single solution cannot be the optimal solution. Instead, a set of solutions (known as Pareto-optimal set) is optimal. Since the evolutionary methods work with a population of solutions, they have been found suitable to find multiple and well-diverse set of Pareto-optimal solutions in one single simulation run [2, 3, 7, 9]. However, till to date, most MOEAs are applied on test problems, where the exact knowledge of the global Pareto-optimal front is available. In solving real-world problems, a straightforward application of an existing MOEA may not guarantee finding the true Pareto optimal set. Moreover, recently a couple of state-of-the-art elitist MOEAs have been found to have deficiencies in truly converging to the exact Pareto-optimal front [8].

A similar difficulty of the true convergence to the optimum solution in single-objective optimization is alleviated by using a hybrid method. In one such method, an evolutionary algorithm (EA) is expected to run for some iterations. Thereafter, a local search method is started from the best solution obtained by the EA. This process is believed to be better for two reasons: (i) most optimum is unimodal (best solvable by a local search approach) in the neighborhood of the optimum and (ii) a hybrid method may use the efforts of both EAs and a local search approach in a way better than either approach alone.

In the proposed hybrid multi-objective optimization, we make use of a local search method and an MOEA. One difficulty with a local search method is that a local search method can only optimize a single objective. Thus, a careful hybridization of the two approaches is necessary to take advantage of better convergence and computational effort. In this paper, we discuss two implementations and compare their performances on three engineering shape design problems. The simulation results provide interesting working behaviors of each hybrid approach.

2 Hybrid Techniques

There are at least two reasons why a hybrid method would be useful in real-world optimization problems, including multi-objective optimization:

1. It ensure better convergence to the global Pareto-optimal front.

2. It demands smaller computational effort than each individual method applied alone.

In the case of real-world problems the knowledge about the Pareto-optimal front is not usually available. Though evolutionary algorithms have shown the potential of reaching close to the global Pareto-optimal front in many problems, it is wise to make use of a specialized method (local search method) to increase the probability of convergence to the global Pareto-optimal front. Since the local search methods have good convergence properties to a local optimal solution, and an EA has overall global perspective, a hybrid method of combining the two approaches is a natural choice. In one approach, an MOEA can be used to find good initial solutions for the local search method, which then can make an attempt to find the solutions even closer to the true Pareto-optimal front. Because of the same reason, the combined use of an MOEA and a local search method may result in a saving of computational effort, if used properly.

In this paper, we discuss two extreme hybrid approaches (i) posteriori approach and (ii) online approach. These are discussed in details in the following sections.

2.1 Posteriori Approach

In this approach, we allow the multi-objective evolutionary algorithm to run for a fixed number of generations. This would produce a number of non-dominated solutions as the outcome. Then, a local search method is started from each of

these solutions independently, as depicted in Figure 1. Since the local search requires a single objective, an aggregated objective function can be formed for each non-dominated solution. The authors suggested one such aggregate objective function in an earlier study [5]. We use the identical objective function here, although the concept can be used for other more generic aggregate functions, such as the Tchebycheff function etc. [2].

As the figure illustrates, the location of each solution in the objective space is used to form a weighted objective function. First, from the extreme non-dominated solutions, a pseudo-weight vector is derived for each intermediate solution. Thereafter, a weighted function is formed using the pseudo-weight vector, providing a direction of search in the objective space. Each solution is thus directed in a different direction and the best solution is found by a local search method.
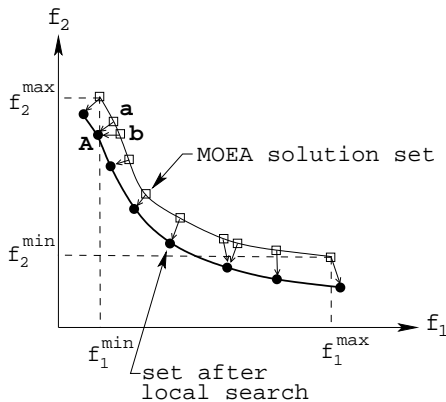

Figure 1: Posteriori approach is illustrated.

Here, the action of MOEAs and the local search method are independent and sequential. The overall computational effort (and function evaluations) is a simple addition of individual computational effort (and function evaluations) allocated to each approach.

2.2 Online Approach

Other extreme of a hybrid approach is the use of a local search procedure in an MOEA. Every solution created by the genetic operators in an MOEA, is modified by the local search procedure before being accepted. Figure 2 illustrates the online approach with randomly-created search directions for the local search approach. Once again a combined objective is used in such a case. Before accepting a solution generated by MOEAs, it is iterated with the local search approach for better convergence properties. Here, the local search approach is embedded in the MOEA. The overall computational effort (and function evaluations) will depend on the extent of local search method applied to each generated solution. Ishibuchi and Murata [6] has implemented one such idea.
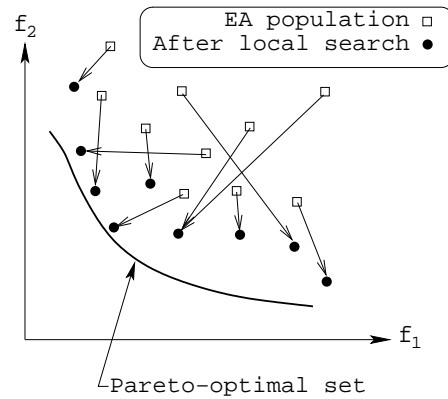

Figure 2: Online approach is illustrated.

The two methods discussed above are the two extreme cases. However, there may exist some other ways of combining a local search and an evolutionary method. The need is to find a balance between the extent of local search and the evolutionary method. The optimal setting of their extent will reduce the computational effort needed in solving a problem.

2.3 Relative Importance

In real-world problem solving, the computational time required for evaluating a solution (we call a function evaluation here) is the main concern for an optimum procedure. For the comparison of the above two approaches, it is, therefore, necessary to allocate a fixed overall number of function evaluations to each approach and then compare the quality of obtained solutions. The quality of solutions obtained by both MOEAs and the local search method will depend on the number of iterations allocated to them. Here, we estimate the iterations to be allocated to each approach in both hybrid approaches.

2.3.1 Posteriori Approach

Let the allowed number of overall function evaluations be F. We allocate F_C number of functions evaluations to each individual during each local search approach. Then, the number of function evaluations that can be allocated to the MOEA will be F_{GA}. Then, we have

$$F_{\mathrm{GA}} = F - F_C \times N. \qquad (1)$$

In each generation, an MOEA takes N functions evaluations. Thus, the maximum number of generations (t_{post}) that can be allowed to the MOEA are given by

$$t_{\mathrm{post}} = \frac{F_{\mathrm{GA}}}{N} = \frac{F - N \times F_c}{N} = \frac{F}{N} - F_C. \qquad (2)$$

It is intuitive that if the extent of local search (or F_C) is more, MOEA is not allocated many iterations (or t_{post} is less).

2.3.2 Online Approach

Here, in each generation of an evolutionary method, we apply the local search on each individual. Then the total number of function evaluations in each generation F_{gen} is given as follows:

$$F_{\text{gen}} = N \times F_C. \tag{3}$$

Thus, an MOEA is allocated

$$t_{\text{online}} = \frac{F}{N \times F_C} \tag{4}$$

number of iterations. This equation shows that the number of MOEA generations in the online approach depends largely on the number of function evaluations to the local search. If F_C is large, t_{online} will be small. In such a case, this method may not allow the evolutionary search to play a major role in the search for optimal solutions.

The simple comparison of equation 2 and equation 4 is given in the following:

$$\frac{t_{\text{post}}}{t_{\text{online}}} = \frac{F_C \times F_{\text{GA}}}{F}. \tag{5}$$

For $F_C \times F_{GA} > F$, we have $t_{\text{post}} > t_{\text{online}}$. Since an optimization method (whether a local search method or an EA) requires a substantial number of function evaluations to choose and compare solutions in finding a near-optimal solution, the product of F_C and F_{GA} is likely to be more than the allocated number of function evaluations, particularly in solving real-world problems. In such situations, the posteriori approach allows more number of MOEA generations than what would be allowed in the online approach.

3 Optimal Engineering Shape Design

We have chosen a number of shape optimization problems to compare the performance of the above two hybrid approaches. Most of the previous studies on the shape design make use of mathematical functions to define the shape of the component and then various parameters of the function are optimized. This method is found to be suited to the classical optimization methods. However, after the advent of genetic algorithms based techniques, there exist a number of studies on optimal shape design where shapes are represented by the presence or absence of the small material elements [1, 4]. The predefined area or the volume is discretized into a number of small regular elements and the task of optimization is to keep some or throw some of these elements which make the shape optimal with respect to the objective function.

Here only two dimensional cases are studied. A rectangular plate is used as a base plate. This is divided into a finite number of small elements (refer to Figure 3). Since the shape is represented by the presence or absence of the elements, which also act as a binary decision variable. The binary string
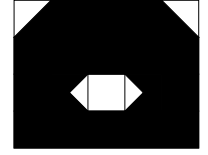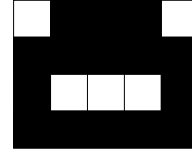

Figure 3: Rectangular plate divided into small elements.

Figure 4: The skeleton of a shape.

Figure 5: Final smoothen-ed shape.

corresponding to the shape shown in the Figure 4 is given as following :

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01110 11111 10001 11111
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Here the presence of the element is denoted by a 1 and the absence is shown by a 0. A left-to-right coding procedure as shown in Figure 3 is followed here. Since the strings are generated at random, it is possible to have some disconnected regions in the rectangular plate. For this case, we use the biggest cluster of the given shape of connected elements (where two elements are defined to be connected if they have at least one common corner). The string is repaired by assigning a 0 at all elements which are not part of the biggest cluster (Lamarckian approach). The shape obtained after finding the connected region are smoothened using triangular elements.

The shapes represented by the binary strings are evaluated by the finite element analysis. For this the shape is further divided into small triangular elements, as we take constant strain triangle as the element for finite element analysis. All the interior elements are divided into two triangles and all the boundary elements (including the elements around a hole) are divided into four small triangles. The boundary triangles are also divided into smaller triangles. The linear shape functions in natural co-ordinates and iso-parametric elements are used.

The maximum stresses and the maximum displacements developed in the body under the action of loads are calculated. In all the applications here: weight and deflection, are taken as two minimization-type objectives. These are two conflicting objectives because a shape with a very small weight produces a large deflection and a shape with densely packed elements (large weight) tends to produce a very small deflection. The maximum stress and deflection developed in the body are restricted to lie within the specified limits decided by the user, by using them as the constraints.

4 Simulation Results

We have used NSGA-II [3] as the MOEA in both hybrid approaches. Since the binary-coded strings are used, we use a bit-wise hill-climbing strategy as the local search approach. We start from the left of the string and flipped every bit, one at a time, to see if it improves the design. If it does, the change is accepted otherwise the bit is restored. For the local search approach, we have considered following three levels:

LS 1 Here, we perform the local search till we reach the end of the binary string.

LS 2 After reaching the end of string we again begin flipping the first bit of the string and move unless we reach the end.

LS 3 Here, the hill-climbing is repeated three times on the complete string.

In all the simulation runs, we have used the following GA parameters.

Population Size	: 30
Crossover Prob.	: 0.95
Mutation Probability	: 1/String Length
Function Evaluations	: $F = 10,000$

For all the problems we have used following material properties:

Plate thickness	: 50 mm
Yield strength	: 150 MPa
Young's modulus	: 200 GPa
Poisson's ratio	: 0.25

4.1 Cantilever Plate

The first problem taken is the design of the cantilever plate, where an end load of $P = 10$ kN is applied. The rectangular plate of the size 60×100 mm^2 is divided into small rectangular elements. Hence, a 60-bit string is taken to represent the shape of the cantilever plate. The problem is solved by both the approaches. The results obtained with the posteriori approach are presented in the Figure 6. The figure shows that
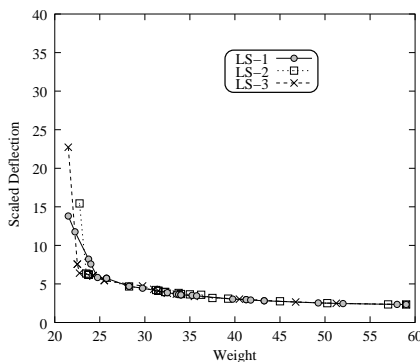

Figure 6: Solutions obtained by using different local search degrees in the posteriori approach for the cantilever plate design problem.

increasing the importance of local search does not have too much effect on the overall performance, except near the minimum weight solutions. With more function evaluations allocated to the local search, better solutions are obtained near the minimum weight solutions. A gradual addition or deletion of
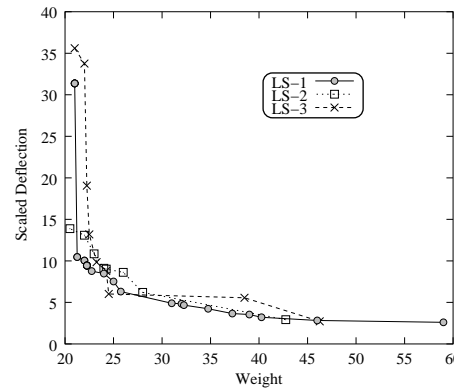

Figure 7: Solution set obtained by using different local search degrees in the online approach for cantilever plate design problem.

crucial elements becomes necessary to find a near minimum weight solution. The steep slope of the non-dominated solutions near minimum weight solutions indicates that removal of one element from the structure can be allowed with a large sacrifice on the deflection, but the removal of the right element is important. A local search is ideal to obtain such a gradual search procedure.

With the online approach, the results are also interesting. Figure 7 shows the non-dominated fronts with the importance of the local search method. As the level of local search is increased, the performance of the online approach deteriorates. In the first level of local search, the emphasis on the evolutionary method is more than that in the third level of local search. The non-dominated front obtained by the third level of local search has worse diversity and convergence than that in the first level of local search. This is entirely different as that obtained in the posteriori method. With a smaller extent of local search, an EA effectively gets more iterations. In this problem, the combination of one round of bit-flipping allows an adequate number of generations for the EA to find a better non-dominated front.
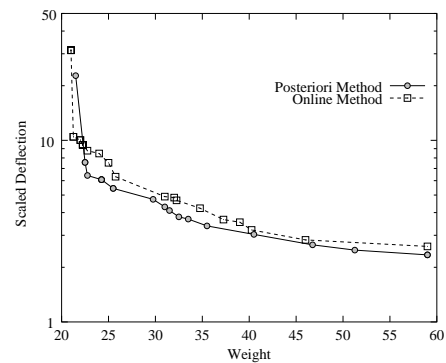

Figure 8: Non-dominated fronts obtained by the best of the posteriori (LS-3) and the online approach (LS-1) on the cantilever beam design problem.

Figure 8 shows the comparison of results obtained by the best of both methods. The posteriori method with the third level of local search is compared with the online approach having the first level of local search. It is clear that the posteriori method has a better diversity and the convergence. However, for small values of weights the convergence of the online approach based search is better.

4.2 Simply-Supported Plate

Next problem is the design of a simply-supported plate. The initial size of the plate is same as that taken in previous problem. A vertical load of $P = 10$ kN is applied on the top middle node of the plate. The results obtained from the two approaches are shown in Figure 9 and Figure 10. In the
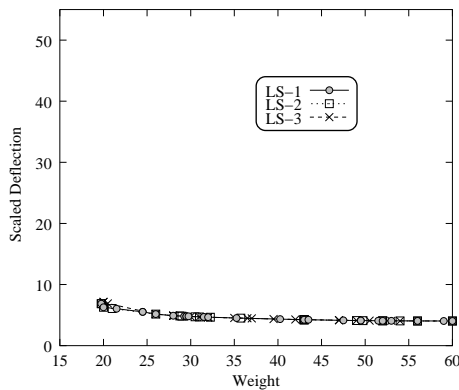
Figure 9: Solutions obtained for simply-supported plate problem by using different local search levels in the posteriori approach.
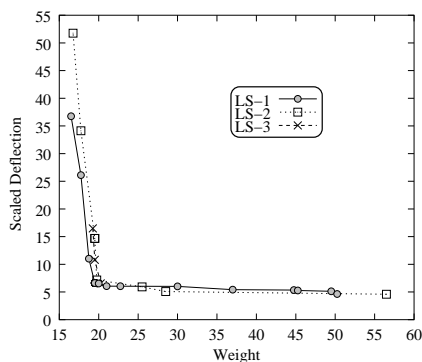
Figure 10: Solution set obtained for simply-supported plate design problem by using different local search levels in the online approach.

posteriori approach, the results obtained by using different level of local search based optimization methods does not show much difference. The spread of solutions obtained in all methods is almost equal. The results obtained by the online approach here also follow a similar trend as in the first

problem. The Pareto-optimal front obtained by the first level of local search and the front obtained by the second level of local search dominates each other in some region and get dominated by other in some other region. However, the performance of the third level of local search method is worst in terms of achieving both diversity and convergence. With the online approach, a small level of local search is better.
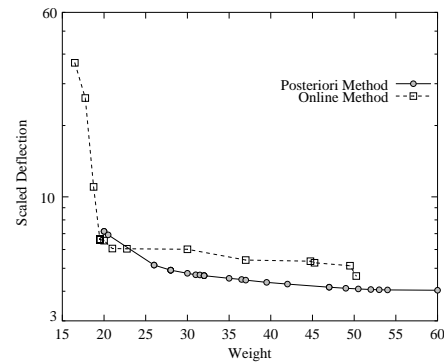
Figure 11: Non-dominated fronts obtained by the best of the posteriori (LS-3) and the online approach (LS-1) on the simply-supported beam design problem.

Figure 11 shows the performance of the best of both approaches. It is clear that the posteriori approach have a better convergence and diversity in a wide range, but for very small weight cases the online approach is able to find better solutions.

4.3 Hoister Plate

Next problem is the design of a weight lifting hoister. The basic design shape again is a rectangular plate. The size of the plate is taken to be 80×60 mm^2. The plate is divided into 48 elements. A hoisting load of 5 kN is applied. Figure 12
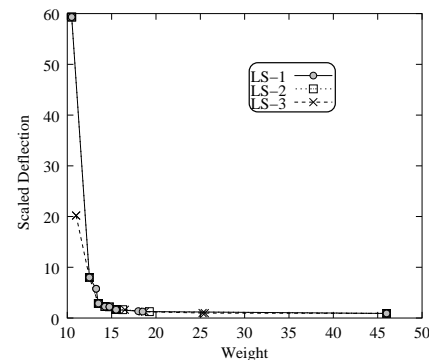
Figure 12: Solutions obtained for hoister design problem by using different local search levels in posteriori approach.

shows the performance of different levels of local search. The non-dominated front for the third level of local search method
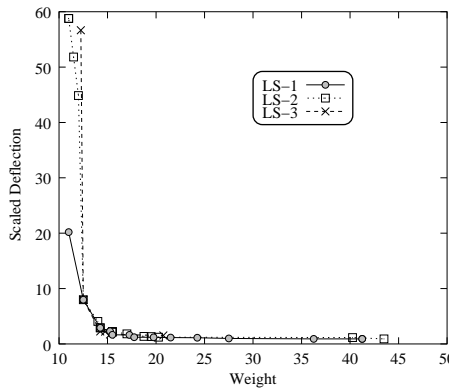
Figure 13: Solution set obtained by using different local search levels in online approach for hoister design problem.

seems to have slightly better convergence than that obtained with the other local searches. Figure 13 shows the relative performance of different level of local search with the online approach. The first level of local search has got the best convergence and reasonably good diversity among the three. However, the results are not as diverse as those obtained by the posteriori approach.
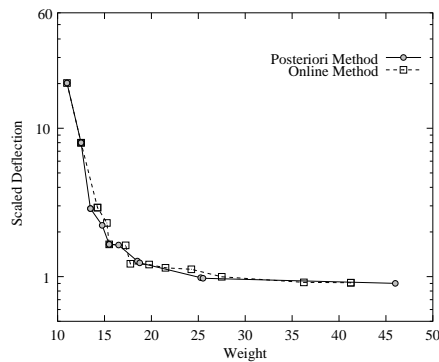

Figure 14: Non-dominated fronts obtained by the best of the posteriori (LS-3) and the online approach (LS-1).

Figure 14 shows the results of the best of the two approaches. The posteriori approach with an extended local search approach has a better convergence than the online approach with the smallest level of local search.

5 Conclusion

Simulation results on three engineering design problems show that the posteriori approach of hybridization is better than the online approach, as the former can obtain better convergence as well as better diversity. The main reason of deterioration of performance of online approach as the emphasis on the local search is increased, is the little emphasis allocated to EA and more emphasis allocated to the local search method. It is also clear from the results that the optimum bal-

ance between the local search and the evolutionary search is essential to achieve the best results – good diversity and convergence to the global Pareto-optimal front. However, it is also evident that the hybrid methods are candidates of being a good and robust algorithm for solving the real-world multi-objective optimization problems.

Acknowledgements

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