Deformation of a layered half-space due to a very long tensile fault

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The problem of the coseismic deformation of an earth model consisting of an elastic layer of uniform thickness overlying an elastic half-space due to a very long tensile fault in the layer is solved analytically. Integral expressions for the surface displacements are obtained for a vertical tensile fault and a horizontal tensile fault. The integrals involved are evaluated approximately by using Sneddon's method of replacing the integrand by a finite sum of exponential terms. Detailed numerical results showing the variation of the displacements with epicentral distance for various source locations in the layer are presented graphically. The displacement field in the layered half-space is compared with the corresponding field in a uniform half-space to demonstrate the effect of the internal boundary. Relaxed rigidity method is used for computing the postseismic deformation of an earth model consisting of an elastic layer of uniform thickness overlying a viscoelastic half-space.

1. Introduction

Tensile fault representation has several important geophysical applications, such as modelling of the deformation fields due to dyke injection in the volcanic region, mine collapse and fluid-driven cracks. Bonafede and Rivalta (1999a) provided analytical solution for the elementary tensile dislocation problem in an elastic medium composed of two welded half-spaces. Subsequently, Bonafede and Rivalta (1999b) derived the solution for the elastic field produced by a vertical tensile crack, opening under the effect of an assigned overpressure within it, in the proximity of the welded boundary between two half-spaces characterized by different elastic parameters.

Weeks *et al* (1968) showed how to express as Fourier integrals the displacement and stress fields due to an edge dislocation in a substrate overlain by an elastic layer in welded contact. Following Weeks *et al* (1968), Savage (1998) obtained the displacement field for an edge dislocation in an earth model consisting of a layer welded to a half-space in the form of a Fourier integral. He used the symbolic computational capabilities of the program MATHEMATICA (Wolfram 1991) to do the algebra and the numerical integration routines in that program to calculate the Fourier integral. In a recent paper, Wang *et al* (2003) gave FORTRAN programs for computation of deformation induced by earthquake sources in a multilayered half-space.

The Airy stress function due to various two-dimensional sources in an unbounded medium was obtained by Singh and Garg (1986). Using these results, Singh *et al* (1997) solved analytically the two-dimensional problem of the deformation of a layered half-space caused by a very long dip-slip fault situated in the overlying layer. The integrals involved were evaluated approximately by replacing the integrand by a finite sum of exponential terms (Sneddon 1951; Ben-Menahem and Gillon 1970).

In the present paper, we consider a homogeneous, isotropic, elastic layer of uniform thickness overlying a homogeneous, isotropic, elastic half-space. We consider a plane strain problem with a very long tensile fault in the layer. It is assumed that the surface of the layer is traction-free and the

Keywords. Coseismic deformation; layered half-space; postseismic deformation; relaxed rigidity; tensile fault.

Proc. Indian Acad. Sci. (Earth Planet. Sci.), **113**, No. 2, June 2004, pp. 235–246

layer and the half-space are in welded contact. The method used by Singh *et al* (1997) is employed to calculate the displacement field caused by horizontal and vertical tensile faults. The variation of the displacement field with the distance from the fault for different source depths is studied numerically. The displacement fields for a uniform half-space and a layered half-space are compared.

Viscoelastic relaxation is generally regarded as the principal source of postseismic deformations. Displacements produced postseismically by time-dependent linear viscoelastic deformation may be calculated by using the correspondence principle. However, in this procedure, a large number of forward and inverse Laplace transforms are involved. Cohen (1980) calculated the displacement field for a vertical rectangular strike-slip fault buried in a viscoelastic layer overlying a viscoelastic half-space by solving the displacement equations for a purely elastic problem and then replacing the elastic moduli by relaxed moduli appropriate to the viscoelastic case. Rundle (1981) used a similar approach for a rectangular thrust fault in a layered model.

Ma and Kusznir (1995) calculated the postseismic displacements and strains for a rectangular dip-slip fault in a three-layer medium using both the correspondence principle and relaxed moduli method, and found that the results were in close agreement. According to Ma and Kusznir (1995), the relaxed moduli method provides an accurate and computationally efficient method of examining postseismic relaxation.

In the present paper, we use the relaxed moduli method for calculating the postseismic deformation of an elastic layer of uniform thickness (representing lithosphere) overlying a viscoelastic half-space (representing asthenosphere) caused by a very long tensile fault in the layer. We assume that the half-space is elastic in dilatation and viscoelastic in distortion. Consequently, only the rigidity μ_2 of the half-space relaxes with time. For the continental earth model $\nu = \mu_2/\mu_1 = 2.22$. We have considered four values for the rigidity of the half-space which correspond to $\nu = 2.22,\ 2.22/5,\ 2.22/10,\ 2.22/20$. The value $\nu = 2.22$ refers to the crust-mantle transition and corresponds to the coseismic deformation and the values $\nu = 2.22/5,\ 2.22/10,\ 2.22/20$ correspond to the postseismic deformation for various points of time. Displacements due to both horizontal and vertical tensile faults are evaluated numerically for the continental earth model for the four values of ν mentioned above.

2. Basic equations

We consider a model consisting of a homogeneous, isotropic, elastic layer of uniform thickness H overlying a homogeneous, isotropic, elastic half-space (figure 1). We place the origin of the Cartesian coordinate system $(x_1,\ x_2,\ x_3) \equiv (x, y, z)$ at the free surface with the x_3-axis vertically downwards. Let λ_1, μ_1 and λ_2, μ_2 be the Lamé constants for the layer and the half-space, respectively. We consider the plane strain problem in which $\partial/\partial x_1 \equiv 0$. The sources considered in this paper are infinitesimally thin strips, delimited by two dislocation lines at an infinitesimal distance ds from each other, with opposite Burgers vectors. The solution for a dislocation of finite thickness can be obtained by the superposition of solutions for thin strips. Let there be a long dislocation parallel to the x_1-axis passing through the point (0, 0, h) of the layer (figure 1). As shown by Singh and Garg (1986), the Airy stress function U_0 for a long dislocation parallel to the x_1-axis passing through the point (0, 0, h) in an infinite medium can be expressed in the form

$$U_0 = \int_0^\infty [(L_0 + M_0 k|z-h|)\sin ky + (P_0 + Q_0 k|z-h|)\cos ky] k^{-1} e^{-k|z-h|} dk, \tag{1}$$

where the source coefficients L_0, M_0, P_0 and Q_0 are independent of k. Singh and Garg (1986) have obtained these source coefficients for various sources.

For a long dislocation parallel to the x_1-axis acting at the point (0, 0, h) of the layer ($h < H$), the expressions for the Airy stress function for the layer, $U^{(1)}$, and the half-space, $U^{(2)}$, are of the form

$$U^{(1)} = U_0 + \int_0^\infty [(L_1 + M_1 kz)\sin ky + (P_1 + Q_1 kz)\cos ky]\, k^{-1} e^{-kz} dk + \int_0^\infty [(L_2 + M_2 kz)\sin ky + (P_2 + Q_2 kz)\cos ky]\, k^{-1} e^{-kz} dk, \tag{2}$$

$$U^{(2)} = \int_0^\infty [(L_3 + M_3 kz)\sin ky + (P_3 + Q_3 kz)\cos ky]\, k^{-1} e^{-kz} dk, \tag{3}$$

where U_0 is given in equation (1) and the unknowns L_1, M_1, etc. are to be determined from the boundary conditions.

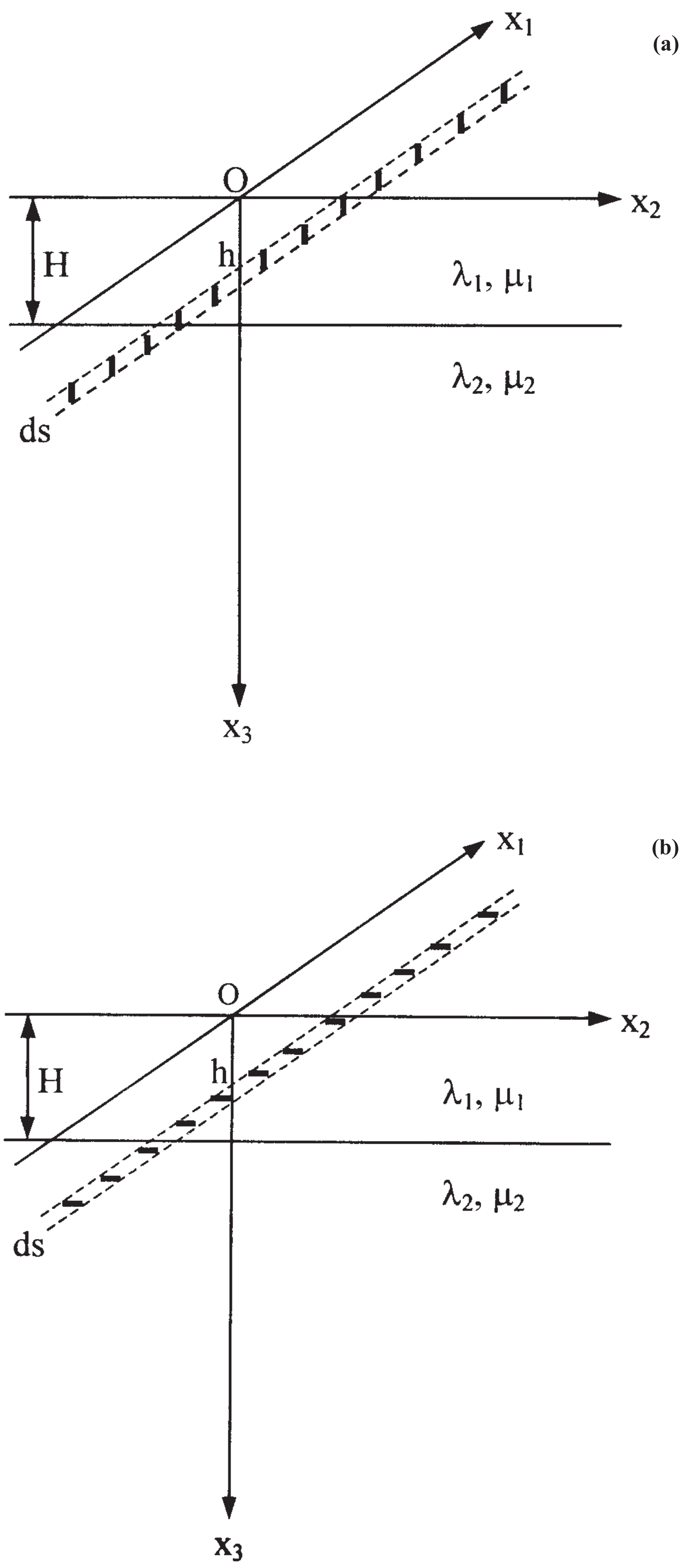


Figure 1. Layer of uniform thickness H overlying a half-space with a dislocation of width ds and of infinite length parallel to the x_1-axis passing through the point (0,0, h) representing a **(a)** vertical fault; **(b)** horizontal fault.

The stresses and displacements in terms of the Airy stress function are given by (Sokolnikoff 1956)

$$\tau_{22}^{(m)} = \frac{\partial^2 U^{(m)}}{\partial z^2}, \quad \tau_{23}^{(m)} = -\frac{\partial^2 U^{(m)}}{\partial y \partial z}, \quad \tau_{33}^{(m)} = \frac{\partial^2 U^{(m)}}{\partial y^2}, \tag{4}$$

$$2\mu_m\, u_2^{(m)} = -\frac{\partial U^{(m)}}{\partial y} + \frac{1}{2\alpha_m}\int \nabla^2 U^{(m)} dy,$$

$$2\mu_m\, u_3^{(m)} = -\frac{\partial U^{(m)}}{\partial z} + \frac{1}{2\alpha_m}\int \nabla^2 U^{(m)} dz, \tag{5}$$

where

$$\alpha_m = \frac{\lambda_m + \mu_m}{\lambda_m + 2\mu_m} = \frac{1}{2(1-\sigma_m)}, \tag{6}$$

$$\nabla^2 U^{(m)} = \tau_{22}^{(m)} + \tau_{33}^{(m)}, \tag{7}$$

σ being the Poisson's ratio. In equations (4) – (7), summation is not taken over m; $m = 1$ is for the layer and $m = 2$ is for the half-space.

We assume that the surface of the layer ($z = 0$) is traction-free and the layer and the half-space are in welded contact along the plane $z = H$ yielding the boundary conditions

$$\tau_{23}^{(1)} = \tau_{33}^{(1)} = 0 \quad \text{at } z = 0,$$

$$\tau_{23}^{(1)} = \tau_{23}^{(2)}, \tau_{33}^{(1)} = \tau_{33}^{(2)} \text{ at } z = H,$$

$$u_2^{(1)} = u_2^{(2)}, u_3^{(1)} = u_3^{(2)} \text{ at } z = H. \tag{8}$$

Let L^-, M^-, P^-, Q^- be the values of L_0, M_0, P_0, Q_0, respectively, valid for $z < h$. Inserting the expressions for the stresses and displacements obtained from equations (1) – (5) into the boundary conditions (8), we obtain two sets of equations, each containing six equations in six unknowns. In one set the unknowns are $L_1, L_2, L_3, M_1, M_2, M_3$ and in the other set the unknowns are $P_1, P_2, P_3, Q_1, Q_2, Q_3$. These two sets of equations have been solved by Singh *et al* (1997). Using the values of the constants L_1, L_2 etc., equations (1) – (5) yield the following expressions for the displacements at $z = 0$ (Singh *et al* 1997)

$$\begin{aligned}
2\mu_1 u_2^{(1)} = \int_0^\infty \Bigg[\Bigg\{ & \frac{1}{\alpha_1 \Delta_0}\big[\{2\delta Z_3(L^- + M^- kh) \\
& -\delta Z_3 M^-\}\, e^{-kh} - 2\delta Z_2\{(L^+ - M^+ kh) \\
& - M^+\} e^{-k(4H-h)} + \delta Z_2 M^- e^{-k(4H+h)} \\
& + \delta^2 Z_1\{2(1+2kH)(L^- + M^- kh) \\
& - 4kHM^-\} e^{-k(2H+h)} \\
& + \{2\delta^2 Z_1(2kH-1)(L^+ - M^+ kh) \\
& + (Z_4 + \delta^2 Z_1(4k^2H^2 - 4kH + 1))M^+\} \\
& \times\, e^{-k(2H-h)}\big] + \frac{M^-}{\alpha_1} e^{-kh}\Bigg\} \cos ky \\
& - \Bigg\{\frac{1}{\alpha_1 \Delta_0}\big[\{2\delta Z_3(P^- + Q^- kh) \\
& - \delta Z_3 Q^-\} e^{-kh} - 2\delta Z_2\{P^+ - Q^+ kh - Q^+\} \\
& \times e^{-k(4H-h)} + \delta Z_2 Q^- e^{-k(4H+h)} \\
& + \delta^2 Z_1\{2(1+2kH)(P^- + Q^- kh) \\
& - 4kHQ^-\} e^{-k(2H+h)} + \{2\delta^2 Z_1(2kH-1) \\
& \times (P^+ - Q^+ kh) + (Z_4 + \delta^2 Z_1(4k^2H^2 \\
& - 4kH + 1))Q^+\} e^{-k(2H-h)}\big] \\
& + \frac{Q^-}{\alpha_1} e^{-kh}\Bigg\} \sin ky\Bigg]\, dk,
\end{aligned} \tag{9}$$

$$\begin{aligned}
2\mu_1 u_3^{(1)} = \int_0^\infty \Bigg[\Bigg\{ & \frac{1}{\alpha_1 \Delta_0}\big[\{2\delta Z_3(L^- + M^- kh) \\
& -\delta Z_3 M^-\}\, e^{-kh} + 2\delta Z_2(L^+ - M^+ kh) \\
& \times e^{-k(4H-h)} - \delta Z_2 M^- e^{-k(4H+h)} \\
& + \delta^2 Z_1\{2(1-2kh)(L^- + M^- kh) \\
& - 2M^-\} e^{-k(2H+h)} + \{2\delta^2 Z_1(2kH+1) \\
& \times (L^+ - M^+ kh) + (Z_4 + \delta^2 Z_1(4k^2H^2 \\
& -1))M^+\} e^{-k(2H-h)}\big] - (M^-/\alpha_1) e^{-kh}\Bigg\} \\
& \times \sin ky + \Bigg\{\frac{1}{\alpha_1 \Delta_0}\big[\{2\delta Z_3(P^- + Q^- kh) \\
& - \delta Z_3 Q^-\} e^{-kh} + 2\delta Z_2(P^+ - Q^+ kh)
\end{aligned}$$

$$\times\, e^{-k(4H-h)} - \delta Z_2 Q^- e^{-k(4H+h)}$$

$$+ \delta^2 Z_1\{2(1-2kH)(P^- + Q^- kh) - 2Q^-\}$$

$$\times\, e^{-k(2H+h)} + \{2\delta^2 Z_1(2kH+1)$$

$$\times\, (P^+ - Q^+ kh)$$

$$+ (Z_4 + \delta^2 Z_1(4k^2H^2 - 1))Q^+\} \times e^{-k(2H-h)}$$

$$\left. \left. -(Q^-/\alpha_1)e^{-kh} \right\} \cos ky \right] dk, \tag{10}$$

where

$$\Delta_0 = -[\delta Z_3 + (Z_4 + \delta^2 Z_1 + 4\delta^2 k^2 H^2 Z_1)e^{-2kH}$$

$$+ \delta Z_2 e^{-4kH}], \tag{11}$$

$$1/\delta = (2/\alpha_1) - 1 = 3 - 4\sigma_1,$$

$$1/\delta_1 = (2/\alpha_2) - 1 = 3 - 4\sigma_2, \quad \nu = \mu_2/\mu_1,$$

$$Z_1 = 4(\nu - 1)(\nu\delta_1 + 1), \quad Z_2 = 4(\nu - 1)(\nu\delta_1 - \delta),$$

$$Z_3 = 4(\nu + \delta)(\nu\delta_1 + 1), \quad Z_4 = 4(\nu + \delta)(\nu\delta_1 - \delta). \tag{12}$$

3. Displacement field due to a tensile fault

We consider two tensile faults: a vertical tensile fault with dislocation in the x_2-direction and a horizontal tensile fault with dislocation in the x_3-direction.

3.1 *Vertical tensile fault*

The source coefficients for a vertical tensile fault with dislocation in the x_2-direction are (Singh and Rani 1991)

$$L^- = L^+ = M^- = M^+ = 0,$$

$$P^- = P^+ = \frac{\alpha_1 \mu_1 bds}{\pi}, \quad Q^- = Q^+ = -\frac{\alpha_1 \mu_1 bds}{\pi},$$

where b is the displacement discontinuity in the direction of the normal to the fault and ds is the width of the line fault. On putting the source coefficients into equations (9) and (10), the integral expressions for the surface displacements are found to be

$$u_2^{(1)} = -\frac{bds}{2\pi}\int_0^\infty \left[\frac{1}{\Delta_0}\{\delta Z_3(3-2kh)e^{-kh}\right.$$

$$- 2\delta Z_2(2+kh)e^{-k(4H-h)} - \delta Z_2 e^{-k(4H+h)}$$

$$+ 2\delta^2 Z_1(1 - kh + 4kH - 2k^2hH)e^{-k(2H+h)}$$

$$+ (\delta^2 Z_1(8kH + 4k^2hH - 2kh - 4k^2H^2 - 3)$$

$$\left. -Z_4)e^{-k(2H-h)}\} - e^{-kh}\right] \sin ky\; dk, \tag{13}$$

$$u_3^{(1)} = \frac{bds}{2\pi}\int_0^\infty \left[\frac{1}{\Delta_0}\{\delta Z_3(3-2kh)e^{-kh}\right.$$

$$+ 2\delta Z_2(1+kh)e^{-k(4H-h)} + \delta Z_2 e^{-k(4H+h)}$$

$$+ 2\delta^2 Z_1(2 - kh - 2kH + 2k^2hH)e^{-k(2H+h)}$$

$$+ (\delta^2 Z_1(4kH + 4k^2hH + 2kh - 4k^2H^2 + 3)$$

$$\left. -Z_4)e^{-k(2H-h)}\} + e^{-kh}\right] \cos ky\; dk. \tag{14}$$

3.2 *Horizontal tensile fault*

The source coefficients for a horizontal tensile fault with dislocation in the x_3-direction are (Singh and Rani 1991)

$$L^- = L^+ = M^- = M^+ = 0,$$

$$P^- = P^+ = Q^- = Q^+ = \frac{\alpha_1 \mu_1 bds}{\pi}.$$

On inserting the source coefficients into equations (9) and (10), the integral expressions for the surface displacements are found to be

$$u_2^{(1)} = -\frac{bds}{2\pi}\int_0^\infty \left[\frac{1}{\Delta_0}\{\delta Z_3(1+2kh)e^{-kh}\right.$$

$$+ 2\delta Z_2 kh e^{-k(4H-h)} + \delta Z_2 e^{-k(4H+h)}$$

$$+ 2\delta^2 Z_1(1 + kh + 2k^2hH)e^{-k(2H+h)}$$

$$+ (\delta^2 Z_1(4k^2H^2 + 2kh - 4k^2hH - 1)$$

$$\left. + Z_4)e^{-k(2H-h)}\} + e^{-kh}\right] \sin ky\; dk, \tag{15}$$

$$u_3^{(1)} = \frac{bds}{2\pi}\int_0^\infty \left[\frac{1}{\Delta_0}\{\delta Z_3(1+2kh)e^{-kh}\right.$$

$$+ 2\delta Z_2(1-kh)e^{-k(4H-h)} - \delta Z_2 e^{-k(4H+h)}$$

$$+ 2\delta^2 Z_1(kh - 2kH - 2k^2hH)e^{-k(2H+h)}$$

$$+ (\delta^2 Z_1(4k^2H^2 + 4kH - 4k^2hH - 2kh + 1)$$

$$+ Z_4)e^{-k(2H-h)}\} - e^{-kh}\Big] \cos ky \; dk. \tag{16}$$

4. Particular case – uniform half-space

Taking the limit $H \to \infty$ in the expressions for the displacement components for the layer of thickness H over a uniform half-space, one can obtain the expressions for the surface displacement components for a uniform half-space. Thus, from equations (13) – (14), we get the following expressions for the displacement components due to a vertical tensile fault

$$u_2 = \frac{bds}{\pi} \int_0^\infty (2 - kh)e^{-kh} \sin ky \; dk, \tag{17a}$$

$$u_3 = -\frac{bds}{\pi} \int_0^\infty (1 - kh)e^{-kh} \cos ky \; dk, \tag{17b}$$

where the superscript (1) is deleted because there is no layer. On integrating, we get

$$u_2 = \frac{bds}{\pi} \left[\frac{2y^3}{(h^2 + y^2)^2} \right], \tag{18}$$

$$u_3 = -\frac{bds}{\pi} \left[\frac{2hy^2}{(h^2 + y^2)^2} \right]. \tag{19}$$

Similarly, taking the limit $H \to \infty$ in equations (15) – (16), the displacement components caused by a horizontal tensile fault in a uniform half-space are

$$u_2 = \frac{bds}{\pi} \int_0^\infty hk \; e^{-kh} \sin ky \; dk, \tag{20a}$$

$$u_3 = -\frac{bds}{\pi} \int_0^\infty (1 + kh)e^{-kh} \cos ky \; dk. \tag{20b}$$

Integrating, we obtain

$$u_2 = \frac{bds}{\pi} \left[\frac{2h^2y}{(h^2 + y^2)^2} \right], \tag{21}$$

$$u_3 = -\frac{bds}{\pi} \left[\frac{2h^3}{(h^2 + y^2)^2} \right]. \tag{22}$$

5. Numerical results and discussion

The integrals appearing in equations (13) – (16) are of the form

$$\int_0^\infty \frac{G}{\Delta_0} e^{-kp} k^q \begin{pmatrix} \cos ky \\ \sin ky \end{pmatrix} dk, \tag{23}$$

where $q = 0, 1, 2$; $G = -\delta Z_3$; $p = h, 2H \pm h, 4H \pm h$. The occurrence of the factor $1/\Delta_0$ in the integrand makes integration by analytical methods difficult. Even numerical integration is not convenient. Following Singh *et al* (1997), we use the approximation

$$\frac{G}{\Delta_0} \approx 1 - (A + Bk^2H^2)e^{-2kH}$$
$$+ (C + \alpha k^n H^n)e^{-\beta kh}, \tag{24}$$

where

$$A = (Z_4 + \delta^2 Z_1)/\delta Z_3, \quad B = 4\delta Z_1/Z_3,$$
$$C = \frac{A^2 + D(A - 1)}{1 + A + D}, \tag{25}$$
$$D = Z_2/Z_3, \quad n = 1, 2, 3 \ldots$$

and $\alpha, \beta (> 2)$ are chosen in such a way so as to ensure a satisfactory fit. The constants α, β and n are to be re-evaluated for each set of values of the parameters σ_1, σ_2 and ν. Using the approximation (24), the integral (23) can be expressed as a linear combination of known integrals. Ben-Menahem and Gillon (1970) found that $n = 2$ yields a satisfactory approximation for $\nu = 2.22$ which corresponds to the continental earth model. The corresponding values of the constants α and β for $\sigma_1 = \sigma_2 = 0.25$ are given in table 1.

Table 1. .

$\nu = \mu_2/\mu_1$	n	α	β
2.22	2	0.495	2.978
2.22/5	3	6.846	4.652
2.22/10	3	36.208	4.795
2.22/20	4	802.922	6.932

We study numerically the variation of the displacement field at the surface with distance from the fault, caused by a vertical tensile fault and a horizontal tensile fault. The effect of source location is also studied. For numerical computation, we define the following dimensionless quantities

$$Y = y/h, \quad T = H/h,$$

$$U_2 = \frac{\pi h}{bds} u_2^{(1)}, \quad U_3 = \frac{\pi h}{bds} u_3^{(1)},$$

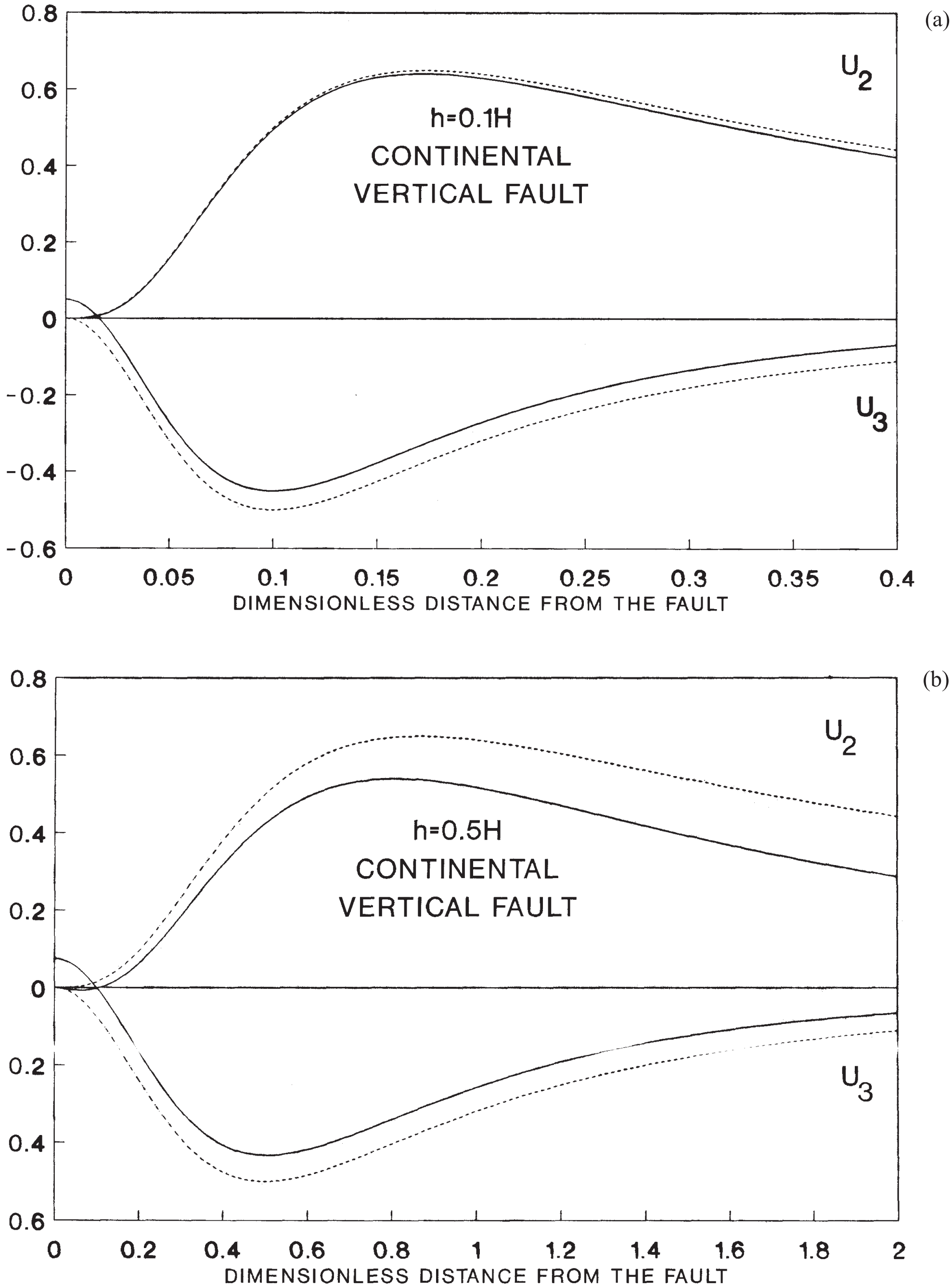


Figure 2 (a–b). (*Continued*)

where h is the distance of the line source from the surface.

Figures 2(a, b, c) show the variation of the dimensionless horizontal displacement U_2 and vertical displacement U_3 at the surface with the dimensionless distance from the fault caused by a vertical tensile fault. These figures are for the continental earth model at three source depths: $h = 0.1H, 0.5H$ and $0.9H$. Broken lines show the variation of the displacement components when the medium is a uniform half-space and the continuous lines show the variation of the displacement components in the case of a layer of uniform thickness H overlying a uniform half-space. The source in the case of layered half-space is in the layer passing through the point (0, 0, h). In the case of a uniform half-space, $U_2 = U_3 = 0$ at $Y = 0$ which is also endorsed by equations (18) and (19). U_2 and U_3 keep the same sign in the case of a uniform half-space. However, in the case of layered half-space, near the origin, U_3 is positive (subsidence) when the source is near the free surface and U_3 is negative (uplift) when the source is near the interface. Figures 2(a, b, c) also show that when the source

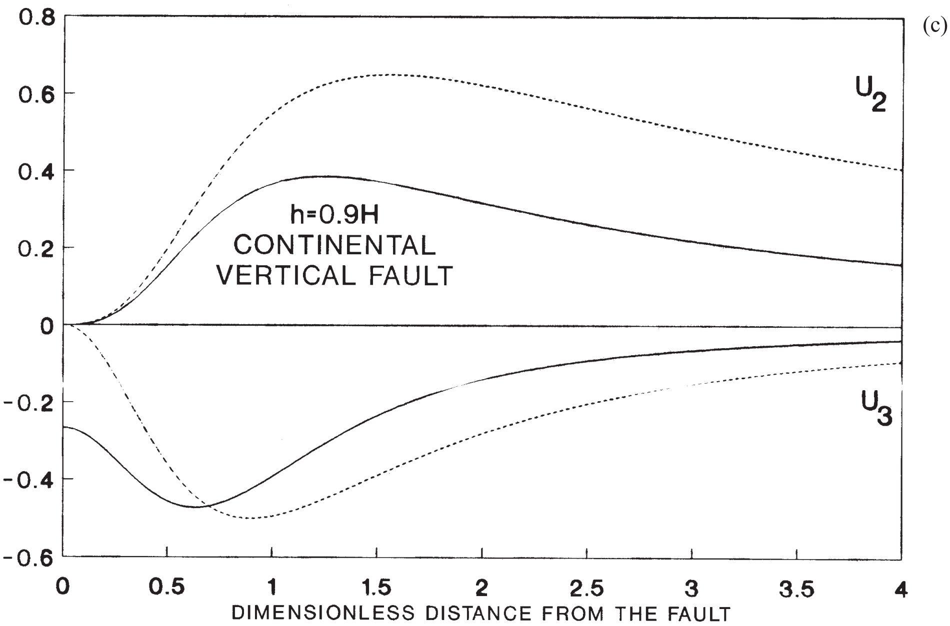


Figure 2 (a–c). Variation of the dimensionless horizontal displacement (U_2) and vertical displacement (U_3) with dimensionless horizontal distance from a vertical tensile fault for $\nu = 2.22$ for three values of the source depth; **(a)** $h = 0.1H$, **(b)** $h = 0.5H$, **(c)** $h = 0.9H$. The continuous lines are for a layered model and the broken lines are for a uniform half-space.

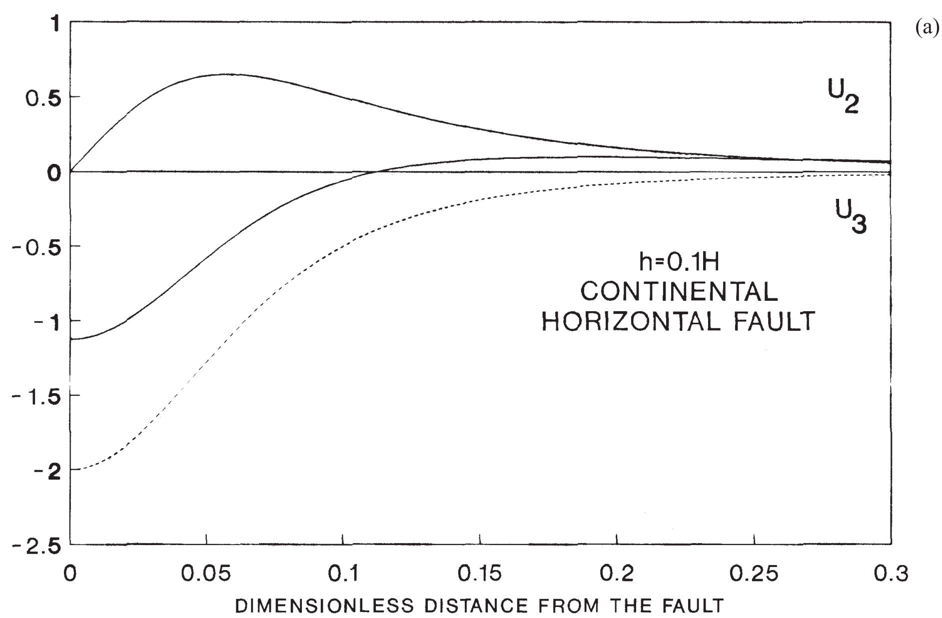
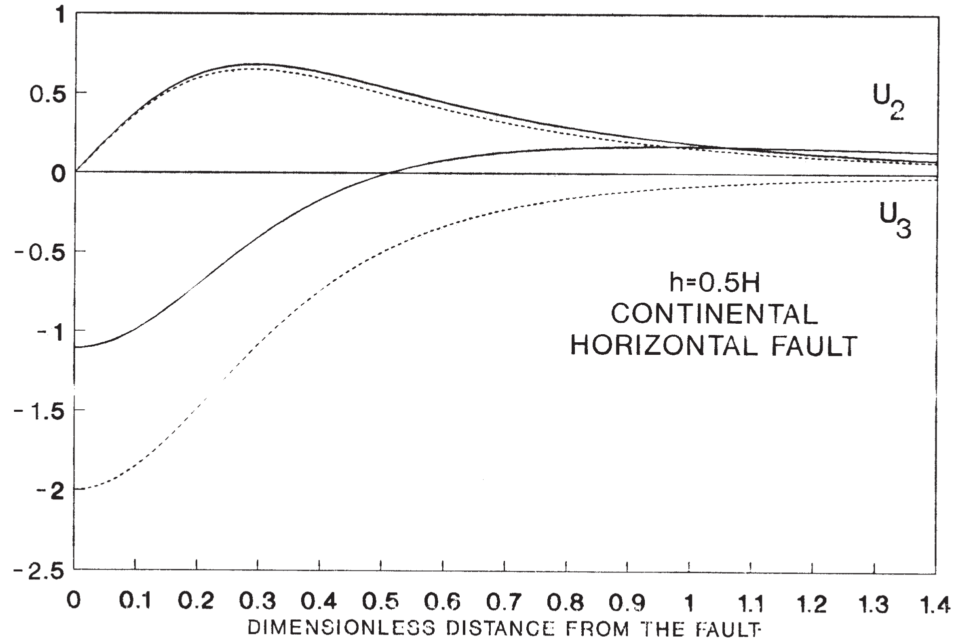


Figure 3 (a). (*Continued*)

is near the free surface the effect of underlying half-space is insignificant, but when the source is near the interface the effect is significant. U_2 and U_3 tend to zero as Y approaches infinity for all the cases.

Figures 3(a, b, c) show the variation of the horizontal displacement U_2 and vertical displacement U_3 at the surface with the distance from the fault caused by a horizontal tensile fault for three source depths: $h = 0.1H$, $0.5H$ and $0.9H$. From equations (21) and (22), $U_2 = 0$ and $U_3 = -2$ at $Y = 0$ for the uniform half-space. However, U_2 is zero and U_3 differs considerably from -2 at $Y = 0$ in the case of a layered half-space. U_2 keeps positive sign for uniform half-space and layered half-space. However, U_3 is negative in the case of uniform half-space, but changes sign in the case of layered half-space as we move away from the fault. As expected, the effect of the underlying medium is significant when the source is near the interface. Moreover, for a horizontal tensile fault, the horizontal component of the displace-



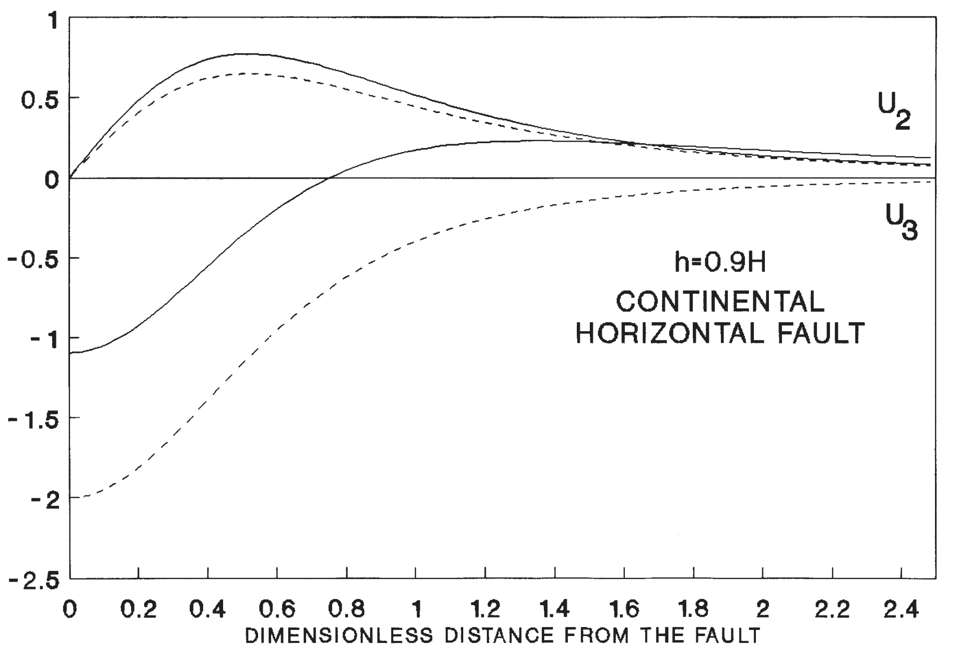


Figure 3 (a–c). Same as in figure 2 but for a horizontal tensile fault.

ment is less sensitive to layering than the vertical component.

For calculating the postseismic deformation, we use the relaxed rigidity method employed earlier by Cohen (1980); Rundle (1981) and Ma and Kusznir (1995). We assume that the layer is purely elastic, but the underlying half-space is viscoelastic. Therefore, for calculating the postseismic deformation, we express the displacements in terms of the rigidity (μ_2) and the bulk modulus (k_2) of the half-space, assume that the bulk modulus k_2 remains constant for all times, but the rigidity μ_2 relaxes with the passage of time. Since the layer is assumed to be elastic, μ_1 and k_1 also remain constant. Consequently, as the rigidity μ_2 of the half-space relaxes with time, the ratio $\nu = \mu_2/\mu_1$ decreases. We have calculated the postseismic deformation for $\nu = 2.22/5$, $2.22/10$ and $2.22/20$. It was found that for $\nu = 2.22$, the value $n = 2$ yields a satisfactory approximation; however, this value is not suitable in the other cases considered. The values of the constants n, α and β which were found suitable

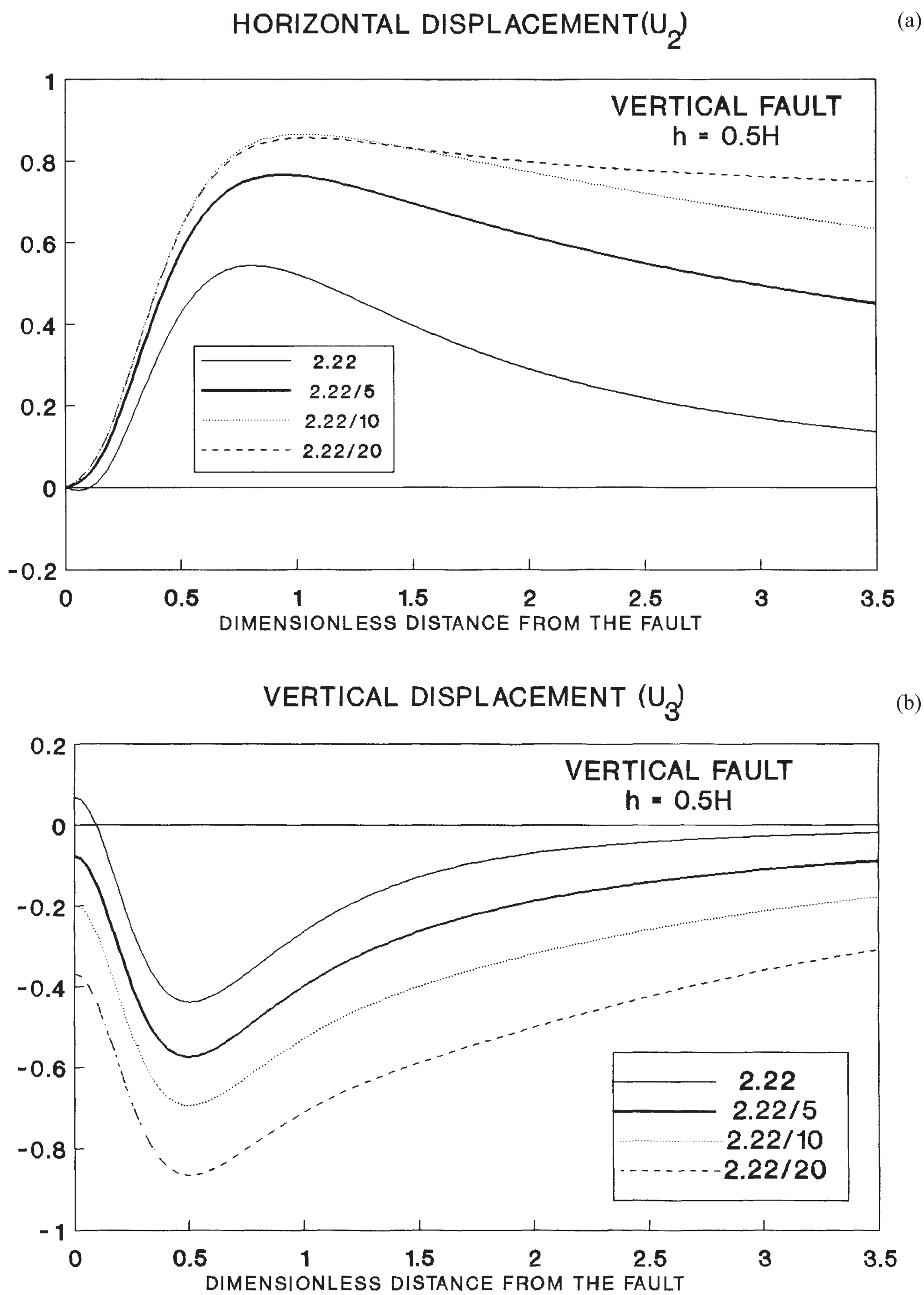


Figure 4. Variation of the dimensionless **(a)** horizontal displacement, **(b)** vertical displacement with the dimensionless horizontal distance from a vertical tensile fault for $h = 0.5H$ for four values of the rigidity ratio $\nu = \mu_2/\mu_1$. The ratio $\nu = 2.22$ corresponds to the coseismic deformation and the ratios $\nu = 2.22/5$, $2.22/10$, $2.22/20$ correspond to the postesismic deformation at various points of time after the earthquake.

for different values of ν are given in table 1. In all cases it was assumed that, before relaxation, $\sigma_1 = \sigma_2 = 0.25$.

We study numerically the variation of the displacement field at the surface with distance from the fault caused by a vertical tensile fault and a horizontal tensile fault for $\nu = 2.22$, $2.22/5$, $2.22/10$, $2.22/20$ and $h = 0.5H$. Figure 4(a) shows the variation of the dimensionless horizontal displacement U_2 at the surface with the dimensionless horizontal distance from the fault for $h = 0.5H$ caused by a vertical tensile fault for four values of the rigidity contrast, viz., $\nu = \mu_2/\mu_1 = 2.22$, $2.22/5$, $2.22/10$, $2.22/20$. The value $\nu = 2.22$ corresponds to the unrelaxed rigidity of the underlying medium and, therefore, gives the coseismic

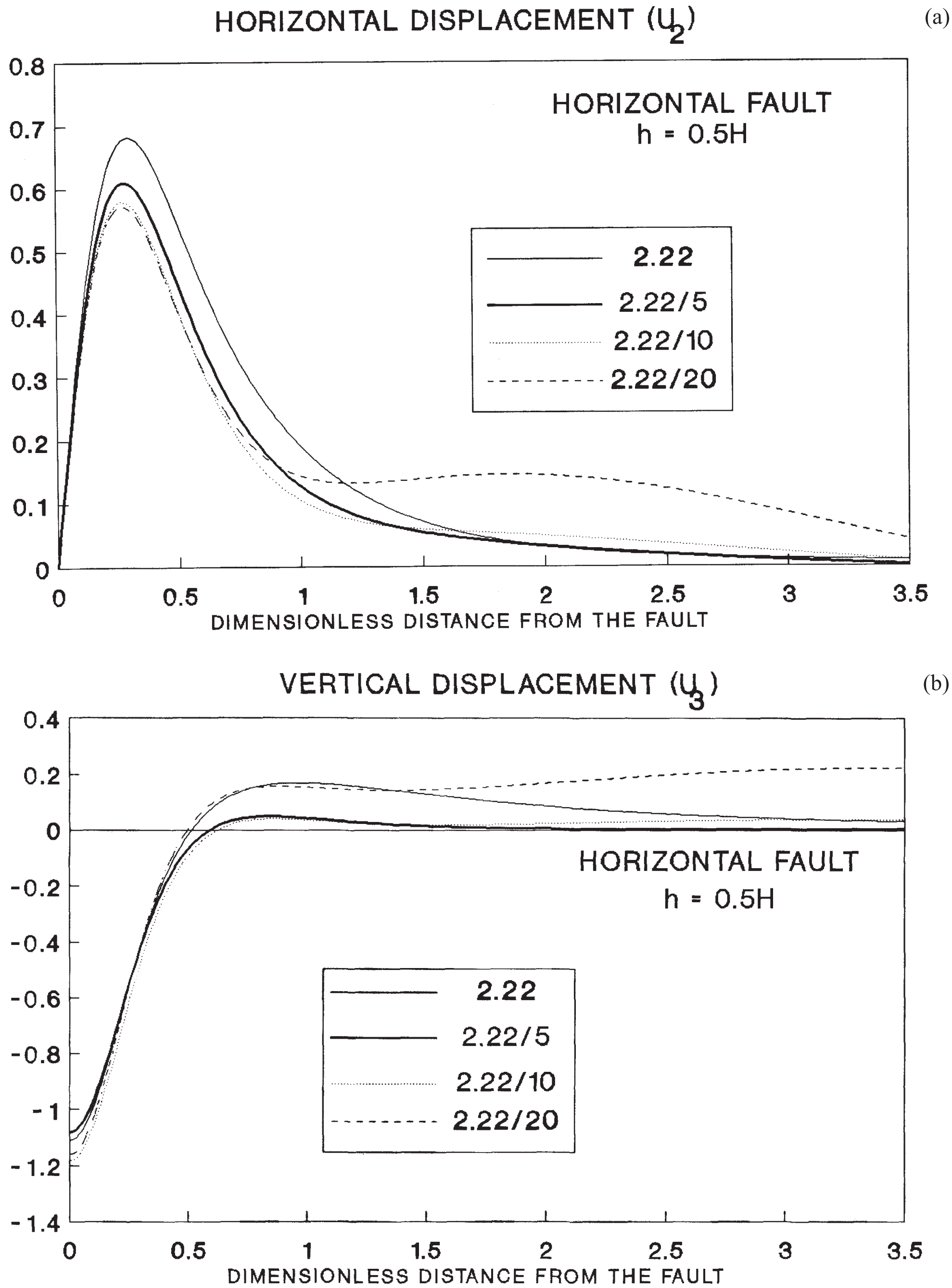


Figure 5. Same as in figure 4 but for a horizontal tensile fault.

displacement. The values $\nu = 2.22/5$, $2.22/10$ and $2.22/20$ correspond to the relaxed rigidity of the underlying medium at different points in time after the earthquake and give the postseismic displacement. We observe that the horizontal displacement vanishes at the origin (epicentre) for all values of ν. For large epicentral distances the magnitude of the horizontal displacement increases with the decrease in the value of ν. The horizontal displacement tends to zero as the epicentral distance tends to infinity for all values of ν. Figure 4(b) shows the variation of the dimensionless vertical displacement U_3 at the surface with the horizontal distance from the fault due to a vertical tensile fault for $h = 0.5H$. For large epicentral distances the magnitude of the vertical displacement increases with the decrease in the value of ν, i.e., for large times. The vertical displacement tends to zero as the epicentral distance approaches infinity for all values of ν.

Figure 5(a) shows the variation of the horizontal displacement U_2 at the surface with the distance from the fault for $h = 0.5H$ due to a horizontal tensile fault. We observe that $U_2 = 0$ at the origin for all values of rigidity contrast. The maximum value of the horizontal displacement decreases with the decrease in the value of ν, i.e., as the rigidity of the lower half-space relaxes with time. Moreover, for small values of ν, i.e., for large times, there appears a bulge (a second maximum) in the curve for the horizontal displacement away from the fault. Figure 5(b) shows the variation of the vertical displacement U_3 with the horizontal distance from the fault due to a horizontal tensile fault for $h = 0.5H$. U_3 tends to zero as the horizontal distance approaches infinity for all values of ν.

Acknowledgement

The authors gratefully acknowledge the financial support received from the Council of Scientific and Industrial Research through the Emeritus Scientist Scheme to SJS and the University Grants Commission through the Teacher Fellowship to MS. The authors thank an anonymous reviewer for useful comments which led to improvements in the paper.

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MS received 9 December 2002; revised 4 August 2003