Proc. Indian Acad. Sci. (Earth Planet. Sci.), Vol. 97, No. 1, July 1988, pp. 11–20. © Printed in India.

Quasi-static deformation of a layered half-space by a long strike-slip fault

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MS received 1 April 1987; revised 30 November 1987

Abstract. Theoretical expressions for the surface displacement and shear stress caused by a long strike-slip dislocation in an elastic layer overlying an elastic half-space are derived and the correspondence principle is used to obtain the quasi-static response when the half-space is Maxwell-viscoelastic. Variation of the surface displacement and shear stress with horizontal distance is studied for various times and vertical extents of the fault. It is seen that the quasi-static response differs significantly from the corresponding elastic response.

Keywords. Correspondence principle; strike-slip fault; Maxwell model; quasi-static deformation; surface displacement; shear stress.

1. Introduction

In quasi-static deformation, the inertial terms in the stress equations of equilibrium can be ignored. The study of quasi-static deformation of a viscoelastic model of the earth caused by a shear dislocation is important to understand the post-seismic deformation following large earthquakes. Singh and Rosenman (1974) derived analytical expressions for the quasi-static surface displacements due to a finite vertical strike-slip fault in a Voigt or Maxwell viscoelastic half-space by applying the correspondence principle of linear viscoelasticity. Rosenman and Singh (1973a, b) obtained the corresponding expressions for the quasi-static surface strains, tilts and stresses. Nur and Mavko (1974) studied the characteristics of time-dependent deformation following an earthquake by considering the problem of a fault in an elastic layer overlying a viscoelastic half-space. Mukhopadhyay and Mukherji (1979) and Mukherji *et al* (1980) studied the stress accumulation near an earthquake fault by considering the antiplane strain problem of a finite displacement dislocation in an elastic layer lying over a Maxwell viscoelastic half-space, in which the shear stress is maintained far away from the fault by tectonic forces.

In our previous paper (Singh and Garg 1985, referred to as paper I hereafter), we discussed the two-dimensional problem of a long displacement dislocation in a multilayered half-space with the help of the Thomson-Haskell matrix method. Explicit expressions for the surface displacements due to dip-slip and strike-slip faults were given.

In the present paper, we first obtain the surface elastostatic field, using the results of paper I, due to a long vertical strike-slip fault in a homogeneous isotropic elastic layer lying over a homogeneous isotropic elastic half-space. The correspondence principle of linear viscoelasticity is then used to find the quasi-static response when the half-space is

Maxwell viscoelastic. In this model, the elastic layer represents the lithosphere and the Maxwell viscoelastic half-space represents the asthenosphere. Curves for surface displacement and shear stress are obtained. These curves differ significantly from the corresponding curves for the elastic case when the vertical extent of the fault is large.

2. Theory

We consider a model consisting of a homogeneous isotropic elastic layer of thickness H lying over a homogeneous isotropic Maxwell viscoelastic half-space. We place the origin of a cartesian coordinate system (x, y, z) at the free surface and the z-axis is drawn into the medium. Let a long vertical strike-slip fault, with strike along the x-axis, be situated on the z-axis at a depth h, $0 \leqslant h \leqslant H$, below the free surface. We first calculate the surface displacement and shear stress due to a long strike-slip fault situated in the corresponding elastic model. The correspondence principle of linear viscoelasticity is then used to obtain the quasi-static response.

2.1 *Elastostatic solution*

In equation (5.9) of paper I, we obtained the surface displacement $u_1(0)$ due to a long strike-slip fault situated in a multilayered elastic half-space. In the present paper, we follow the notation used in paper I. We have

$$u_1(0) = \frac{1}{\pi}\mu B_1 \int_0^\infty \left(\frac{V_{12}E_{21} - V_{22}E_{11}}{E_{21}}\right) \sin ky \, dk, \tag{1}$$

where the matrices $[E]$ and $[V]$ are defined in paper I. μ is rigidity of the source layer and $B_1 = \Delta u_1 \, dh$ where Δu_1 is the dislocation and dh the fault width.

For an elastic layer over an elastic half-space, equations (3.29) and (3.31) of paper I yield the following elements for the matrix $[E]$:

$$\begin{aligned} E_{11} &= [\mathrm{ch}\,(kH) + \beta\,\mathrm{sh}\,(kH)]\exp(-kH), \\ E_{12} &= [\mathrm{ch}\,(kH) - \beta\,\mathrm{sh}\,(kH)]\exp(kH), \\ E_{21} &= -\mu_1[\mathrm{sh}\,(kH) + \beta\,\mathrm{ch}\,(kH)]\exp(-kH), \\ E_{22} &= \mu_1[\beta\,\mathrm{ch}\,(kH) - \mathrm{sh}\,(kH)]\exp(kH), \end{aligned} \tag{2}$$

where μ_1 is the rigidity of the layer, μ_2 the rigidity of the half-space and

$$\beta = \mu_2/\mu_1. \tag{3}$$

Similarly, the matrix $[V]$ reduces to

$$[V] = \begin{bmatrix} \mathrm{ch}\,(kh) & -\mu_1^{-1}\,\mathrm{sh}\,(kH) \\ -\mu_1\,\mathrm{sh}\,(kh) & \mathrm{ch}\,(kh) \end{bmatrix}. \tag{4}$$

Equations (1), (2) and (4) yield

$$u_1 = \frac{B_1}{\pi}\int_0^\infty \left[\frac{\mathrm{ch}\,(kH - kh) + \beta\,\mathrm{sh}\,(kH - kh)}{\mathrm{sh}\,(kH) + \beta\,\mathrm{ch}\,(kH)}\right] \sin ky \, dk. \tag{5}$$

Expressing the hyperbolic functions in terms of the exponential functions, (5) becomes

$$u_1 = \frac{B_1}{\pi}\int_0^\infty \exp(-kh)\left[\frac{1 + r\exp\{-2k(H-h)\}}{1 - r\exp(-2kH)}\right]\sin ky\, dk, \tag{6}$$

where

$$r = (1-\beta)/(1+\beta) = (\mu_1 - \mu_2)/(\mu_1 + \mu_2). \tag{7}$$

Expanding the denominator in (6) in a power series and integrating term by term, we obtain

$$u_1 = \frac{B_1}{\pi}\left[\frac{y}{y^2+h^2} + \sum_{n=1}^{\infty} r^n \left\{\frac{y}{y^2+(2nH-h)^2} + \frac{y}{y^2+(2nH+h)^2}\right\}\right]. \tag{8}$$

Equation (8) gives the surface displacement parallel to the fault caused by a long vertical strike-slip fault situated in an elastic layer of thickness H lying over an elastic half-space.

The displacement due to a finite vertical strike-slip fault with vertical extent $0 \leqslant h \leqslant d \leqslant H$ (figure 1) is obtained from (8) by integrating with respect to h from 0 to d. We find

$$u_1 = \frac{\Delta u_1}{\pi}\left[\tan^{-1}\left(\frac{d}{y}\right) + \sum_{n=1}^{\infty} r^n \times \left\{\tan^{-1}\left(\frac{2nH+d}{y}\right) - \tan^{-1}\left(\frac{2nH-d}{y}\right)\right\}\right]. \tag{9}$$

The corresponding shear stress is given by

$$p_{12} = \mu_1 \frac{\partial u_1}{\partial y} = \frac{\Delta u_1}{\pi}\mu_1\left[\frac{-d}{y^2+d^2} + \sum_{n=1}^{\infty} r^n \times \left\{\frac{2nH-d}{y^2+(2nH-d)^2} - \frac{2nH+d}{y^2+(2nH+d)^2}\right\}\right]. \tag{10}$$

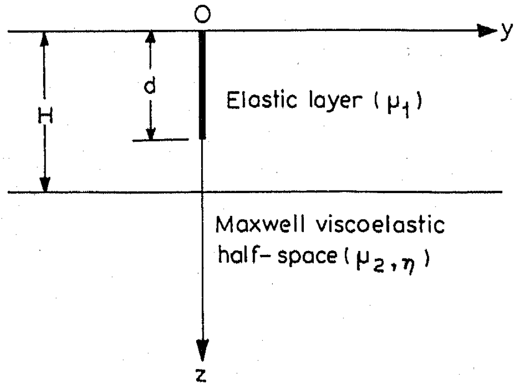

Figure 1. Section of the model by the plane $x = 0$.

2.2 *Viscoelastic solution*

We write (9) and (10) in the form

$$u_1 = \frac{1}{\pi}\left[N_0 \tan^{-1}(d/y) + \sum_{n=1}^{\infty} N_n \tan^{-1}\left(\frac{2dy}{y^2 + 4n^2H^2 - d^2}\right)\right], \tag{11}$$

$$p_{12} = \frac{1}{\pi}\left[N_0\left(\frac{-d}{y^2+d^2}\right) + \sum_{n=1}^{\infty} N_n \times \left\{\frac{2nH-d}{y^2+(2nH-d)^2} - \frac{2nH+d}{y^2+(2nH+d)^2}\right\}\right], \tag{12}$$

where

$$N_0 = \Delta u_1, N_n = \Delta u_1\left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^n. \tag{13}$$

We now use the correspondence principle (Fung 1965) to obtain the quasi-static deformation field for a model consisting of an elastic layer lying over a Maxwell viscoelastic half-space. For the elastic layer

$$p_{12} = 2\mu_1 e_{12}. \tag{14}$$

For the Maxwell viscoelastic half-space

$$\dot{e}_{12} = (1/2\mu_2)\dot{p}_{12} + [(1/\eta)p_{12}], \tag{15}$$

where η is the viscosity and the dot (˙) signifies time-differentiation. Taking the Laplace transform of (15), we obtain

$$s\bar{e}_{12} = (s/2\mu_2)\bar{p}_{12} + [(1/\eta)\bar{p}_{12}], \tag{16}$$

where s is the Laplace transform variable. We may write (16) in the form

$$\bar{p}_{12} = 2\mu_2^*\bar{e}_{12}, \tag{17}$$

where

$$\mu_2^* = s\mu_2/(s + 2\tau^{-1}), \tag{18}$$

is the transform rigidity and $\tau = \eta/\mu_2$ is the relaxation time. Time dependence of the dislocation source is taken to be a step-function, i.e.

$$\Delta u_1(t) = U_0 H(t), \tag{19}$$

where U_0 and $H(t)$ are respectively the dislocation and the Heaviside step function. Then

$$\overline{\Delta u_1(t)} = U_0/s. \tag{20}$$

In order to obtain the Laplace transformed solution of the viscoelastic problem, it is only necessary to replace μ_2 and Δu_1 by μ_2^* and $\overline{\Delta u_1}$ respectively in the corresponding elastic solution. From (11) and (12) we notice that μ_2 and Δu_1 appear only in the expressions for N_0 and N_n. Therefore, the Laplace transformed solution of the viscoelastic problem is obtained from (11) and (12) on replacing N_0 and N_n by $\bar{N}_0$ and $\bar{N}_n$ respectively, where, from (13), (18) and (20)

$$\bar{N}_0 = (U_0/s), \quad \bar{N}_n = U_0 G_n(s), \tag{21}$$

$$G_n(s) = (Bs + A)^n / s(s + A)^n, \tag{22}$$

$$A = 2\mu_1/(\mu_1 + \mu_2)\tau, \quad B = (\mu_1 - \mu_2)/(\mu_1 + \mu_2). \tag{23}$$

In order to find the inverse Laplace transform of $G_n(s)$, we use a transform integral listed in Erdélyi (1954). We find that

$$L^{-1}[G_n(s)] = 1 + \exp(-At) \sum_{m=1}^{n} \frac{F_{2m}(-A)}{(n-m)!(m-1)!} t^{n-m}, \tag{24}$$

where

$$F_{2m}(s) = (d^{m-1}/ds^{m-1})[(Bs + A)^n/s]. \tag{25}$$

Equations (11), (12), (21) and (24) yield ($t > 0$)

$$u_1 = \frac{U_0}{\pi}\left[\tan^{-1}\left(\frac{d}{y}\right) + \sum_{n=1}^{\infty}\left\{1 + \exp(-At)\sum_{m=1}^{n}\frac{F_{2m}(-A)}{(n-m)!(m-1)!}t^{n-m}\right\} \times \tan^{-1}\left(\frac{2dy}{y^2 + 4n^2H^2 - d^2}\right)\right], \tag{26}$$

$$p_{12} = \frac{U_0}{\pi}\mu_1\left[\frac{-d}{y^2 + d^2} + \sum_{n=1}^{\infty}\left\{1 + \exp(-At)\sum_{m=1}^{n}\frac{F_{2m}(-A)}{(n-m)!(m-1)!}t^{n-m}\right\} \times \left\{\frac{2nH - d}{y^2 + (2nH - d)^2} - \frac{2nH + d}{y^2 + (2nH + d)^2}\right\}\right]. \tag{27}$$

Putting $\mu_1 = \mu_2$ in (27), we get the solution obtained by Bonafede *et al* (1984) for the shear stress p_{12} as a particular case.

3. Particular case

Let $\quad \mu_1 = \mu_2 = \mu$ (say). (28)

Then $\quad A = \tau^{-1}, \quad B = 0,$ (29)

and $\quad F_{2m}(-A) = -(m-1)!/\tau^{n-m}.$ (30)

Equations (26) and (27) give ($t > 0$)

$$u_1 = \frac{U_0}{\pi}\left[\tan^{-1}(d/y) + \sum_{n=1}^{\infty}\left\{1 - \exp(-t/\tau)\sum_{k=0}^{n-1}\frac{(t/\tau)^k}{k!}\right\} \times \tan^{-1}\left(\frac{2dy}{y^2 + 4n^2H^2 - d^2}\right)\right], \tag{31}$$

$$p_{12}=\frac{U_0}{\pi}\mu\left[\frac{-d}{y^2+d^2}+\sum_{n=1}^{\infty}\left\{1-\exp(-t/\tau)\sum_{k=0}^{n-1}\frac{(t/\tau)^k}{k!}\right\}\right.$$
$$\left.\times\left\{\frac{2nH-d}{y^2+(2nH-d)^2}-\frac{2nH+d}{y^2+(2nH+d)^2}\right\}\right]. \tag{32}$$

Equations (31) and (32) give respectively the quasi-static surface displacement parallel to the fault and the horizontal shear stress caused by a long vertical strike-slip fault $(-\infty<x<\infty, 0\leqslant z\leqslant d\leqslant H)$ in an elastic layer lying over a Maxwell viscoelastic half-space.

The case $t=0$ corresponds to the elastic problem. Equations (7), (9), (10) and (28) then yield

$$u_1=\frac{U_0}{\pi}\tan^{-1}(d/y), \quad p_{12}=\frac{U_0}{\pi}\mu\left(\frac{-d}{y^2+d^2}\right), \tag{33}$$

where $\Delta u_1=U_0$.

Since we have taken $\mu_1=\mu_2=\mu$, (33) gives the field due to a long vertical strike-slip fault $(-\infty<x<\infty, 0\leqslant h\leqslant d)$ situated in an elastic half-space.

4. Numerical results

We wish to study the variation of the displacement and the stress fields generated by a long strike-slip fault in an elastic layer lying over a Maxwell viscoelastic half-space. In (31) and (32), we have obtained the expressions for the parallel quasi-static displacement and the shear stress. For numerical computation, we define the dimensionless quantities α, T, Y, U_1 and P_{12} through the relations

$$d=\alpha H, \quad t=T\tau, \quad y=YH,$$
$$u_1=(U_0/\pi)U_1, \quad p_{12}=(U_0\mu/\pi H)P_{12}. \tag{34}$$

Using (34), equations (31) and (32) yield $(T>0)$

$$U_1=\tan^{-1}(\alpha/y)+\sum_{n=1}^{\infty}\left[1-\exp(-T)\sum_{k=0}^{n-1}\frac{T^k}{k!}\right]$$
$$\times\tan^{-1}\left(\frac{2\alpha Y}{Y^2+4n^2-\alpha^2}\right), \tag{35}$$

$$P_{12}=\frac{-\alpha}{Y^2+\alpha^2}+\sum_{n=1}^{\infty}\left[1-\exp(-T)\sum_{k=0}^{n-1}\frac{T^k}{k!}\right]$$
$$\times\left[\frac{2n-\alpha}{Y^2+(2n-\alpha)^2}-\frac{2n+\alpha}{Y^2+(2n+\alpha)^2}\right], \tag{36}$$

where U_1, P_{12}, T and Y are respectively the dimensionless displacement, shear stress, time and horizontal distance. In the elastic case, (33) gives

$$U_1=\tan^{-1}(\alpha/y), \quad P_{12}=-\alpha/(Y^2+\alpha^2). \tag{37}$$

Since

$$\left[\exp(-T)\sum_{k=0}^{n-1}\frac{T^k}{k!}\right] < 1$$

for all n and $T > 0$, it is obvious that the infinite series appearing on the right hand side of (35) converges at least as rapidly as the infinite series $\sum(1/n^2)$. Similarly, noting that

$$\frac{2n-\alpha}{Y^2+(2n-\alpha)^2} - \frac{2n+\alpha}{Y^2+(2n+\alpha)^2} = \frac{2\alpha\{4n^2-(Y^2+\alpha^2)\}}{[Y^2+(2n-\alpha)^2][Y^2+(2n+\alpha)^2]}$$

the infinite series appearing on the right hand side of (36) also converges at least as rapidly as the infinite series $\sum(1/n^2)$. In our numerical computation, we found that the first 10 terms of the infinite series are adequate.

Figures 2–4 show the variation of the dimensionless surface displacement U_1 parallel to the fault with the dimensionless horizontal distance Y from the fault for three values of the vertical extent of the source namely $d = H, H/2$ and $H/10$ and three values of the dimensionless time $T(T = 0, 1$ and $10)$. For all values of d and T, $U_1 = \pi/2$ when $Y = 0$ (see equations (35) and (37)). The graphs for $T = 0$ correspond to the elastic case. From (37), we find that, in the elastic case, the parallel horizontal displacement $U_1 \to 0$ for all values of d as $Y \to \infty$. We note that the deviation of the viscoelastic solution from the elastic solution increases as d increases for a given value of Y. Similarly, the deviation of the viscoelastic solution from the elastic solution increases as Y increases for a fixed value of d.

Figures 5–7 exhibit the variation of the dimensionless horizontal shear stress P_{12} with the dimensionless horizontal distance Y from the fault for three values of the vertical extent of the source, namely, $d = H$, $H/2$, $H/10$ and different values of the dimensionless time T. The graphs for $T = 0$ correspond to the elastic case for which $P_{12} = -H/d$ when $Y = 0$. Also, $P_{12} \to 0$ as Y approaches infinity [see equation (37)]. For $d = H$, the graphs for $T = 1$ and $T = 5$ are quite different from the graph for the elastic case (figure 5). For $d = H/2$, there is only a slight difference between the graphs

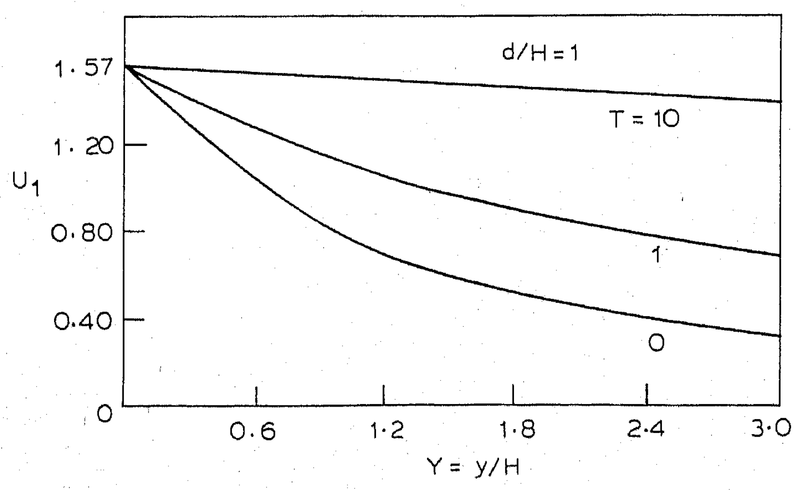

Figure 2. Variation of parallel displacement U_1 with horizontal distance Y when $d = H$.

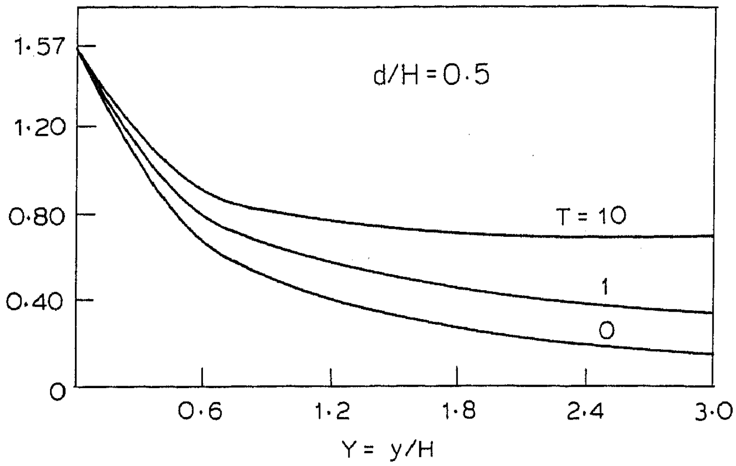

Figure 3. Variation of parallel displacement U_1 with horizontal distance Y when $d = H/2$.

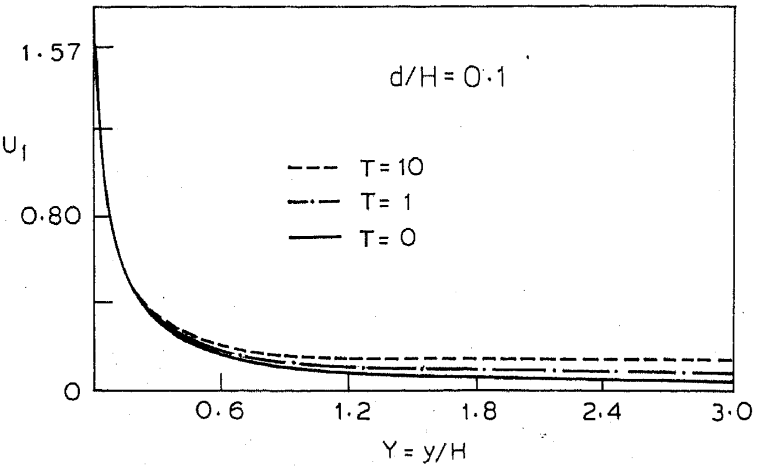

Figure 4. Variation of parallel displacement U_1 with horizontal distance Y when $d = H/10$.

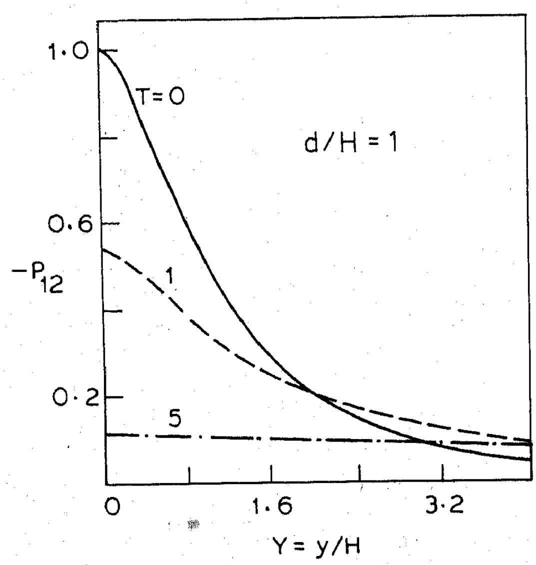

Figure 5. Variation of shear stress $-P_{12}$ with horizontal distance Y when $d = H$.

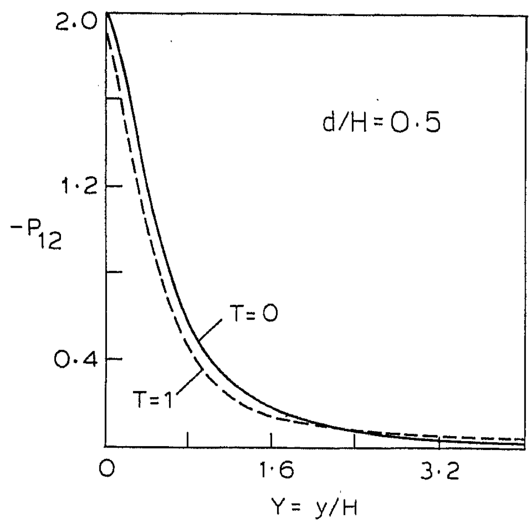

Figure 6. Variation of shear stress $-P_{12}$ with horizontal distance Y when $d = H/2$.

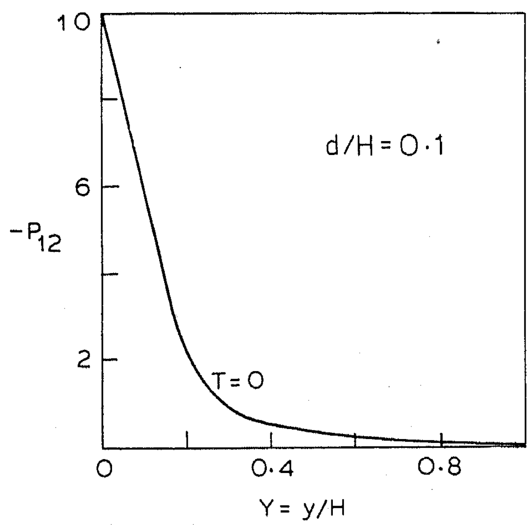

Figure 7. Variation of shear stress $-P_{12}$ with horizontal distance Y when $d = H/10$.

for the viscoelastic case and the elastic case (figure 6). For $d = H/10$, the graphs for the viscoelastic case almost coincide with the corresponding graph for the elastic case (figure 7).

5. Discussion

Equations (9) and (10) give the surface displacement and the surface shear stress respectively caused by a long vertical strike-slip fault in a homogeneous isotropic layer of thickness H lying over a homogeneous isotropic elastic half-space. These results are in complete agreement with the corresponding results of Rybicki (1971) who used the method of images to obtain these results while we have obtained them directly. The advantage of our method is that it can also be used when there is more than one layer lying over the half-space.

We have used the correspondence principle of linear viscoelasticity to obtain the quasi-static displacement and stress fields for a model consisting of an elastic layer lying over a Maxwell viscoelastic half-space. Our results are particularly useful for studying the time-dependent post-seismic deformation associated with large earthquakes. Mukhopadhyay and Mukherji (1979) and Mukherji *et al* (1980) discussed the problem of a long strike-slip fault in an elastic layer lying over a viscoelastic half-space without using the correspondence principle. In addition, these authors included the effect of initial stress and assumed that the shear stress is maintained far away from the fault by tectonic forces. The results of Mukhopadhyay and Mukherji (1979) and Mukherji *et al* (1980) are useful for studying the stress accumulation near an earthquake fault. The correspondence principle is normally applicable if the material is stress-free and strain-free before the fault movement, and we have made this assumption in our model. Similarly, since our main interest was to determine the displacement and the stress fields generated by the fault movement, we have not considered the shear stress maintained far away from the fault by tectonic forces.

Acknowledgement

One of the authors (SJS) is grateful to the Council of Scientific and Industrial Research, New Delhi for financial support.

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