A Domain-Specific Crossover and a Helper Objective for Generating Minimum Weight Compliant Mechanisms

Deepak Sharma*   Kalyanmoy Deb   N. N. Kishore

KanGAL Report Number K2008001
Indian Institute of Technology Kanpur
Kanpur-208016, U.P., India
{dsharma,deb,nnk}@iitk.ac.in

ABSTRACT

A domain-specific crossover with NSGA-II algorithm is proposed for generating the compliant mechanisms. The new-crossover divides a design-domain into sub-domains and exchanges the areas of sub-domains between two-parents of GA population. A single objective analysis of minimizing the weight of structure is performed first and topology is named as *reference*-design. Thereafter, a bi-objective optimization problem is formulated with primary and helper objectives of minimizing the weight and maximizing the diversity of structures with respect to *reference*-design. The diversity is evaluated by comparing the dissimilarities in bit value at each gene position of binary strings of reference-design and GA evolved structure. Motivation behind the use helper-objective and new-crossover is to generate a diverse set of non-dominated NSGA-II solutions which accomplish a task of tracing a user-defined path. Therefore, both optimization problems are subjected to constraints of allowed deviation at precision points of a prescribed path with a constraint on stress, and are solved using a local search based NSGA-II procedure. During the optimization, compliant mechanisms can have any support and loading positions with varying input-displacement magnitude within their respective domains. Obtained results are further compared with the results of another-crossover operator based study which was used to generate compliant mechanisms.

Categories and Subject Descriptors

Real-World Applications [**Design/Synthesis and Mechanical Engineering**]: Compliant Mechanism.

1. INTRODUCTION

Compliant mechanisms are flexible elastic structures which can deform to transmit the force and/or generating some desired path on the application of applied load. Compliant mechanisms have shown many advantages over pseudo-rigid-body mechanisms as jointless and monolithic structures, involved less friction, wear and noise [12], ease of manufacturing without assembly, light weight devices [2] etc. Applications of compliant mechanisms are in the area of product design, off-shore structures, smart structures, MEMS [1] etc.

In this paper, a few important studies incorporating a binary (0-1) representation of material are presented. In the early studies, a genetic algorithm (GA) was used for structural topology optimization using single point crossover and bitwise mutation operators [4]. Extending the idea of using GA, the structural optimization problems with different kinds of objective functions and constraints were solved. Later, a two-dimensional crossover operator which divides the design domain into four rectangular sub-domains was used for further improving the GA evolved designs [3, 10, 13]. After the feasibility of GA-based optimal designs, the study [14] emphasized on its flexibility and demonstrated its potentialities over the classical optimization methods. A special two-dimensional crossover operator and a mutation operator were introduced to solve the various structural topology optimization problems.

Later, a novel morphological representation scheme using the arrangement of skeleton and flesh to define a structure was introduced for designing the structural and compliant mechanism problems [22, 21, 24]. The chromosome string was designed to store the control points of the Bezier curves and the associated thickness of each curve. Genetic algorithm was used as a global optimizer for designing and synthesis of path generation compliant mechanism problem which was defined by minimizing the deviation between the distance of desired and corresponding actual trajectories after dividing them into N precision points [23]. Different ways of performing the crossover operator were suggested in the studies using morphological representation scheme of strings. Using the NSGA-II algorithm [7], the large displacement [18] and path generating [17, 16] compliant mechanisms were designed in which individual multiple least square objectives of actual and desired output responses and a Fourier shape descriptor based objective functions were used to compare the global shape, size and orientation of the desired and actual paths.

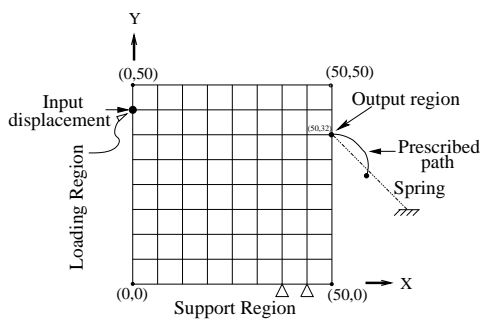Using a row/column-wise crossover operator, a local search

---

Figure 1: Design domain with loading, output and support regions.

based genetic algorithm procedure was introduced for shape optimization [9, 8] and also used for generating the topologies of path generating complaint mechanisms [19, 20]. An optimization formulation for path generating complaint mechanisms was introduced by imposing the constraint on the precision points of a prescribed path with different sets of bi-objective functions.

With the aim of generating the non-dominated minimum weight topologies of compliant mechanism, a domain-specific crossover operator with NSGA-II algorithm and a helper objective in the problem formulation of path tracing compliant mechanism are used in the present study. Using a local search based NSGA-II procedure, single and bi-objective optimization problems are solved and the obtained results are compared with another crossover based study for compliant mechanism designs. All structures are designed to accomplish a task of tracing a user-defined path which is checked by imposing the constraints of allowed deviation at each precision point of a prescribed path. The problem formulation, a local search based NSGA-II procedure and obtained results are presented in the subsequent sections.

2. BI-OBJECTIVE PROBLEM FORMULATION

Consider a design domain of 50 mm by 50 mm in Figure 1. The given design domain is divided into three regions of interest. The first region is called support region where the structure is supported (restrained, with zero displacement) whereas, in the second region (loading region) some specified load (input displacement) is applied. The output region is the third region of interest, that is, the point on the structure which traces out the desired path defined by user. The task of tracing a user-defined path is checked by imposing the constraints on the precision points. Therefore, a prescribed path is divided into N number of precision points as shown in Figure 2. A fixed percent of deviation (η) is allowed between the precision points (i) of a prescribed path and corresponding points (ia) obtained on the actual path after FE analysis of a structure. Hence the problem is subject to N number of constraints on the basis of the number of precision points representing the prescribed path, along with the constraints on stress and weight as given in Equation 1. In the implied stress constraint, maximum stress developed in the structure should be less the flexural yield stress of the material. For providing some resistance at the output region and for some work meant to be done, a spring of constant
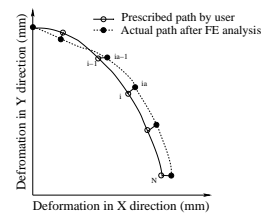

Figure 2: Precision points representing a prescribed path and corresponding points after FE analysis of a structure.
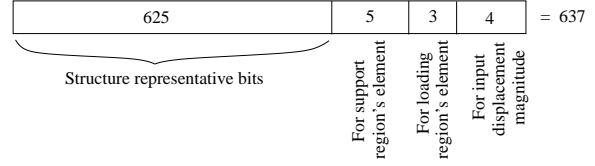

Figure 3: A binary string representation.

stiffness (κ) is attached [19, 20].

The present study is dealt with single and bi-objective optimization problems of compliant mechanism designs which are evolved using a local search based evolutionary multi-objective (NSGA-II) procedure. For generating topologies, a domain-specific crossover operator is coupled with the NSGA-II algorithm which helps in evolving a diverse set of non-dominated solutions. These solutions based designs are then further improved by a local search method. The Equation 1 shows a formulation of bi-objective problem which includes a primary objective of minimizing the weight with a helper objective of maximizing the diversity of GA evolved structures with respect to the *reference* design. The *reference* design is found by a single objective analysis which minimizes the weight of structure subjected to the constraints of allowed deviation at the precision points and on stress. This inclusion of helper objective of diversity will help in generating a diverse set of non-dominated topologies of compliant mechanism in one run of optimization procedure. Here, the diversity of structures are evaluated by comparing the dissimilarities in the bit value at each gene position of the binary strings representing the *reference* design and a structure evolved from the GA population [20].

Minimize: Weight of structure,
Maximize: Diversity of structure,
subject to:
$$1 - \frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta \times \sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}} \geq 0, i = 1, 2, ..., N$$
$$\sigma_{flexural} - \sigma \geq 0,$$
$$3 \times weight\ of\ reference\ design - weight\ of\ structure \geq 0,$$
(1)

where $\eta = 15\%$ is allowed deviation, and $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the structure, respectively.

3. A LOCAL SEARCH BASED EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION PROCEDURE
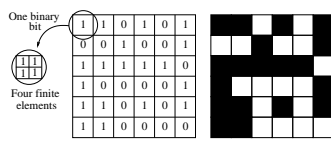
Figure 4: Representation of structure using binary string and material.

Figure 5: Sub-divided domain for crossover.

Among different multi-objective optimization algorithms, the elitist non-dominated sorting genetic algorithm (known as NSGA-II) [7] is popularly used. For two objective problems, it is shown to have a good convergence property to the global Pareto-optimal front as well as to maintain the diversity of population on the Pareto-optimal front. As topology optimization of compliant mechanism problem is non-linear and discrete in nature, NSGA-II with local search procedure is used in the present study. A population of 240, crossover probability of 0.95 and mutation probability of (1/string length) are assigned and NSGA-II is run for a maximum of 100 generations. For each NSGA-II population member, a binary string length of 637 bits is used in which first 625 bits are used to represent a structure (representing 25×25 grids) and additional 12 bits are decoded to determine the support and loading region's elements, and the magnitude of input displacement boundary condition. Figure 3 shows a pictorial view of a binary string. Here, the 12 additional bits are further divided into three sets of five, three and four bits. First five bits indicate the support region's element number, whereas the three bits help in determining the loading region's element number. The decoded value of last four bits are used to evaluate the range of input displacement magnitude which varies from 1 mm to 16 mm at a step of 1 mm.

3.1 Representation Scheme
Continuum structure is discretized by 4 node rectangular elements and each element is represented either by 0 or 1, where 1 signifies the presence of material and 0 represents the void. This makes a binary string which is copied to two dimensional array as per the sequence shown in Figure 4. In the present study, one bit of the binary string represents four element for FE analysis with same gene value as shown in Figure 4.

3.2 Connectivity Analysis
In the representation scheme, values (0-1) of the finite elements are assigned at random. This ensures the material present in the search domain does not follow any particular pattern. To make the designs meaningful, the first task is to find clusters of material at all three regions (support region, loading region and output region) and check the connectivity among them. When, all three regions are connected either directly or indirectly to each other, given procedure fills those elements with the material which are void and surrounded by the neighboring eight elements of material. If two elements generate a point connection, then the given procedure puts one extra material at the nearby element (according to the nature of connectivity) to eliminate the problem of high stress at the point connectivity. If any cluster of material which is not a part of any clusters of three regions of interest as mentioned above, then it is deleted from
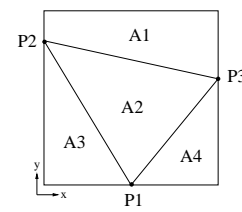
the structure (assigned '0' to each element of this cluster). Hence, the type of representation and connectivity eliminates the problems of 'checker-board' pattern and 'floating' elements of material.

3.3 Parallel Computing
A distributed computing platform is used to reduce the computational time of designing and synthesis of compliant mechanisms. In this parallelization process, the root processor initializes a random population and performs the NSGA-II operators, like selection, crossover and mutation operators, Pareto-optimal front ranking etc. on the population and replaces it with good individuals. Slave processors calculate the values of objective function and constraints and send them to the root processor. The above process is repeated till the termination criterion of NSGA-II is met. The parallel implementation of NSGA-II is done in the context of FE analysis through $ANSYS$ software which consumes the maximum time of the optimization procedure [19, 20]. A MPI based Linux cluster with 24 processors is used in the present study to solve the computationally extensive evaluation procedure of compliant mechanisms.

3.4 GA Operators
In this paper, a domain-specific crossover operator is proposed which is coupled with the NSGA-II algorithm for compliant mechanism topologies. In the given crossover operator, the design domain is divided into the sub-domains (A1, A2, A3 and A4) as shown in Figure 5. For the same, points P1, P2 and P3 are randomly generated at their respective sides of the given design domain which help in dividing the design domain into four sub-domains. For each sub-domain, a coin is flipped to decide whether to exchange the corresponding sub-domain between the two parents. Therefore, an area of the exchanged sub-domain between the two parents varies and depends on the randomly generated points which incorporates the stochasticity in the operator.

In the present study, results of the proposed crossover analysis is also compared with the analysis of another row/column-wise crossover which has shown a successfully applications in shape optimization [9, 8] and in the topology optimization of compliant mechanisms designs[19, 20]. In this recombination operator, two parent solutions are selected and a coin is flipped to decide either for row-wise or column-wise crossover. If a row crossover is done, a row is chosen with an equal probability of (P_{xover}/no. of rows) for swapping. The same is done if a column-wise crossover has to be done. During crossover, a random number is generated to identify the number of rows (columns) to be swapped and then, another generated random number helps in getting the first row (column) number of patches. A range of row (column)

index is calculated and swapped with other parent. For both of the crossover analyses, mutation is done with a low probability on the each bit of a string to change from a void to a filled or from a filled to a void element. Detailed discussion of the row/column-wise crossover and mutation operators are given in these literature [11, 5]. In the present study, a single point crossover and bit-wise mutation operators are used for additional 12 bits (refer Figure 3).

3.5 Clustering Procedure

The number of feasible solutions after the NSGA-II run is equal to or less than the population size. But actually at the end of NSGA-II run, there are not many distinct solutions. So it is not advisable to represent all solutions to the end users. But for the convergence near to the global Pareto-optimal front, a GA needs a fairly large number of population members and generations depending on the problem complexity. To get a meaningful idea of the type of solutions at the end of a NSGA-II run, a clustering procedure is used. The neighboring solutions are grouped together and solutions from each group representing that zone of the Pareto-optimal front are selected as representative solutions [25].

3.6 Local Search Method

The local search method used here is a combination of evolutionary and classical methods. It is a variant of classical hill climbing process. As a single objective function is needed for the hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. The scaled single objective function is minimized in the present study and it is shown in Equation 2.

$$F(x) = \sum_{j=1}^{n} \frac{\overline{w}_j^x (f_{j_{max}}^x - f_j^x)}{f_{j_{max}}^x - f_{j_{min}}^x}, \qquad (2)$$

where, f_j^x is j^{th} objective function, $f_{j_{min}}^x$ and $f_{j_{max}}^x$ are minimum and maximum values of jth objective function in the population respectively, n is number of objectives and $\overline{w}_j^x$ is the corresponding weight to the jth objective function which is computed as:

$$\overline{w}_j^x = \frac{(f_{j_{max}}^x - f_j^x) \setminus (f_{j_{max}}^x - f_{j_{min}}^x)}{\sum_{k=0}^{M} (f_{k_{max}}^x - f_k^x) \setminus (f_{k_{max}}^x - f_{k_{min}}^x)}, \qquad (3)$$

where M is the number of representative solutions after clustering procedure.

In the Equation 2, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space.

First the weighted sum of the scaled fitness of a selected representative solution after the clustering procedure is executed as given in Equation 2. One bit of representative solution is mutated at a time and the design is extracted from the new string. This new string is now ready for FE analysis and after an $ANSYS$ simulation, objective functions and constraint functions are evaluated. If the new design does not satisfy the constraints, then the change in the new

string is discarded and old values are restored. Otherwise, the weighted sum of scaled fitness of new string is calculated and compared with the old string. In case of mutating a '0' to '1' , a change is only accepted when the weighted sum of scaled fitness of new string is strictly better than that in the old string, else it is rejected. For the case of mutating '1' to '0', if the weighted sum of scaled fitness of new string is better than or equal to the old string's weighted sum value, then it is accepted else discarded the change. In the case of rejection, the previous bit values are restored.

Before mutating any bit, a binary string is converted into a two-dimensional array and checked for the element having a material. Then, one by one, all nine neighboring bits including its own bit value are mutated. If a change brings an improvement in scaled fitness, then the change is accepted. This process is repeated till all bits are mutated once. If there is no change in the values of weighted sum of scaled fitness, the local search is terminated. In the same way, all representative solutions are mutated to achieve a local search. As we discussed in Section 3.1, that one binary bit represents four elements for FE analysis, therefore the local search is performed on these elements. A detailed discussion of the local search is given in the literature [5, 6].

4. ANALYSES RESULTS AND COMPARISONS

The topologies of compliant mechanism tracing curvilinear trajectory are generated using the local search based NSGA-II procedure with the proposed and obtained solutions are further compared with the results of row/column-wise crossover operators analysis. Design domain of compliant mechanism (50 mm by 50 mm) is discretized with 50 by 50 rectangular elements in x and y directions respectively. A material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and Poisson ratio of 0.40, is assumed for the synthesis of compliant mechanism. Here, a prescribed path is divided into five precision points and the trajectory traced by output point of a structure is evaluated through a geometric nonlinear FE analysis using $ANSYS$. During the FE analysis, a small region near the support position is declared as plastic zone and is not considered for stress constraint evaluation. A spring of constant stiffness ($\kappa = 0.4$ KN/m) is attached to the output port for providing some resistance to simulate a real application. After the NSGA-II run, six solutions are selected as representative solutions with the help of a clustering procedure.

As the structure can evolve at any support and loading regions with varying input displacement magnitude, GA operations on the 12 bits assist in developing the non-dominated topologies. In both examples, output region is fixed with fixed direction of input displacement boundary condition and a prescribed (curvilinear) path. Here, the input displacement is applied in equal steps at the loading region in x direction. Figure 1 shows the coordinates of output point $(50, 32)$ which traces out the prescribed trajectory. It also shows the support region at the bottom of structure and loading region at the right hand side of structure.

4.1 Proposed Crossover Based Analysis

Using a proposed crossover operator with NSGA-II algorithm, single and bi-objective optimization problems of compliant mechanism designs are solved. In a single objective
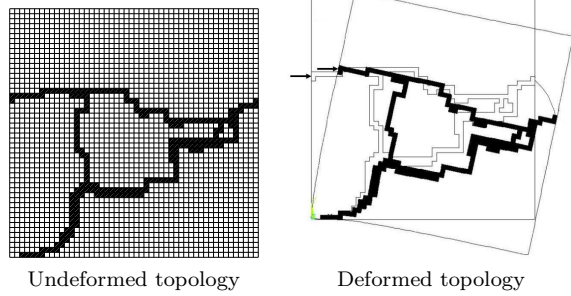
Undeformed topology   Deformed topology

Figure 6: A *reference* design tracing curvilinear path.

Figure 7: A two-objective space of NSGA-II and local search solutions using proposed crossover operator.

Figure 8: Representing minimum weight solutions and *reference* design.

analysis, the objective of minimizing the weight of structure is dealt and the topology evolved after a local search is named as *reference* design. However, a bi-objective optimization problem considers a primary objective of minimizing the weight with a helper objective of maximizing the diversity of GA evolved structures with respect to the *reference* design. As discussed in Section 3.1 that one binary bit represents four finite elements, therefore the diversity of structures can vary from 0 to 2500 depending on dissimilarity in the bit value of finite elements. In the present work, the inclusion of an helper objective assists in evolving the 'trade-off' solutions whereas, the proposed crossover operator helps in finding a diverse set of non-dominated NSGA-II solutions.

4.1.1 Reference Design

For generating the *reference* design, a single objective analysis of minimum weight of the structure subjected to the constraints on stress and of allowed deviation at precision points is performed using the local search based NSGA-II procedure. After the local search, the *reference* design of 0.6985 gms is evolved and its deformed and undeformed topologies are shown in Figure 6. The design is supported at fourth element of the support region and is loaded at 44th loading region's element. The evolved *reference* design requires 6 mm of input displacement for tracing a user-defined curvilinear path.

4.1.2 Bi-Objective Optimization Analysis

With an attempt of generating non-dominated topologies of complaint mechanism in one run of optimization procedure, a bi-objective optimization problem is solved with the primary and helper objectives of minimizing the weight and maximizing the diversity of GA evolved structures with respect to the *reference* design. The formulation of the given bi-objective optimization is given in Equation 1.

As the *reference* design is used for evaluating the diversity, value of its own diversity comes out to be zero and its position in the two objective space corresponding to its weight is shown in Figure 7 with the NSGA-II solution of a single objective study for which its diversity is evaluated with respect to its *reference* design. A significant improvement of a NSGA-II solution can be seen after a local search and hence, results in minimum weight design of compliant mechanism. Figure 7 also shows the NSGA-II and local search solutions of bi-objective optimization problem of the compliant mech-
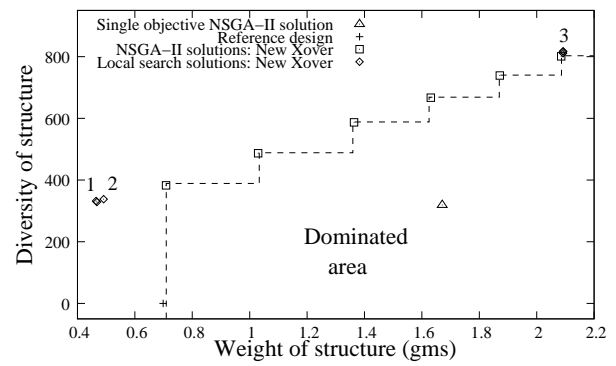
anism tracing curvilinear trajectory. These NSGA-II solutions are nicely distributed through out the objective space but after the local search, solutions are grouped at the extremes of the non-dominated front. One group signifies the primary objective of minimum weight structures whereas, the other group depicts the maximum diverse structures with respect to the *reference* design. When the NSGA-II solutions of single and bi-objective analyses are observed, a few of the NSGA-II solutions of bi-objective problem are better than the single-objective solution in terms of their weight. When a local local search (as explained in Section 3.6) is employed on them then the solutions corresponding to primary objective of minimum weight of structure are found to be better than single-objective minimum weight *reference* design.
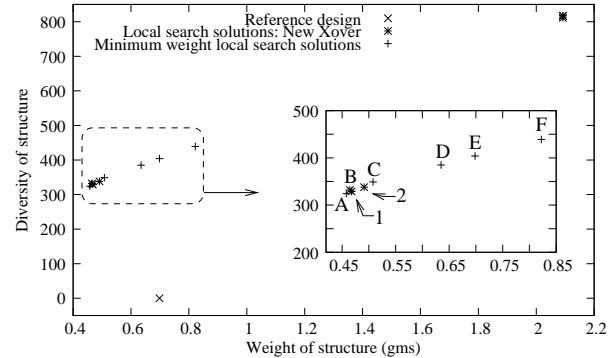
Another attempt is also made by performing a local search of all the representative NSGA-II solutions for minimizing the weight of structures alone. As discussed in Section 3.6, the pseudo-weights of all representative solutions are assigned as per Equation 3 which provides a relative importance factor for each objective corresponding to each solution. In this case, a pseudo-weight which indicates the minimization of weight during a local search, is allocated and all these solutions after a local search are referred as minimum weight local search solutions. Figure 8 is drawn to show the position of *reference* design, local search solutions as per
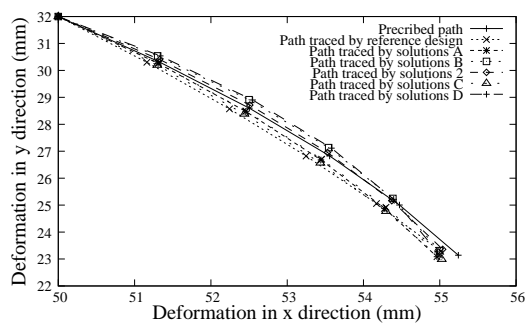
Figure 9: Prescribed path and path traced by the *reference* **design and local search solutions.**

Table 1: Time taken by given optimization procedure.

Problem	NSGA-II time (hrs)	Local search time (hrs)
Reference design	5.47	22.20
Bi-objective optimization problem	5.48	Solution A: 15.75 Solution B: 14.71 Solution 2: 10.67 Solution C: 13.13 Solution D: 16.04

the pseudo-weights calculation in Equation 3 and minimum weight local search solutions. Solutions **A** to **D** represent light weight structures with respect to the *reference* design whereas, solutions **A** comes out to be a minimum weight topology of compliant mechanism. *Reference* design has a weight of 0.6985 gms whereas, solutions **A** has a weight of 0.4579 gms. Solutions **B** and 1 signify the same solution because of the calculated (refer Equation 3) and allocated pseudo-weights are similar in both case. Therefore with an inclusion of helper objective, the local search based optimization procedure evolves solutions **A**, **B** (or 1), 2, **C** and **D** as lighter weight compliant mechanism topologies with respect to the *reference* design. It reveals a fact that the helper objective not only helps in evolving diverse topologies in one run of optimization but also generates 'trade-off' solutions near minimum weight structure. On the other hand, a single objective optimization study only deals with the optimization of one objective and may result in a premature suboptimal solution. The use of second objective as a helper objective is clear from this study. Interested readers may find more such advantages in current compiled book [15].

The paths traced by the *reference* design and local search solutions **A**, **B**, 2, **C** and **D** are shown in Figure 9 which shows an accomplishment of the task of tracing a prescribed path. All these structures are deformed to 10.49% in x direction and 17.72% in y direction at the output point with respect to the size of design domain. The computational times taken by the given optimization procedure for solving the *reference* design and two-objective optimization problems are shown in Table 1. It depicts that the parallel computing used in here helps in reducing the computational time of the compliant mechanism design problem. The local search is performed individually in different processors which takes a con-
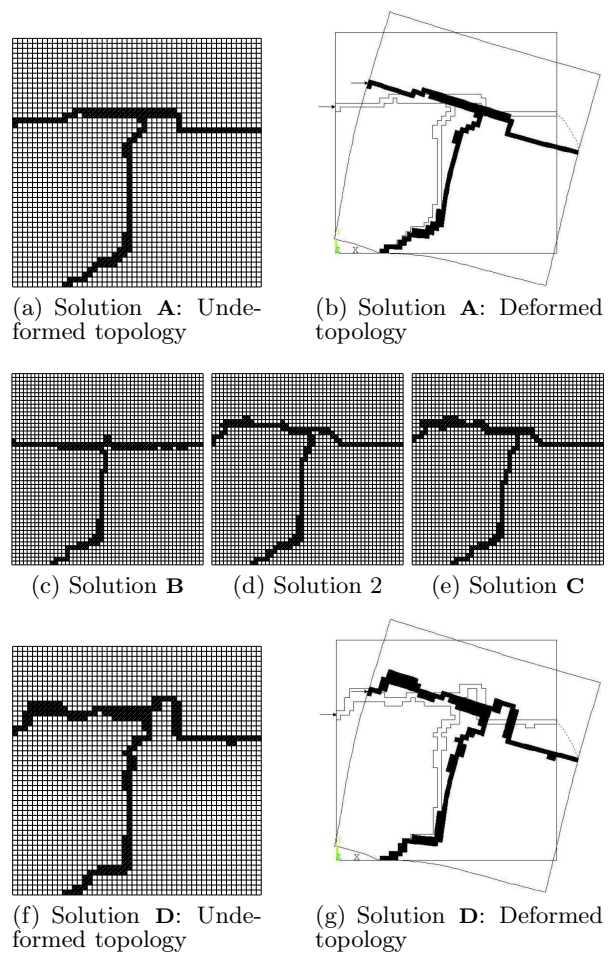

(a) Solution **A**: Undeformed topology

(b) Solution **A**: Deformed topology

(c) Solution **B**

(d) Solution 2

(e) Solution **C**

(f) Solution **D**: Undeformed topology

(g) Solution **D**: Deformed topology

Figure 10: Minimum weight topologies of compliant mechanism tracing curvilinear path.

siderable amount of time of a given optimization procedure to improve the solutions.

The present study finds a set of 'trade-off' solutions from light to heavy weight structures with respect to the diversity but the aim of the study is to generate minimum weight compliant mechanism designs. Therefore, the non-dominated lighter weight topologies with respect to the *reference* design are presented in Figure 10. It is interested to note that the topology of *reference* design (refer Figure 6) consists loop like structure whereas, the minimum weight designs of bi-objective problem show simple shape topologies and easy to manufacture. The deformed and undeformed topologies of extreme solutions **A** and **D** of the non-dominated front is also shown in figures 10(a), 10(b), 10(f) and 10(g) in which topology of solution **A** gets deformed through out the element members joining support and other regions whereas, the topology **D** is deformed more at few places which act like the flexible hings to accomplish the task of tracing a user-defined path.

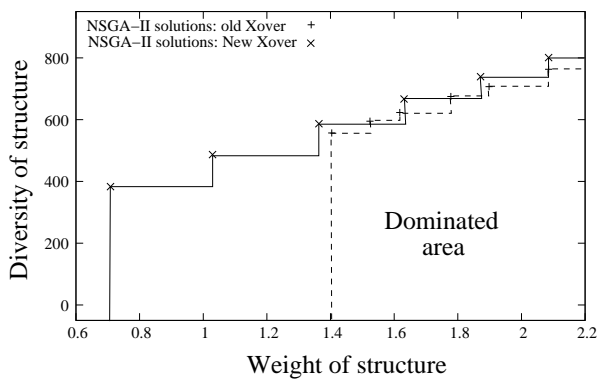## 4.2 Comparison With Another Crossover Based Results

Figure 11: NSGA-II solutions of crossover operators based analyses.

(1) Support region   (2) Loading region   (3) Input displacement magnitude

Figure 12: Growth of feasible non-dominated solutions of domain-specific crossover based analysis.

(1) Support region   (2) Loading region   (3) Input displacement magnitude

Figure 13: Growth of feasible solutions of single objective analysis.

A comparative study of NSGA-II solutions using proposed crossover and an another crossover operators based analyses, is done in the present section. Using another crossover operator which works on exchanging the row/column between two parents [5, 6], the problems of compliant mechanisms have successfully solved which generate straight line and curvilinear paths using a local search based NSGA-II algorithm [19, 20]. Therefore for comparison, the same bi-objective formulation of primary and helper objectives (refer Section 2) are used for solving the path tracing compliant mechanism for which the diversity is evaluated with the same generated *reference* design in Section 4.1.1. As Figure 11 shows that the proposed crossover based analysis generates a better distributed set of NSGA-II solutions. This distributed set of NSGA-II solutions helps in providing a good platform for a local search and hence, in one optimization run different topologies of compliant mechanism can be evolved for performing the prescribed task.

For more insight into the NSGA-II algorithm's behavior, the growth of feasible non-dominated solutions for both crossover based analyses are presented with respect to the positions of support and loading regions, and also, with varying magnitudes of input displacement. Considering the proposed crossover based analysis, Figure 12 shows that initially, few feasible non-dominated designs are supported at different positions but after the few more generation, one support position becomes dominated. Similar outcomes on the loading location and on input displacement magnitude are also found after the NSGA-II run. For this analysis, the first few feasible solutions are appeared at ninth iteration of NSGA-II and after the completion of its generations, all evolved structures are supported at 10th element and loaded at 32nd element with the magnitude of 7 mm input displacement. Here, the growth of feasible NSGA-II solution of single-objective analysis is presented in Figure 13 which shows that the design are supported and loaded with one particular element in the early iteration of NSGA-II and are optimized for minimum weight solutions during NSGA-II run. In this case, first few feasible solutions appear at sixth iteration of NSGA-II.

When the growth of feasible non-dominated solutions are drawn using old operator (refer Figure 14), all evolved structures after the NSGA-II are supported at the second element
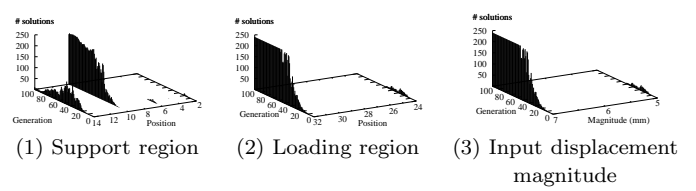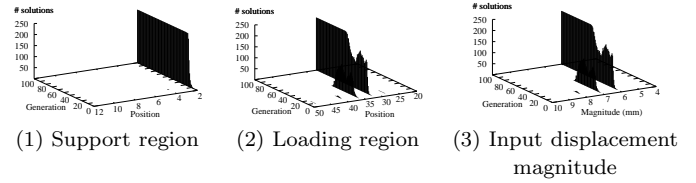
and loaded at 40th element with 8 mm of input displacement. In this case, first few feasible solutions appear at 27th iterations of NSGA-II. Here also, one position of element always dominates the others for support and loading region as in the case of previous analysis. But Figure 14 also indicates that during the NSGA-II run, one region always dominates others and no single intermediate design is evolved with different support and loading regions with varied input displacement magnitude. Based on these analyses, it is concluded that the single-objective study of minimizing weight alone and bi-objective study with NSGA-II using old crossover operator fails to maintain more than one combination of loading position, support position and displacement magnitude, thereby a procedure converges to a sub-optimal structure. On the other hand, NSGA-II with new crossover maintains multiple structures with different topologies which helps in finding a widely distributed set of non-dominated solutions.

5. CONCLUSIONS

In the present work, a domain-specific crossover is proposed and successfully implemented for generating the minimum weight topologies of path tracing compliant mechanisms. The proposed crossover operator with local search based NSGA-II procedure shows its capability of evolving the compliant mechanism designs from the random initial population. An inclusion of helper objective with primary objective also helps evolving a set of minimum weight de-
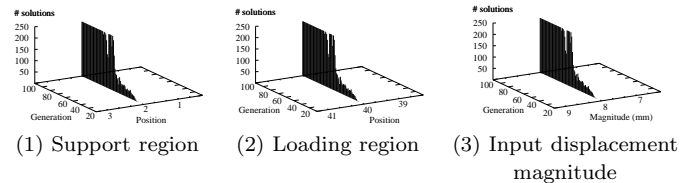

(1) Support region   (2) Loading region   (3) Input displacement magnitude

Figure 14: Growth of feasible non-dominated solutions of another crossover based analysis.

signs which are better than the single objective *reference* design in one run of optimization procedure. While comparison with another crossover operator based results, the proposed crossover based NSGA-II solutions represent a better distributed set of solutions. Because of the complexity of path tracing compliant mechanism problem as well as from the behavior of optimization procedure, not many distinct topologies are found but the minimum weight compliant mechanism designs with different connectivity and shapes of stiffeners joining support, loading and output region are evolved and presented. The computational time of solving compliant mechanism problem is also reduced by the parallel implementation of NSGA-II, whereas the local search improves the NSGA-II solutions in a considerable amount of time of the optimization procedure. A selection pressure for choosing the different positions of support and loading can be incorporated in the optimization procedure as a future work for generating the diverse topologies. Also, if an optimization formulation for path tracing complaint mechanism is relaxed with a fewer number of constraints then it may also result in developing distinct topologies.

6. ACKNOWLEDGMENTS

7. REFERENCES

[1] G. K. Ananthasuresh. *Optimal Synthesis Methods for MEMS*. Kluwer Academic Publishers, 2003.

[2] G. K. Ananthasuresh and S. Kota. Designing compliant mechanism. *Mechanical Engineering*, 117(11):93–96, November 1995.

[3] C. D. Chapman and M. J. Jakiela. Genetic algorithm-based structural topology design with compliance and topology simplification considerations. *ASME, Journal of Mechanical Design*, 118:89–98, March 1996.

[4] C. D. Chapman, K. Saitou, and M. J. Jakiela. Genetic algorithms as an approach to configuration and topology design. *ASME, Journal of Mechanical Design*, 116:1005–1012, December 1994.

[5] S. Chaudhuri. Optimal shape design of mechanical components using local search based multi-objective genetic algorithm. Master's thesis, Indian Institute of Technology Kanpur, 2002.

[6] K. Deb. *Multi-objective optimization using evolutionary algorithms*. Chichester, UK: Wiley, 2001.

[7] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

[8] K. Deb and S. Chaudhuri. Automated discovery of innovative designs of mechanical components using evolutionary multi-objective algorithms. In N. Nedjah and L. de Macedo M, editors, *Evolutionary Machine Design: Methodology and Applications*, chapter 6, pages 143–168. Nova Science Publishers, Inc, New York, 2005.

[9] K. Deb and T. Goel. A hybrid multi-objective evolutionary approach to engineering shape design. In *Proceedings of the First International Conference on Evolutionary Multi-Criterion Optimization (EMO-01)*, pages 385–399, 2001.

[10] J. W. Duda and M. J. Jakiela. Generation and classification of structural topologies with genetic algorithm speciation. *ASME, Journal of Mechanical Design*, 119:127–131, March 1997.

[11] T. Goel. Optimal shape design of mechanical components for single and multi-objectives using genetic algorithms.

[12] L. L. Howell and A. Midha. A method for the design of compliant mechanisms with small-length flexural pivots. *ASME, Journal of Mechanical Design*, 116:280–290, March 1994.

[13] M. J. Jakiela, C. D. Chapman, J. Duda, A. Adewuya, and K. Saitou. Continuum structural topology design with genetic algorithms. *Computer Methods in Applied Mechanics and Engineering*, 186:339–356, March 2000.

[14] C. Kane and M. Schoenauer. Topological optimum design using genetic algorithms. *Control and Cybernetics*, 25(5), 1996.

[15] J. Knowles, D. Corne, and K. Deb. *Multiobjective Problem Solving from Nature From Concepts to Applications (Natural computing series)*. Springer, 2008.

[16] A. K. Rai, A. Saxena, and N. D. Mankame. Synthesis of ptah generation compliant mechanisms using initially curved frame elements. *ASME, Journal of Mechanical Design*, 129:1056–1063, October 2007.

[17] A. Saxena. Synthesis of compliant mechanisms for path generation using genetic algorithm. *ASME, Journal of Mechanical Design*, 127:745–752, July 2005.

[18] A. Saxena. Topology design of large displacement compliant mechanisms with multiple material and multiple output ports. *Structural and Multidisciplinary Optimization*, 30:477–490, 2005.

[19] D. Sharma, K. Deb, and N. N. Kishore. Evolving path generation compliant mechanisms (PGCM) using local-search based multi-objective genetic algorithm. In *International Conference on Trends in Product Life Cycle, Modeling, Simulation and Synthesis (PLMSS)*, pages 227–238, December 2006.

[20] D. Sharma, K. Deb, and N. N. Kishore. Towards generating diverse topologies of path tracing compliant mechanisms using a local search based multi-objective genetic algorithm procedure. Technical Report KanGAL Report No.2007010., Indian Institute of Technology Kanpur, India, 2007.

[21] K. Tai and S. Akhtar. Structural topology optimization using a genetic algorithm with a morphological geometric representation scheme. *Structural and Multidisciplinary Optimization*, 30:113–127, 2005.

[22] K. Tai and T. H. Chee. Design of structures and compliant mechanisms by evolutionary optimization of morphological representations of topology. *ASME, Journal of Mechanical Design*, 122:560–566, December 2000.

[23] K. Tai, T. H. Chee, and T. Ray. Design synthesis of path generating compliant mechanisms by evolutionary optimization of topology and shape. *ASME, Journal of Mechanical Design*, 124:492–500, September 2002.

[24] K. Tai and J. Prasad. Target-matching test problem for multiobjective topology optimization using genetic algorithm. *Structural and Multidisciplinary Optimization*, 34:333–345, 2007.

[25] E. Zitzler. *Evolutionary Agorithms for Multiobjective Optimization: Methods and Applications*. PhD thesis, Zürich, Switzerland: Swiss Federal Institute of Technology ETH, 1999.