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Plane Strain Quasi-static Deformation of a Poroelastic Half-space in Welded Contact with an Elastic Half-space due to Tensile Faulting

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ABSTRACT: An analytical solution is obtained of the problem of the plane strain quasi-static deformation of a composite consisting of a poroelastic half-space in welded contact with an elastic half-space caused by a long tensile fault located in the elastic half-space. The solution obtained is in the Fourier-Laplace transform domain. Two integrations are required to be performed to get the solution in the space-time domain. Schapery's formula is used for the Laplace inversion and extended Simpson's formula for the Fourier inversion. The diffusion of the pore pressure and the consolidation rate are studied numerically. It is found that the consolidation rate is not affected much by the compressibility of the fluid constituents of the poroelastic material.

1 Introduction

Tensile fault representation has several geophysical applications, such as modelling of the deformation field due to a dyke injection in the volcanic region, mine collapse and fluid driven cracks. Bonafede & Rivalta (1999) studied the problem of a long vertical tensile fault in two homogeneous, isotropic, elastic half-spaces in welded contact. Kumar *et al.* (2005) obtained the corresponding solution for a tensile fault of arbitrary dip. Rani & Singh (2007) solved the plane strain problem of the quasi-static deformation of a homogeneous, isotropic, poroelastic half-space in welded contact with a homogeneous, isotropic, elastic half-space due to a vertical dip-slip fault located in the elastic half-space. The aim of the present paper is to solve the corresponding problem for a horizontal tensile fault with dislocation in the vertical direction. Explicit analytical expressions for the pore pressure and the displacements are obtained in the transform domain. The diffusion of the pore pressure with time and horizontal distance are studied numerically.

2 Theory

Consider two homogeneous isotropic half-spaces which are welded along the horizontal plane z = 0. The upper half-space (z > 0) is poroelastic and the lower half-space (z < 0) is perfectly elastic. The homogeneous, isotropic, poroelastic half-space can be characterized by five poroelastic parameters. Let these parameters be: the shear modulus (G), the drained Poisson's ratio (ν), the undrained Poisson's ratio (ν_u), the Skempton's coefficient (B) and the hydraulic diffusivity (c).

A plane strain problem for the poroelastic half-space can be solved in terms of Biot's stress function F such that (Biot, 1956; Singh & Rani, 2006)

$$\sigma_{xx} = \frac{\partial^2 F}{\partial z^2}, \quad \sigma_{zz} = \frac{\partial^2 F}{\partial x^2}, \quad \sigma_{xz} = -\frac{\partial^2 F}{\partial x \partial z} \tag{1}$$

$$\nabla^2(\nabla^2 F + 2\eta p) = 0 \tag{2}$$

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\left[\nabla^2 F + \frac{3}{(1+\nu_u)B} p\right] = 0 \tag{3}$$

where σ denotes the stress tensor, p the excess fluid pore pressure and

$$\eta = \frac{3(\nu_u - \nu)}{2B(1-\nu)(1+\nu_u)} \tag{4}$$

the poroelastic stress coefficient.

The Laplace transform of the solution of the governing equations (1) to (3), which is bounded as z → ∞, can be expressed in the form (Singh & Rani, 2006)

$$p = -\int_0^\infty \left(\frac{s}{2c\eta} B_1 e^{-mz} - \xi k^2 B_3 e^{-kz} \right) \cos kx \, dk \tag{5}$$

$$\sigma_{xx} = \int_0^\infty \left[m^2 B_1 e^{-mz} + (B_2 - 2B_3 + B_3 kz) k^2 e^{-kz} \right] \cos kx \, dk \tag{6}$$

$$\sigma_{zz} = -\int_0^\infty \left[B_1 e^{-mz} + (B_2 + B_3 kz) e^{-kz} \right] \cos kx \, k^2 \, dk \tag{7}$$

$$\sigma_{xz} = -\int_0^\infty \left[m B_1 e^{-mz} + (B_2 - B_3 + B_3 kz) k e^{-kz} \right] \sin kx \, k \, dk \tag{8}$$

where s is the Laplace transform variable, B_1, B_2 and B_3 are arbitrary constants and

$$m = \left(k^2 + s/c \right)^{1/2}, \qquad \xi = \frac{2}{3}(1+\nu_u) B . \tag{9}$$

Here, we have selected the solution in which p is an even function of x. The displacements found by integrating the coupled constitutive relations are given by

$$2Gu_x = \int_0^\infty \left[B_1 e^{-mz} + \{ B_2 + B_3 (2\nu_u - 2 + kz) \} e^{-kz} \right] \sin kx \, k \, dk \tag{10}$$

$$2Gu_z = \int_0^\infty \left[m B_1 e^{-mz} + \{ B_2 + B_3 (1 - 2\nu_u + kz) \} k e^{-kz} \right] \cos kx \, dk \tag{11}$$

The homogeneous, isotropic, elastic half-space can be characterized by two elastic parameters. Let these parameters be the shear modulus (G') and the Poisson's ratio (ν). A plane strain problem for the isotropic elastic half-space can be solved in terms of the Airy stress function Φ such that

$$\sigma'_{xx} = \frac{\partial^2 \Phi}{\partial z^2}, \quad \sigma'_{zz} = \frac{\partial^2 \Phi}{\partial x^2}, \quad \sigma'_{xz} = -\frac{\partial^2 \Phi}{\partial x \partial z} \tag{12}$$

$$\nabla^2 \nabla^2 \Phi = 0 \tag{13}$$

where σ' is the stress tensor.

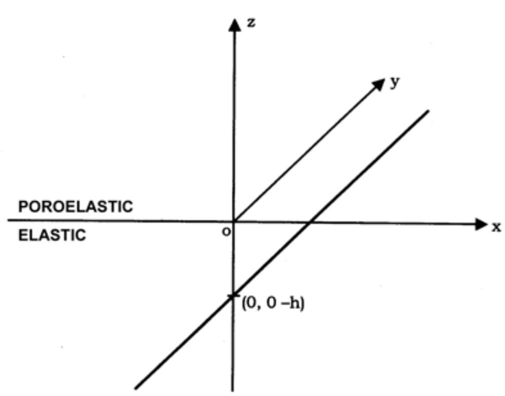

Figure 1. Geometry of the problem. A line source parallel to the y-axis passes through the point (0, 0, -h) of the elastic half-space.

Let there be a long horizontal tensile fault (with displacement discontinuity in the vertical direction) parallel to the y-axis passing through the point (0, 0, -h) of the elastic half-space (Figure 1). A suitable solution of the biharmonic equation (13) for the elastic half-space, with a line source parallel to the y-axis, is of the form.

$$\Phi = \Phi_0 + \int_0^\infty (C_1 + C_2 kz) e^{kz} \cos kx \, dk \qquad (14)$$

where C_1, C_2 are arbitrary constants and Φ_0 is the Airy stress function for the line source. For a horizontal tensile fault with a dislocation of magnitude b, we have (Singh & Garg, 1986)

$$\Phi_0 = \frac{G' bds}{2(1 - v')\pi} \int_0^\infty \left(\frac{1}{k} + \in Z \right) e^{-\in kZ} \cos kx \, dk \qquad (15)$$

where, $Z = z + h$, $\in = \pm 1$, ds is the width of the long fault. The upper sign is for $Z > 0$ and the lower sign is for $Z < 0$. The stresses follow from equation (12) and the displacements are obtained by integrating the constitutive relations (Rani *et al.*, 1991).

3 Solution

Since the half-spaces are assumed to be in welded contact, the boundary conditions are

$$\sigma_{xz} = \sigma'_{xz}, \qquad \sigma_{zz} = \sigma'_{zz}, \qquad (16)$$
$$u_x = u_x', \qquad u_z = u_z'$$

at z = 0. If we assume the interface to be impermeable, the hydraulic boundary condition implies

$$\partial p / \partial z = 0 \quad \text{at } z = 0 . \qquad (17)$$

The five boundary conditions mentioned in equations (16) and (17) can be used to find the five constants B_1, B_2, B_3, C_1 and C_2 which occur in equations (5) to (11) and (14). Proceeding as in Rani & Singh (2007), we obtain the following expressions for the displacements and the pore pressure. In the poroelastic half-space ($z > 0$)

$$u_x = \frac{\alpha' bds}{2\pi\theta} \int_0^\infty \left[\Omega_1 Q e^{-kh-mz} + \{ -P_2 (1 + 2kh) + Q [4\nu_u - 3 - (k + m)\Omega_1 + 2kz] \} \frac{e^{-kZ}}{2k} \right] \sin kx \; k \; dk \qquad (18)$$

$$u_z = \frac{\alpha' bds}{2\pi\theta} \int_0^\infty \left[\Omega_1 Q m e^{-kh-mz} - \{ P_2 (1 + 2kh) + Q [4\nu_u - 3 + (k + m)\Omega_1 - 2kz] \} \frac{e^{-kZ}}{2} \right] \cos kx \; dk \qquad (19)$$

$$p = \frac{\alpha' G' bds}{2\pi\eta} \int_0^\infty \Omega Q (k + m) \left[e^{-kh-mz} - \frac{m}{k} e^{-kZ} \right] \cos kx \; dk . \qquad (20)$$

In the elastic half-space ($z < 0$)

$$u'_x = \frac{\alpha' bds}{2\pi} \int_0^\infty \left[(-1 + 2\nu' + \in kZ) e^{-\in kZ} + \{ \Omega_2 + P_1 \left[1 - 2\nu' + k (z + 3h - 4\nu' h) + 2k^2 hz \right] \} e^{-k(h-z)} \right] \sin kx \; dk \qquad (21)$$

$$u'_z = \frac{\alpha' bds}{2\pi} \int_0^\infty \left[(2 - 2\nu' + \in kZ) \in e^{-\in kZ} - \{ \Omega_2 - P_1 \left[2 - 2\nu' - k (z - 3h + 4h\nu') - 2k^2 hz \right] \} e^{-k(h-z)} \right] \cos kx \; dk \qquad (22)$$

where

$$\theta = G/G', \quad \alpha' = \frac{1}{2(1-\nu')}$$

$$P_1 = \frac{1-\theta}{1+3\theta-4\theta\nu'}, \quad P_2 = P_1 - 1$$

$$P_3 = \frac{4\nu_u - 3 - \theta}{1-\theta}, \quad \gamma = \frac{2(\nu-\nu_u)}{1-\nu}$$

$$Q = \frac{P_2}{P_1(P_3+\Omega)}, \quad \Omega = \frac{k^2\gamma}{m(m+k)}$$

$$\Omega_1 = \frac{\Omega}{k-m}, \quad \Omega_2 = \frac{P_2 + Q + Q\Omega}{2}. \tag{23}$$

4 Numerical results and discussion

The solution obtained is in the Laplace-Fourier transform domain. We have used Schapery's approximate formula (Schapery, 1962) for the Laplace transform inversion. The Fourier transform inversion has been performed numerically by using the extended Simpson's rule. For numerical computations we assume the poreelastic half-space to be Ruhr Sandstone for which ν = 0.12, ν_u = 0.31, B = 0.88 and take θ = G/G' = 2, ν' = 0.25. We define the following dimensionless quantities:

$$P = (h^2/G'bds)\,p, \quad W = (hu_z)/bds, \quad T = 2\,c\,t/h^2. \tag{24}$$

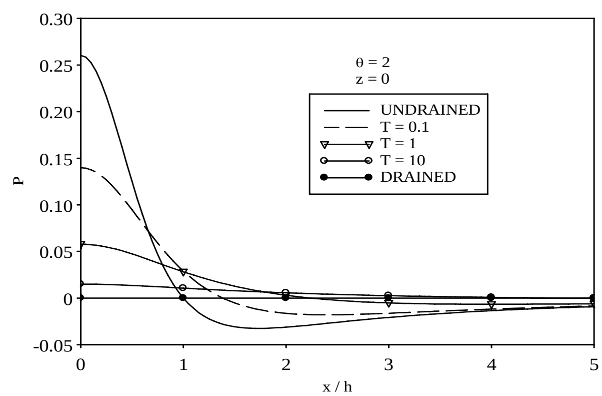

Figure 2. Variation of the pore pressure with horizontal distance. Undrained case corresponds to T → 0^+ and drained case that to T → ∞.

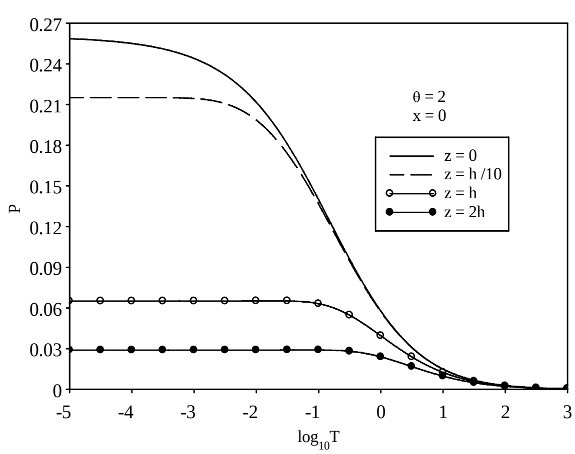

Figure 3. Diffusion of the pore pressure with time.

Figure 2 shows the variation of the pore pressure at the interface for the poroeleastic half-space with horizontal distance for five values of T. The undrained case refers to t → 0^+ and the drained one corresponds t → ∞ (P = 0 in the drained state). Figure 3 shows the diffusion of the pore pressure with time for x = 0 and z = 0, h/10, h, 2h. As the distance from the interface increases, the magnitude of the pore pressure decreases. Furthermore, as t tends to infinity, the pore pressure tends to zero at all depths. Figure 4 shows the effect of ν_u on the vertical displacement at the origin. For a poroelastic medium with incompressible fluid constituents, ν_u = 0.5. We notice that the compressibility of the fluid constituents has no effect on the vertical displacement in the drained state (T → ∞). In the undrained state (T → 0^+), the vertical displacement for the incompressible case is smaller than the vertical displacement for the compressible cases. The effect of ν_u on the consolidation rate $\overline{W}$ at the origin is studied in Figure 5. The consolidation rate is defined by the relation.

$$\overline{W}(0,0,T) = [W(0,0,T) - W(0,0,0^+)] \big/ [W(0,0,\infty) - W(0,0,0^+)] \qquad (25)$$

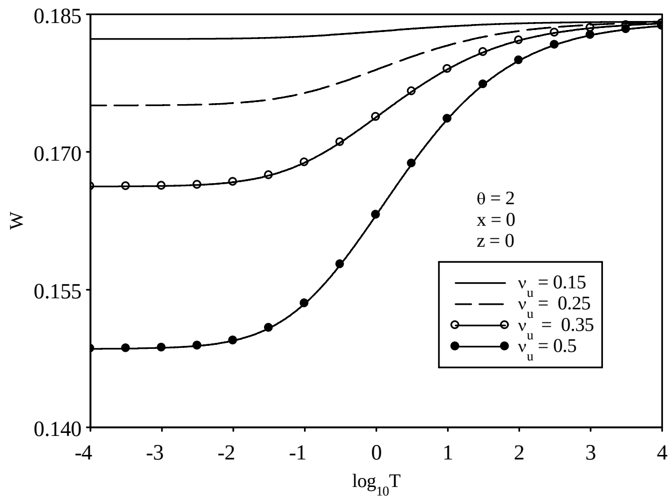

Figure 4. Effect of the compressibility of the fluid constituents of the poroelastic half-space on the vertical displacement at the origin. The case ν_u = 0.5 refers to an incompressible poroelastic half-space.

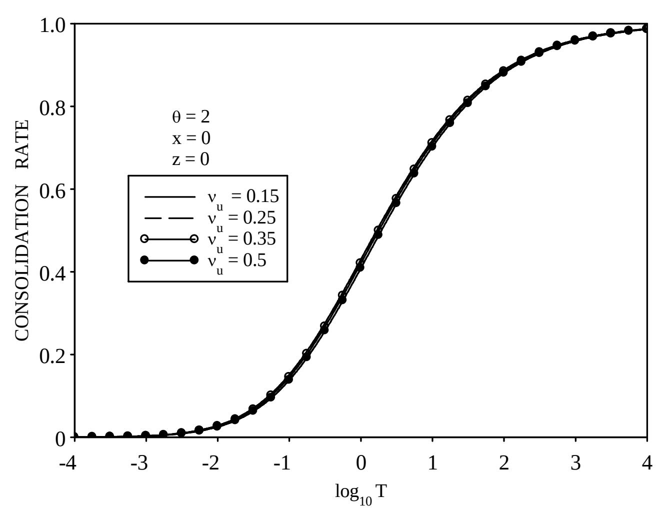

Figure 5. Effect of ν_u on the consolidation rate. The compressibility of the fluid constituents of the poroelastic half-space has very little effect on the consolidation rate.

Figure 5 shows that ν_u has only an insignificant effect on the consolidation rate, i.e. the consolidation rate is not much affected by the compressibility of the fluid constituents of the poroelastic material. In the drained and undrained states, the consolidation rate for the compressible half-space is the same as the consolidation rate for the incompressible half-space.

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