Tunneling conductance of graphene NIS junctions

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(Dated: February 6, 2008)

We show that in contrast to conventional normal metal-insulator-superconductor (NIS) junctions, the tunneling conductance of a NIS junction in graphene is an oscillatory function of the effective barrier strength of the insulating region, in the limit of a thin barrier. The amplitude of these oscillations are maximum for aligned Fermi surfaces of the normal and superconducting regions and vanishes for large Fermi surface mismatch. The zero-bias tunneling conductance, in sharp contrast to its counterpart in conventional NIS junctions, becomes maximum for a finite barrier strength. We also suggest experiments to test these predictions.

PACS numbers: 74.45+c, 74.78.Na

Graphene, a two-dimensional single layer of graphite, has been recently fabricated by Novoselov *et. al.* [1]. This has provided an unique opportunity for experimental observation of electronic properties of graphene which has attracted theoretical attention for several decades [2]. In graphene, the energy bands touch the Fermi energy at six discrete points at the edges of the hexagonal Brillouin zone. Out of these six Fermi points, only two are inequivalent; they are commonly referred to as K and K' points [3]. The quasiparticle excitations about these K and K' points obey linear Dirac-like energy dispersion. The presence of such Dirac-like quasiparticles is expected to lead to a number of unusual electronic properties in graphene including relativistic quantum hall effect with unusual structure of Hall plateaus [4]. Recently, experimental observation of the unusual plateau structure of the Hall conductivity has confirmed this theoretical prediction [5]. Further, as suggested in Ref. [6],the presence of such quasiparticles in graphene provides us with an experimental test bed for Klein paradox [7]

Another, less obvious but nevertheless interesting, consequence of the existence Dirac-like quasiparticles can be understood by studying tunneling conductance of a normal metal-superconductor (NS) interface of graphene [8]. Graphene is not a natural superconductor. However, superconductivity can be induced in a graphene layer in the presence of a superconducting electrode near it via proximity effect [8, 9, 10]. It has been recently predicted [8] that a graphene NS junction, due to the Dirac-like energy spectrum of its quasiparticles, can exhibit specular Andreev reflection in contrast to the usual retro reflection observed in conventional NS junctions [11, 12]. Such specular Andreev reflection process leads to qualitatively different tunneling conductance curves compared to conventional NS junctions [8]. However, the effect of the presence of a potential barrier between the normal and superconducting regions in graphene NS junction has not been studied so far.

In this letter, we study the tunneling conductance of a normal metal-insulator-superconductor (NIS) junction of graphene in the limit of thin barrier. We show that in contrast to the conventional NIS junctions, the tunneling conductance of a graphene NIS junction is an oscillatory function of the effective barrier strength. The amplitude of these oscillations is maximum for aligned Fermi surfaces of the normal and superconducting regions and vanishes for large Fermi surface mismatch. In particular, we point out that the zero-bias conductance, in complete contrast to its behavior in conventional NIS junctions, reaches its maximum value for a finite barrier strength. By using the fact that the effective barrier height can be tuned experimentally by changing a gate voltage [5, 6], we suggest an experimental setup where these effects can be observed. We also point out that our analysis reproduces the results of previous work on graphene NS junctions as a special case [8].

We consider a NIS junction in a graphene sheet occupying the xy plane with the normal region extending $x = -\infty$ to $x = -d$ for all y. The region I, modeled by a barrier potential V_0, extends from $x = -d$ to $x = 0$ while the superconducting region occupies $x \geq 0$. Such a local barrier can be implemented by either using the electric field effect or local chemical doping [5, 6]. The region $x \geq 0$ is to be kept close to an superconducting electrode so that superconductivity is induced in this region via proximity effect [8, 9]. In the rest of this work, we shall assume that the barrier region has sharp edges on both sides. This condition requires that $d \ll 2\pi/k_F$, where k_F is the Fermi wave-vector for graphene, and can be realistically created in experiments [6]. The NIS junction can then be described by the Dirac-Bogoliubov-de Gennes (DBdG) equations [8]

$$\begin{pmatrix} \mathcal{H}_a - E_F + U(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & E_F - U(\mathbf{r}) - \mathcal{H}_a \end{pmatrix} \psi_a = E\psi_a. \tag{1}$$

Here, $\psi_a = (\psi_{A\,a}, \psi_{B\,a}, \psi^*_{A\,\bar{a}}, -\psi^*_{B\,\bar{a}})$ are the 4 component wavefunctions for the electron and hole spinors, the index a denote K or K' for electron/holes near K and K' points, $\bar{a}$ takes values $K'(K)$ for $a = K(K')$, E_F denote the Fermi energy, A and B denote the two inequivalent sites in the hexagonal lattice of graphene, and the Hamiltonian $\mathcal{H}_a$ is given by

$$\mathcal{H}_a = -i\hbar v_F \left(\sigma_x \partial_x + \mathrm{sgn}(a)\sigma_y \partial_y\right). \tag{2}$$

In Eq. 2, v_F denotes the Fermi velocity of the quasiparticles in graphene and sgn(a) takes values $\pm$ for $a = K(K')$.

The pair-potential $\Delta(\mathbf{r})$ in Eq. 1 connects the electron and the hole spinors of opposite Dirac points. We have modeled the pair-potential as $\Delta(\mathbf{r}) = \Delta_0 \exp(i\phi)\theta(x)$, where Δ_0 and ϕ are the amplitude and the phase of the induced superconducting order parameter respectively and θ is the Heaviside step function. The potential $U(\mathbf{r})$ gives the relative shift of Fermi energies in normal, insulating and superconducting regions of graphene and can be modeled as $U(\mathbf{r}) = -U_0\theta(x) + V_0\theta(-x)\theta(x+d)$. At this stage, we introduce the dimensionless barrier strength

$$\chi = V_0 d/\hbar v_F, \tag{3}$$

which is going to play a key role in the subsequent analysis. In particular, we define a thin barrier as one with $V_0 \to \infty$ and $d \to 0$ such that χ remains finite. For the NS junction studied in Ref. [8], $\chi = 0$. The gate potential U_0 can be used to tune the Fermi surface mismatch between the normal and the superconducting regions. Notice that the mean-field conditions for superconductivity is satisfied as long as $\Delta_0 \ll (U_0 + E_f)$; thus, in principle, for large U_0 one can have regimes where $\Delta_0 \geq E_f$ [8].

Eq. 1 can be solved in a straightforward manner to yield the wavefunction ψ in the normal, insulating and the superconducting regions. In the normal region, for electron and holes traveling the $\pm x$ direction with a transverse momentum $k_y = q$ and energy ϵ, the wavefunctions are given by

$$\begin{aligned}
\psi_N^{e\pm} &= \left(1, \pm e^{\pm i\alpha}, 0, 0\right) \exp\left[i\left(\pm k_n x + qy\right)\right], \\
\psi_N^{h\pm} &= \left(0, 0, 1, \mp e^{\pm i\alpha'}\right) \exp\left[i\left(\pm k'_n x + qy\right)\right], \\
\sin(\alpha) &= \frac{\hbar v_F q}{\epsilon + E_F}, \quad \sin(\alpha') = \frac{\hbar v_F q}{\epsilon - E_F},
\end{aligned} \tag{4}$$

where for the electron wavefunctions $k_n = (\epsilon + E_F)\cos(\alpha)/\hbar v_F$ and α is the angle of incidence of the electron. Similarly for the hole wavefunctions, $k'_n = (\epsilon - E_F)\cos(\alpha')/\hbar v_F$ with angle of incidence α'. Note that for an Andreev process to take place, the maximum angle of incidence for an electron is given by $\alpha_c = \arcsin\left[|\epsilon - E_F| / (\epsilon + E_F)\right]$ [8].

In the barrier region, one can similarly obtain $\psi_B^{e\pm} = \left(1, \pm e^{\pm i\theta}, 0, 0\right) \exp\left[i\left(\pm k_b x + qy\right)\right]$ and $\psi_B^{h\pm} = \left(0, 0, 1, \mp e^{\pm i\theta'}\right) \exp\left[i\left(\pm k'_b x + qy\right)\right]$ for electron and holes moving along $\pm x$. Here the angle of incidence of the electron(hole) $\theta(\theta')$ is given by $\sin\left[\theta(\theta')\right] = \hbar v_F q / \left[\epsilon + (-)(E_F - V_0)\right]$ and $k_b(k'_b) = \left[\epsilon - (+)(E_F - V_0)\right]\cos\left[\theta(\theta')\right]/\hbar v_F$. Note that in the limit of thin barrier, $\theta, \theta' \to 0$ and $k_b d, k'_b d \to \chi$.

In the superconducting region, the BdG quasiparticles are mixtures of electron and holes. Consequently, the wavefunctions of the BdG quasiparticles moving along $\pm x$ with transverse momenta q and energy ϵ has the form

$$\begin{aligned}
\psi_S^{\pm} &= \left(e^{\mp i\beta}, \mp e^{\pm i(\gamma - \beta)}, e^{-i\phi}, \mp e^{i(\pm\gamma - \phi)}\right), \\
&\quad \times \exp\left[i\left(\pm k_s x + qy\right) - \kappa x\right], \\
\sin(\gamma) &= \hbar v_F q / (E_F + U_0),
\end{aligned} \tag{5}$$

where $k_s = \sqrt{\left[\left(U_0 + E_F\right)/\hbar v_F\right]^2 - q^2}$ and γ is the angle of incidence for the quasiparticles. Here $\kappa^{-1} = (\hbar v_F)^2 k_s / \left[(U_0 + E_F)\Delta_0 \sin(\beta)\right]$ is the localization length and β is given by

$$\begin{aligned}
\beta &= \cos^{-1}\left(\epsilon/\Delta_0\right) \quad \text{if } |\epsilon| < \Delta_0, \\
&= -i\cosh^{-1}\left(\epsilon/\Delta_0\right) \quad \text{if } |\epsilon| > \Delta_0,
\end{aligned} \tag{6}$$

Note that for $|\epsilon| > \Delta_0$, κ becomes imaginary and the quasiparticles can propagate in the bulk of the superconductor.

Let us now consider an electron incident on the barrier from the normal side with an energy ϵ and transverse momentum q. The wave functions in the normal, insulating and superconducting regions, taking into account both Andreev and normal reflection processes, can then be written as [12]

$$\begin{aligned}
\Psi_N &= \psi_N^{e+} + r\psi_N^{e-} + r_A \psi_N^{h-}, \quad \Psi_S = t\psi_S^{+} + t'\psi_S^{-}, \\
\Psi_B &= p\psi_B^{e+} + q\psi_B^{e-} + m\psi_B^{h+} + n\psi_N^{h-},
\end{aligned} \tag{7}$$

where r and r_A are the amplitudes of normal and Andreev reflections respectively, t and t' are the amplitudes of electron-like and hole-like quasiparticles in the superconducting region and p, q, m and n are the amplitudes of electron and holes in the barrier. These wavefunctions must satisfy the appropriate boundary conditions:

$$\Psi_N|_{x=-d} = \Psi_B|_{x=-d}, \quad \Psi_B|_{x=0} = \Psi_S|_{x=0}. \tag{8}$$

Notice that these boundary conditions, in contrast their counterparts in standard NIS interfaces, do not impose any constraint on derivative of the wavefunctions at the boundary. Thus the standard delta function potential approximation for thin barrier [12] can not be taken the outset, but has to be taken at the end of the calculation.

Using the boundary conditions (Eq. 8), one can now solve for the coefficients r and r_A in Eq. 7. After some straightforward but cumbersome algebra, we find that in the limit of thin barriers, the expressions for r t, t', and r_A depend on the dimensionless coefficient χ as

$$\begin{aligned}
r &= \frac{\cos(\chi)\left(e^{i\alpha} - \rho\right) - i\sin(\chi)\left(1 - \rho e^{i\alpha}\right)}{\cos(\chi)\left(e^{-i\alpha} + \rho\right) + i\sin(\chi)\left(1 + \rho e^{-i\alpha}\right)}, \\
t' &= \frac{\cos(\chi)\left(1 + r\right) - i\sin(\chi)\left(e^{i\alpha} - re^{-i\alpha}\right)}{\Gamma e^{-i\beta} + e^{i\beta}}, \quad t = \Gamma t', \\
r_A &= \frac{t'\left(\Gamma + 1\right)e^{-i\phi}}{\cos(\chi) - ie^{-i\alpha'}\sin(\chi)},
\end{aligned} \tag{9}$$

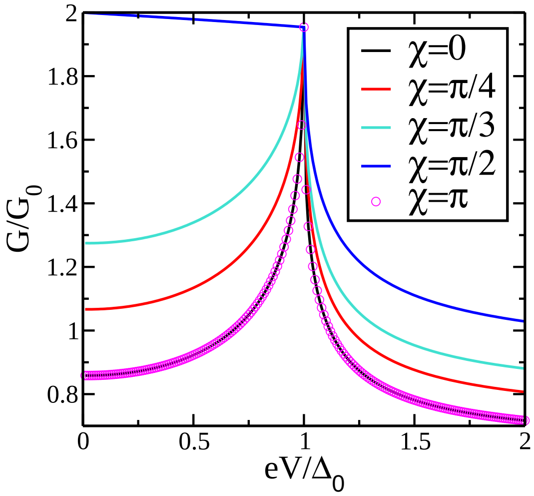

FIG. 1: Plot of tunneling conductance of a NIS junction graphene as a function of bias voltage for different effective barrier strengths for $U_0 = 0$ and $\Delta_0 = 0.01E_F$. Note that the curves for $\chi = 0$ (black line) and $\chi = \pi$(pink circles) coincide reflecting π periodicity.

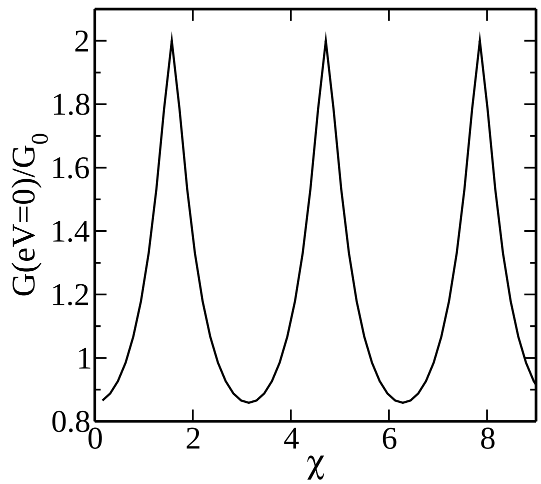

FIG. 2: Plot of zero-bias tunneling conductance as a function of effective barrier potential χ for $U_0 = 0$ and $\Delta_0 = 0.01E_F$. The π periodic oscillatory behavior with maxima at $\chi = (n+1/2)\pi$ is to be contrasted with the monotonous decay of $G(eV = 0)$ with increasing χ in conventional NIS junctions

where the parameters Γ and ρ can be expressed in terms of γ, β, α, and α' (Eqs. 4, 5, and 6) as

$$\Gamma = \frac{e^{-i\gamma} - \eta}{e^{i\gamma} + \eta}, \quad \eta = \frac{e^{-i\alpha'}\cos(\chi) - i\sin(\chi)}{\cos(\chi) - ie^{-i\alpha'}\sin(\chi)},$$
$$\rho = \frac{e^{-i(\gamma-\beta)} - \Gamma e^{i(\gamma-\beta)}}{\Gamma e^{-i\beta} + e^{i\beta}}. \qquad (10)$$

The tunneling conductance of the NIS junction can now be expressed in terms of r and r_A by [12]

$$G(eV) = G_0 \int_0^{\alpha_c} \left(1 - |r|^2 + |r_A|^2 \frac{\cos(\alpha')}{\cos(\alpha)}\right) \cos(\alpha)\, d\alpha, \qquad (11)$$

where $G_0 = 4e^2 N(eV)/h$ is the ballistic conductance of metallic graphene, eV denotes the bias voltage, and $N(eV) = (E_f + \epsilon)w/(\pi\hbar v_F)$ denotes the number of available channels for a graphene sample of width w. Note that for $eV \ll E_F$, G_0 is a constant. Eq. 11 can be evaluated numerically to yield the tunneling conductance of the NIS junction for arbitrary parameter values.

Eqs. 9 and 10 represent the key result of this work. From these equations, we find that in contrast to conventional NIS junctions [12], both r and r_A are *oscillatory functions of the effective barrier potential* χ *for any angle of incidence* $\alpha < \alpha_c$. Consequently, we expect $G(eV)$ (Eq. 11) to demonstrate oscillatory behavior as a function of χ with a period π. We also note that our work reproduces the results of Ref. 8 as a special case when $\chi = n\pi$ for any integer n.

Let us now consider the regime where the Fermi surfaces of the normal metal and the superconductor is aligned ($U_0 = 0$) and $\Delta_0 \ll E_F$. A plot of the tunneling conductances as a function of the bias voltage eV for different barrier strength χ, shown in Fig. 1, confirms the π periodic oscillatory behavior. The oscillation amplitude is maximum for zero-bias ($eV = 0$), as shown in Fig. 2, and vanishes at the gap edge ($eV = \Delta_0$). From Fig. 1, we find two noteworthy features. First, the tunneling conductance at the gap edge reaches a value close to $2G_0$ independent of the barrier strength, and second, the subgap tunneling conductance becomes close to $2G_0$ when $\chi = (n+1/2)\pi$ for any integer n and any $eV \le \Delta_0$.

Both of the above-mentioned features can be understood by noting that when $eV \ll E_F$, $\alpha \simeq -\alpha' \simeq \gamma$ (Eq. 4). In this limit, using Eqs. 9 and 10, it can be shown that the reflection amplitude, for $eV \le \Delta_0$, becomes $r \simeq \mathcal{N}\left[\cos(\chi) + i\sin(\chi)\cos(\alpha)\right]/\mathcal{D}$, where $\mathcal{D} = \left[\cos(\alpha)\left[\cos(\beta)\cos(2\chi) + i\sin(\beta)\right] + 2\sin(2\chi)\sin(\beta)\sin^2(\alpha)\right]$ and

$$\mathcal{N} = 2\sin(\alpha)\left[\sin(\chi+\beta) - \sin(\chi-\beta)\right]. \qquad (12)$$

Note that $\mathcal{N}$ and hence r vanishes and when $\alpha = 0$ or $\sin(\chi+\beta) = \sin(\chi-\beta)$. The former condition ($\alpha = 0$) is a manifestation of the well-known Klein paradox [7] and occurs since scattering of normal-incident Dirac electrons from a potential barrier can not change their chirality [6]. This effect, however, is not manifested easily in the tunneling conductance since G receives contribution from electron approaching the barrier with all possible incidence angles $\alpha \le \alpha_c$. The latter equality ($\sin(\chi+\beta) = \sin(\chi-\beta)$) represents condition for transmission resonance ($r = 0$ and $|r_A| = 1$) of a graphene NIS interface and has the solutions $\beta = 0$ and $\chi = (n+1/2)\pi$.

Such transmission resonances occur for all barrier strengths χ and angle of incidence α when $eV = \Delta_0$ (*i.e.*

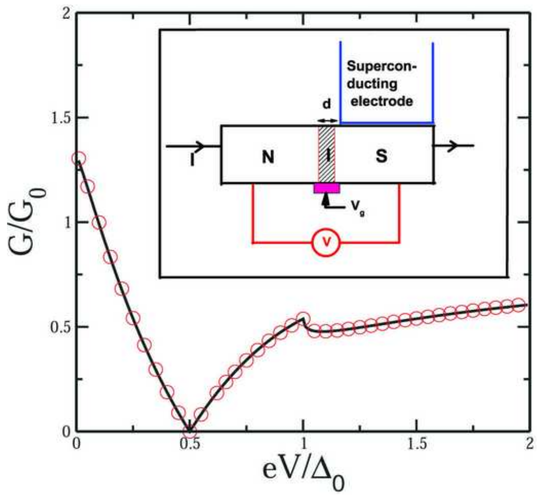

FIG. 3: Tunneling conductance as a function of bias voltage for effective barrier strengths $\chi = 0$ (solid line) and $\chi = \pi/2$ (open circle) for $U_0 = 25E_F$ and $\Delta_0 = 2E_F$. The tunneling conductance becomes barrier independent and vanishes for $eV = E_F$. The inset shows a schematic experimental setup. The dashed region sees a variable gate (shown as pink filled region) voltage V_0 which creates the barrier. Additional gate voltage U_0, which may be applied on the superconducting side, and the current source is not shown to avoid clutter.

$\beta = 0$). Consequently, $G(eV = \Delta_0) \simeq 2G_0$ independent of the barrier strength, as is also well-known for conventional NIS junctions [12]. The novel aspect of a graphene NIS junction comes from the second class of solution of the transmission resonance condition: $\chi = (n + 1/2)\pi$. At these special values of the barrier, $G \simeq 2G_0$ for any subgap voltage as long as $eV \ll E_F$ [13]. This feature, clearly seen in Figs. 1 and 2, is in sharp contrast to conventional NIS junction where $G(eV < \Delta_0)$ always decreases with increasing barrier strength [12]. For $\chi \neq (n + 1/2)\pi$ or $eV \neq \Delta_0$, $r \neq 0$ and consequently $|r_A| < 1$ so that $G < 2G_0$. In particular, r reaches a maxima leading to minimum value of subgap tunneling conductance for $\chi = n\pi$. Thus, in contrast to conventional NS junctions, $G(eV = 0) < 2G_0$ for $\chi = 0$ in graphene NS junctions as noted earlier in Ref. [8].

Next, we briefly explore the regime where $\Delta_0 \geq E_F$ and $U_0 \gg E_F$, so that $\Delta_0 \ll (U_0 + E_F)$ [8]. Here the tunneling conductance, shown in Fig. 3 for $\Delta_0 = 2E_F$ and $\chi = 0, \pi/2$, becomes independent of barrier strength. This can be intuitively understood from the fact that a large Fermi surface mismatch acts as an effective barrier for the electrons tunneling at the interface [12] which makes the presence of an additional barrier irrelevant. Further, as seen from Fig. 3, the tunneling conductance vanishes for $eV = E_F = 0.5\Delta_0$ due to the absence of Andreev reflection since $\alpha_c = 0$ at this bias-voltage. Our results in this limit therefore becomes identical to those for NS junction studied in Ref. [8].

Finally, we discuss possible experimental setup (shown in inset of Fig. 3) to test our predictions. In the experiment, the local barrier can be fabricated using methods of Ref. [5]. The easiest experimentally achievable regime corresponds to $\Delta_0 \ll E_F$ with aligned Fermi surfaces for the normal and superconducting regions. We suggest measurement of tunneling conductance curves at zero-bias ($eV = 0$) in this regime. Our prediction is that the zero-bias conductance will show an oscillatory behavior with change of effective bias voltage with maxima(minima) when χ becomes odd(even) integer multiples of $\pi/2$. In graphene, typical Fermi energy is $E_F \simeq$ 80meV and the Fermi-wavelength is $\lambda = 2\pi/k_F \simeq 100$nm [5, 6]. For realization of the thin and sharp barriers discussed in this work, one needs $d/\lambda << 1$ and $V_0/E_F \gg 1$. Effective barrier strengths of $500 - 1000$meV and barrier widths of $d \simeq 20 - 10$ nm, which can be achieved in realistic experiments [5, 6], shall therefore meet the demands of the proposed experimental setup. To observe the oscillatory behavior of the zero-bias tunneling conductance, it would be necessary to change χ in small steps $\delta\chi$. For barriers of a fixed width, with values of $d/\lambda = 0.1$ and $V_0/E_F = 10$, this would require changing V_0 in steps of approximately 12meV which corresponds to $\delta\chi = 0.1$.

In conclusion, we have a presented a theory of tunneling conductance of graphene NIS junctions. We have demonstrated that the tunneling conductance exhibits novel π periodic oscillatory behavior as a function of barrier strength of the junction and have suggested experiments to observe this effect. The authors thank S.M. Bhattacharjee, A. Ghosh, P.K. Mohanty,M. Khan, T. Senthil and V. B. Shenoy for valuable discussions and Graduate Associateship Program at Saha Institute which made this work possible.

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