

Deformation of two welded elastic half-spaces due to a long inclined tensile fault

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The calculation of the deformation caused by shear and tensile faults is necessary for the investigation of seismic and volcanic sources. The solution of the two-dimensional problem of a long inclined shear fault in two welded half-spaces is well known. The purpose of this note is to present the corresponding solution for a tensile fault. Closed-form analytical expressions for the Airy stress function for a tensile line source in two welded half-spaces are first obtained. These expressions are then integrated analytically to derive the Airy stress function for a long tensile fault of arbitrary dip and finite width. Closed-form analytical expressions for the displacements and stresses follow immediately from the Airy stress function. These expressions are suitable for computing the displacement and stress fields around a long inclined tensile fault near an internal boundary.

1. Introduction

Tensile fault representation has several very important geophysical applications, such as modelling of the deformation field due to a dyke injection in the volcanic region, mine collapse and fluid driven crack. Recent studies have shown that a large number of earthquake sources cannot be represented by the double-couple source mechanism which models a shear fault. According to Sipkin (1986), the non-double-couple mechanism might be due to tensile failure under high fluid pressure.

The problem of a tensile fault in a uniform half-space has been studied by Maruyama (1964), Davis (1983), Yang and Davis (1986), Bonafede and Danesi (1997), Singh and Singh (2000) and Singh *et al* (2002). The aim of the present note is to study the deformation of two homogeneous, isotropic, elastic half-spaces in welded contact caused by a long tensile fault of arbitrary dip and finite width, using the Airy function approach. The corresponding problem of an inclined dip-slip fault has been discussed by Rani and Singh (1992). Bonafede

and Rivalta (1999) have studied the problem of a long vertical tensile fault in two welded half-spaces, using a Galerkin vector approach. It is useful to generalize the results of Bonafede and Rivalta (1999) to the case of a tensile fault of arbitrary dip.

We begin with the closed-form expressions for the Airy stress function for an arbitrary line source in two welded half-spaces given by Singh *et al* (1992). Following Singh and Singh (2000), we obtain the Airy stress function for a long tensile fault of arbitrary dip and finite width. The expressions for the displacements and stresses follow from the stress function. The results of the present study may find applications in extracting, from geodetic and seismic data, information on the position, depth, magma content and geometry of a buried dyke. The stresses induced by dyke opening are thought to be responsible for the seismicity generally observed prior to an eruption, for inducing isotropic moment tensor components and for causing changes in the principal stress directions. Availability of results for an arbitrarily dipping tensile fault is of immense help in such studies.

Keywords. Deformation; dyke injection; long tensile fault; volcanic source; welded half-spaces.

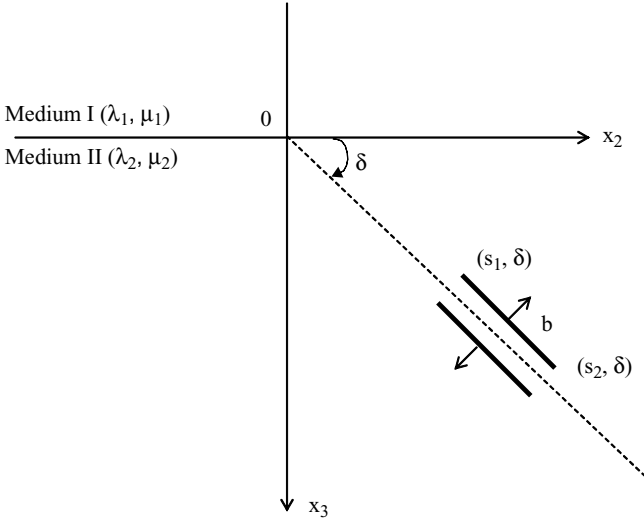


Figure 1. Two half-spaces in welded contact with a long inclined tensile fault in the lower half-space. The Cartesian coordinates of a point on the fault are (y_2, y_3) and its polar coordinates (s, δ) , where δ is the dip angle and $s_1 \leq s \leq s_2$.

2. Theory

Let the Cartesian coordinates be denoted by (x_1, x_2, x_3) with the x_3 -axis vertically downwards. Consider two homogeneous, isotropic, elastic half-spaces that are welded along the plane $x_3 = 0$. The upper half-space ($x_3 < 0$) is called medium I and the lower half-space ($x_3 > 0$) is called medium II, with elastic constants λ_1, μ_1 and λ_2, μ_2 , respectively (figure 1). In the following, the superscript (1) denotes quantities related to medium I and the superscript (2) denotes those related to medium II.

We consider a plane strain approximation parallel to the x_2x_3 -plane so that

$$u_1 \equiv 0, \quad \partial/\partial x_1 \equiv 0. \quad (1)$$

We define the Airy stress function U through the relations

$$\begin{aligned} \tau_{22} &= U_{,33}, & \tau_{23} &= -U_{,23}, & \tau_{33} &= U_{,22}, \\ \nabla^2 \nabla^2 U &= 0, \end{aligned} \quad (2)$$

where τ_{ij} are the components of stress and $U_{,33} = \partial^2 U / \partial x_3^2$, etc.

Singh *et al* (1992) obtained closed-form analytical expressions for the Airy stress function at any point of either of two homogeneous, isotropic, elastic half-spaces in welded contact caused by various two-dimensional sources in terms of certain source coefficients. These source coefficients have been given by Singh and Rani (1991). It has been shown by Singh and Singh (2000) that, for a line

source, the Airy stress function for a long inclined tensile fault is given by

$$U = U_{22} \sin^2 \delta - U_{23} \sin 2\delta + U_{33} \cos^2 \delta, \quad (3)$$

where δ is the dip of the fault (figure 1), U_{22} , U_{33} and U_{23} are the Airy stress functions for a vertical tensile fault ($\delta = \pi/2$), a horizontal tensile fault ($\delta = 0$) and a vertical dip-slip fault. Using the values of the source coefficients given by Singh and Rani (1991), we obtain the following expressions for the Airy stress function due to a long tensile fault (line source) parallel to the x_1 -axis and passing through the point (y_2, y_3) of the lower half-space (medium II) which is in welded contact with the upper half-space (medium I):

$$\begin{aligned} U^{(1)} &= \left(\frac{\alpha_2 \mu_2 b d s}{\pi} \right) \left[-D_1 \ln R + \cos 2\delta \left\{ \left(\frac{C_1 + D_1}{2} \right) \right. \right. \\ &\quad \times \ln R + \left. \left. \frac{(D_1 y_3 + C_1 x_3)(y_3 - x_3)}{R^2} \right\} \right. \\ &\quad - \sin 2\delta \left\{ \left(\frac{C_1 + D_1}{2} \right) \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - x_3} \right) \right. \\ &\quad \left. \left. - \frac{(D_1 y_3 + C_1 x_3)(x_2 - y_2)}{R^2} \right\} \right], \end{aligned} \quad (4)$$

$$\begin{aligned} U^{(2)} &= \left(\frac{\alpha_2 \mu_2 b d s}{\pi} \right) \left[-\ln R - C_2 \ln S \right. \\ &\quad + \frac{2x_3(x_3 + y_3)C_2}{S^2} + \cos 2\delta \left\{ \frac{(x_3 - y_3)^2}{R^2} \right. \\ &\quad - D_2 \ln S - \frac{C_2(x_3^2 - y_3^2 + 2x_3 y_3)}{S^2} \\ &\quad \left. \left. + \frac{4C_2 x_3 y_3 (x_3 + y_3)^2}{S^4} \right\} \right. \\ &\quad - \sin 2\delta \left\{ \left(\frac{(x_2 - y_2)(x_3 - y_3)}{R^2} \right) \right. \\ &\quad - D_2 \tan^{-1} \left(\frac{x_2 - y_2}{x_3 + y_3} \right) + \frac{C_2(x_2 - y_2)(x_3 - y_3)}{S^2} \\ &\quad \left. \left. - \frac{4C_2 x_3 y_3 (x_2 - y_2)(x_3 + y_3)}{S^4} \right\} \right], \end{aligned} \quad (5)$$

where

b = displacement discontinuity,

ds = width of the line source,

δ = dip angle,

(x_2, x_3) = receiver location,

$$R^2 = (x_2 - y_2)^2 + (x_3 - y_3)^2,$$

$$S^2 = (x_2 - y_2)^2 + (x_3 + y_3)^2,$$

$$\beta = \mu_1/\mu_2,$$

$$\alpha_1 = (\lambda_1 + \mu_1)/(\lambda_1 + 2\mu_1),$$

$$\alpha_2 = (\lambda_2 + \mu_2)/(\lambda_2 + 2\mu_2),$$

$$C_1 = 2\beta[\alpha_2(1 - \beta - 2/\alpha_1)]^{-1},$$

$$C_2 = (\beta - 1)(1 - \beta + 2\beta/\alpha_2)^{-1},$$

$$D_1 = 1 + C_2,$$

$$D_2 = -(C_1 + D_1)/2. \quad (6)$$

We put (see figure 1)

$$y_2 = s \cos \delta, \quad y_3 = s \sin \delta. \quad (7)$$

Inserting the values of y_2 and y_3 from equation (7) into equations (4) and (5) and integrating over s between the limits (s_1, s_2) , we obtain the following expressions for the Airy stress function for a long inclined tensile fault of finite width $L = s_2 - s_1$:

$$\begin{aligned} U^{(1)} = & \left(\frac{\alpha_2 \mu_2 b}{2\pi} \right) \left[\{ (C_1 + D_1) \cos 2\delta - 2D_1 \} s \ln R \right. \\ & + (D_1 - C_1)(x_2 \cos \delta + x_3 \sin \delta) \ln R \\ & - (C_1 + D_1)(x_2 \sin \delta - x_3 \cos \delta) \\ & \times \tan^{-1} \left(\frac{s - x_2 \cos \delta - x_3 \sin \delta}{x_2 \sin \delta - x_3 \cos \delta} \right) \\ & + s(D_1 - C_1 \cos 2\delta) - (C_1 + D_1)s \\ & \left. \times \sin 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{s \sin \delta - x_3} \right) \right] \Bigg\|, \quad (8) \end{aligned}$$

$$\begin{aligned} U^{(2)} = & \left(\frac{\alpha_2 \mu_2 b}{\pi} \right) \left[(x_3 \sin \delta + x_2 \cos \delta - s) \ln R \right. \\ & + \{ (C_2 + D_2)(x_2 \cos \delta + x_3 \sin \delta) \\ & - s(C_2 + D_2 \cos 2\delta) \} \ln S + \{ (1 + C_2 + 2D_2) \\ & \times \cos^2 \delta - D_2 \} s + \frac{2C_2}{S^2} \{ s(x_2 \cos \delta - x_3 \sin \delta) \\ & - (x_2^2 + x_3^2) \} x_3 \sin \delta + D_2(x_2 \sin \delta - x_3 \cos \delta) \\ & \times \tan^{-1} \left(\frac{s + x_3 \sin \delta - x_2 \cos \delta}{x_2 \sin \delta + x_3 \cos \delta} \right) \\ & \left. + sD_2 \sin 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{x_3 + s \sin \delta} \right) \right] \Bigg\|, \quad (9) \end{aligned}$$

where

$$R^2 = (x_2 - s \cos \delta)^2 + (x_3 - s \sin \delta)^2,$$

$$S^2 = (x_2 - s \cos \delta)^2 + (x_3 + s \sin \delta)^2,$$

$$f(s) \Big| = f(s_2) - f(s_1). \quad (10)$$

3. Stresses and displacements

From equations (2), (8) and (9), we obtain the following expressions for the stresses in two half-spaces:

$$\begin{aligned} \tau_{22}^{(1)} = & \left(\frac{\alpha_2 \mu_2 b}{2\pi} \right) \left[\{ D_1(x_3 \sin \delta - x_2 \cos \delta) \right. \\ & - C_1(3x_2 \cos \delta + 5x_3 \sin \delta) \} \frac{1}{R^2} \\ & + \{ 3C_1 + D_1(2 \cos 2\delta - 1) \} \frac{s}{R^2} \\ & + 4(C_1 x_3 + D_1 s \sin \delta)(x_2 \cos \delta + x_3 \sin \delta - s) \\ & \left. \times \left(\frac{x_3 - s \sin \delta}{R^4} \right) \right] \Bigg\|, \quad (11) \\ \tau_{23}^{(1)} = & - \left(\frac{\alpha_2 \mu_2 b}{2\pi} \right) \left[\{ C_1(3x_3 \cos \delta - x_2 \sin \delta) \right. \\ & \left. + D_1(x_2 \sin \delta + x_3 \cos \delta) \} \frac{1}{R^2} \right] \Bigg\| \end{aligned}$$

$$+ 4(C_1 x_3 + D_1 s \sin \delta)(x_2 \sin \delta - x_3 \cos \delta) \\ \times \left(\frac{x_3 - s \sin \delta}{R^4} \right) \Bigg\| \Bigg\|, \quad (12)$$

$$\tau_{33}^{(1)} = \left(\frac{\alpha_2 \mu_2 b}{2\pi} \right) \left[\{D_1(x_2 \cos \delta - x_3 \sin \delta - s) \right. \\ \left. - C_1(x_2 \cos \delta + 3x_3 \sin \delta - s)\} \frac{1}{R^2} \right. \\ \left. + 4(C_1 x_3 + D_1 s \sin \delta)(x_2 \sin \delta - x_3 \cos \delta) \right. \\ \left. \times \left(\frac{x_2 - s \cos \delta}{R^4} \right) \right] \Bigg\| \Bigg\|, \quad (13)$$

$$\tau_{22}^{(2)} = \left(\frac{\alpha_2 \mu_2 b}{\pi} \right) \left[(x_3 \sin \delta - x_2 \cos \delta + s \cos 2\delta) \frac{1}{R^2} \right. \\ \left. + 2(x_3 \sin \delta + x_2 \cos \delta - s) \times \frac{(x_2 - s \cos \delta)^2}{R^4} \right. \\ \left. + \{D_2(x_3 \sin \delta - x_2 \cos \delta + s) \right. \\ \left. + C_2(3x_3 \sin \delta + x_2 \cos \delta) \right. \\ \left. - C_2(1 + 2 \sin^2 \delta) s\} \frac{1}{S^2} \right. \\ \left. - 2C_2(x_2 \cos \delta + x_3 \sin \delta - s) \frac{(x_3 + s \sin \delta)^2}{S^4} \right. \\ \left. + 8C_2 x_2 s \sin^2 \delta \frac{(x_2 - s \cos \delta)}{S^4} \right. \\ \left. + 12C_2 x_3 \sin \delta (x_2 \cos \delta - x_3 \sin \delta - s) \frac{s}{S^4} \right. \\ \left. + 16C_2 x_3 \sin \delta (x_3 \sin \delta - x_2 \cos \delta + s) \right. \\ \left. \times \frac{s(x_3 + s \sin \delta)^2}{S^6} \right] \Bigg\| \Bigg\|, \quad (14)$$

$$\tau_{23}^{(2)} = - \left(\frac{\alpha_2 \mu_2 b}{\pi} \right) \left[(x_2 \sin \delta - x_3 \cos \delta) \frac{1}{R^2} \right. \\ \left. + 2(x_3 \cos \delta - x_2 \sin \delta) \frac{(x_3 - s \sin \delta)^2}{R^4} \right.$$

$$+ D_2(x_2 \sin \delta + x_3 \cos \delta) \frac{1}{S^2} \\ + C_2(x_2 \sin \delta - x_3 \cos \delta) \frac{1}{S^2} \\ + 2C_2 \{ (x_3 \cos \delta - x_2 \sin \delta)(x_3 + s \sin \delta) \\ - 2x_3 s \sin 2\delta \} \frac{(x_3 + s \sin \delta)}{S^4} \\ - 4C_2 x_3 \sin \delta (x_3 \cos \delta + x_2 \sin \delta) \frac{s}{S^4} \\ + 16C_2 x_3 \sin \delta (x_3 \cos \delta + x_2 \sin \delta) \\ \times \frac{s(x_3 + s \sin \delta)^2}{S^6} \Bigg\| \Bigg\|, \quad (15)$$

$$\tau_{33}^{(2)} = \left(\frac{\alpha_2 \mu_2 b}{\pi} \right) \left[(x_2 \cos \delta - x_3 \sin \delta - s \cos 2\delta) \frac{1}{R^2} \right. \\ \left. + 2(x_3 \sin \delta + x_2 \cos \delta - s) \frac{(x_3 - s \sin \delta)^2}{R^4} \right. \\ \left. + (C_2 + D_2)(x_2 \cos \delta - x_3 \sin \delta) \frac{1}{S^2} \right. \\ \left. - (D_2 + C_2 \cos 2\delta) \frac{s}{S^2} \right. \\ \left. + 2C_2(x_2 \cos \delta + x_3 \sin \delta - s) \frac{(x_3 + s \sin \delta)^2}{S^4} \right. \\ \left. + 12C_2 x_3 \sin^2 \delta \frac{s(x_3 + s \sin \delta)}{S^4} \right. \\ \left. - 2C_2 x_3 \sin 2\delta \frac{s(x_2 - s \cos \delta)}{S^4} \right. \\ \left. - 16C_2 x_3 \sin \delta (x_3 \sin \delta - x_2 \cos \delta + s) \right. \\ \left. \times \frac{s(x_3 + s \sin \delta)^2}{S^6} \right] \Bigg\| \Bigg\|. \quad (16)$$

Corresponding to the stresses given by equations (11)–(16), the displacements are found by integrating the stress-strain relations (Sokolnikoff 1956; Singh and Garg 1985). We find

$$\begin{aligned}
2\mu_1 u_2^{(1)} = & \left(\frac{\alpha_2 \mu_2 b}{2\pi} \right) \left[\cos \delta \left(C_1 - D_1 - \frac{2C_1}{\alpha_1} \right) \right. \\
& \times \ln R + (D_1 + C_1) \sin \delta \\
& \times \tan^{-1} \left(\frac{s - x_2 \cos \delta - x_3 \sin \delta}{x_2 \sin \delta - x_3 \cos \delta} \right) \\
& + \frac{2D_1 s (x_2 - s \cos \delta)}{R^2} \\
& - \frac{2C_1}{\alpha_1} \sin \delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{x_3 - s \sin \delta} \right) \\
& - (C_1 + D_1) s \cos 2\delta \frac{(x_2 - s \cos \delta)}{R^2} \\
& + (C_1 - D_1) (x_2 \cos \delta + x_3 \sin \delta) \\
& \times \frac{(x_2 - s \cos \delta)}{R^2} + (C_1 + D_1) (x_3 - s \sin \delta) \\
& \left. \times \left(\frac{(x_2 \sin \delta - x_3 \cos \delta - s \sin 2\delta)}{R^2} \right) \right] \Bigg\|, \quad (17)
\end{aligned}$$

$$\begin{aligned}
2\mu_1 u_3^{(1)} = & \left(\frac{\alpha_2 \mu_2 b}{2\pi} \right) \left[\sin \delta \left(C_1 - D_1 - \frac{2C_1}{\alpha_1} \right) \ln R \right. \\
& - (C_1 + D_1) \cos \delta \\
& \times \tan^{-1} \left(\frac{s - x_2 \cos \delta - x_3 \sin \delta}{x_2 \sin \delta - x_3 \cos \delta} \right) \\
& + 2D_1 s \frac{(x_3 - s \sin \delta)}{R^2} - \frac{2C_1}{\alpha_1} \cos \delta \\
& \times \tan^{-1} \left(\frac{x_3 - s \sin \delta}{x_2 - s \cos \delta} \right) - (C_1 + D_1) s \\
& \times \cos 2\delta \frac{(x_3 - s \sin \delta)}{R^2} + (C_1 - D_1) \\
& \left. \times (x_2 \cos \delta + x_3 \sin \delta) \frac{(x_3 - s \sin \delta)}{R^2} \right] \Bigg\|, \quad (18)
\end{aligned}$$

$$\begin{aligned}
& + (C_1 + D_1) (x_2 - s \cos \delta) \\
& \times \frac{(x_3 \cos \delta - x_2 \sin \delta + s \sin 2\delta)}{R^2} \Bigg\|, \quad (18) \\
2\mu_2 u_2^{(2)} = & \left(\frac{\alpha_2 \mu_2 b}{\pi} \right) \left[\left(\frac{1}{\alpha_2} - 1 \right) \cos \delta \ln R \right. \\
& - \left(C_2 + D_2 - \frac{C_2}{\alpha_2} \right) \cos \delta \ln S + \frac{\sin \delta}{\alpha_2} \\
& \times \tan^{-1} \left(\frac{x_2 - s \cos \delta}{x_3 - s \sin \delta} \right) + \frac{C_2 \sin \delta}{\alpha_2} \\
& \times \tan^{-1} \left(\frac{x_2 - s \cos \delta}{x_3 + s \sin \delta} \right) - D_2 \sin \delta \\
& \times \tan^{-1} \left(\frac{x_3 \sin \delta - x_2 \cos \delta + s}{x_2 \sin \delta + x_3 \cos \delta} \right) \\
& + C_2 x_3 s \sin 2\delta \frac{1}{S^2} \\
& + D_2 (x_2 \sin \delta - x_3 \cos \delta - s \sin 2\delta) \\
& \times \frac{(x_3 + s \sin \delta)}{S^2} - (x_2 - s \cos \delta) \\
& \times (x_2 \cos \delta + x_3 \sin \delta - s) \left(\frac{C_2}{S^2} + \frac{1}{R^2} \right) \\
& - \frac{2C_2 s \sin \delta (x_2 \sin \delta + x_3 \cos \delta)}{\alpha_2 S^2} \\
& - D_2 (x_2 \cos \delta + x_3 \sin \delta - s \cos 2\delta) \\
& \times \frac{(x_2 - s \cos \delta)}{S^2} + 4C_2 x_3 \sin \delta \\
& \left. \times (x_3 \sin \delta - x_2 \cos \delta + s) \frac{s(x_2 - s \cos \delta)}{S^4} \right] \Bigg\|, \quad (19)
\end{aligned}$$

$$\begin{aligned}
2\mu_2 u_3^{(2)} = & \left(\frac{\alpha_2 \mu_2 b}{\pi} \right) \left[\left(\frac{1}{\alpha_2} - 1 \right) \sin \delta \ln R \right. \\
& - \left(C_2 + D_2 - \frac{C_2}{\alpha_2} \right) \sin \delta \ln S + \frac{\cos \delta}{\alpha_2}
\end{aligned}$$

$$\begin{aligned}
& \times \tan^{-1} \left(\frac{x_3 - s \sin \delta}{x_2 - s \cos \delta} \right) + \frac{C_2}{\alpha_2} \cos \delta \\
& \times \tan^{-1} \left(\frac{x_3 + s \sin \delta}{x_2 - s \cos \delta} \right) + D_2 \cos \delta \\
& \times \tan^{-1} \left(\frac{s + x_3 \sin \delta - x_2 \cos \delta}{x_2 \sin \delta + x_3 \cos \delta} \right) \\
& - C_2 \sin \delta + \frac{2C_2}{\alpha_2} \sin \delta \\
& \times (x_3 \sin \delta - x_2 \cos \delta + s) \frac{s}{S^2} \\
& - (x_3 \sin \delta + x_2 \cos \delta - s) \frac{(x_3 - s \sin \delta)}{R^2} \\
& - D_2(x_2 \sin \delta - x_3 \cos \delta - s \sin 2\delta) \\
& \times \frac{(x_2 - s \cos \delta)}{S^2} \\
& - D_2(x_2 \cos \delta + x_3 \sin \delta - s \cos 2\delta) \\
& \times \frac{(x_3 + s \sin \delta)}{S^2} - 2C_2 x_3 \sin^2 \delta \frac{s}{S^2} \\
& + C_2(x_2 \sin \delta - x_3 \cos \delta)(x_2 - s \cos \delta) \frac{1}{S^2} \\
& + 4C_2 x_3 \sin \delta (x_3 \sin \delta - x_2 \cos \delta + s) \\
& \times \frac{s(x_3 + s \sin \delta)}{S^4} \Big] \Big\| . \tag{20}
\end{aligned}$$

4. Discussion

Equations (11)–(20) yield the displacement and stress fields at any point of the two welded half-space caused by a long tensile fault of dip δ and width $L = s_2 - s_1$ located in one of the half-spaces. These closed-form analytical expressions are very convenient for computing the displacements and stresses at any point of the medium. The results for a long tensile fault in a uniform half-space (medium II) follow as a particular case on putting $\mu_1 = 0$. From equation (6), we note that, this implies $\beta = 0$,

$C_2 = -1$, $C_1 = D_1 = D_2 = 0$. It has been verified that the results obtained on putting $\mu_1 = 0$ in equations (8), (9), (14)–(16), (19) and (20) coincide with the corresponding results of Singh *et al* (2002) for a uniform half-space. (There is a printing error in the expression for u_2 given by Singh *et al* (2002). In the first line of the expression for u_2 , $(3 - 2\sigma)$ should be replaced by $(1 - 2\sigma)$.)

The displacements given by equations (17)–(20) have been obtained by integrating the strains. This integration process introduces arbitrary additive constants. However, these constants pose no problem since the integrals involved are definite integrals and the definition

$$f(s) \Big| = f(s_2) - f(s_1)$$

cancels the constants mutually. The function $\tan^{-1}(\text{---})$ appearing in the expressions for the displacements does create problems. The angle generated by this function should be so shifted as to make the displacements continuous outside the dislocated portion of the fault plane, i.e., for $s < s_1$ and $s > s_2$ of figure 1 [see, e.g., Bonafede and Rivalta (1999), p. 343].

It has been verified that the stresses and displacements given in equations (12)–(20) satisfy the necessary continuity conditions, i.e.

$$\tau_{23}^{(1)} = \tau_{23}^{(2)}, \quad \tau_{33}^{(1)} = \tau_{33}^{(2)},$$

$$u_2^{(1)} = u_2^{(2)}, \quad u_3^{(1)} = u_3^{(2)},$$

at $x_3 = 0$.

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