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A NOTE ON ORIENTIFOLDS AND F-THEORY

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ABSTRACT

An orientifold of Type-IIB theory on a $K3$ realized as a Z_2 orbifold is constructed which corresponds to F-theory compactification on a Calabi-Yau orbifold with Hodge numbers $(51, 3)$. The T-dual of this model is analogous to an orbifold with discrete torsion in that the action of orientation reversal has an additional phase on the twisted sectors, and both 9-branes and 5-branes carry orthogonal gauge groups. An orientifold of the Z_3 orbifold and its relation to F-theory is briefly discussed.

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Orientifolds are a generalization of orbifolds in which the orbifold symmetry is a combination of a spacetime symmetry and orientation reversal on the worldsheet [1,2,3,4]. These techniques have significantly enlarged the set of string vacua that can be studied perturbatively. Several new string vacua can now be constructed as orientifolds which exhibit novel dynamical phenomena and have interesting nonperturbative duals in M-theory, F-theory, or heterotic string theory.

One important application of orientifolds is in the construction of models in six dimensions with $N = 1$ supersymmetry. The dynamics of these theories offers many surprises like the appearance of tensionless strings which can cause a phase transition in which the number of tensor multiplets changes [5,6,7], or the appearance of enhanced gauge symmetry when an instanton shrinks to zero scale size [8]. Orientifolds are useful in understanding some aspects of these phenomena perturbatively. For instance, the models with multiple tensor multiplets are inaccessible using usual Calabi-Yau compactifications which give only a single tensor multiplet. However, one can easily construct orientifolds [9,10,11,12,13] with multiple tensor multiplets at special points in this moduli space. By turning on the moduli in the tensor multiplets one can move away from these special points and thus explore different regions of the moduli space that are separated by phase boundaries. Some of these models [10] are known to have M-theory duals[14,15]. The extra tensor multiplets which arise in M-theory from the addition of M-theory 5-branes occur perturbatively in the dual orientifold. Similarly, small instantons, which cannot be described as a conformal field theory in heterotic compactifications, have a perturbative description in terms of a Dirichlet 5-brane in the dual orientifold [8,16]. In particular, the enhanced $Sp(k)$ symmetry when k small instantons coincide can be understood in terms of coincident 5-branes with a specific symplectic projection in the open string sector that is determined by the consistency of the world-sheet theory.

Another more recent application of orientifolds is in connection with F-theory [17,18,19]. F-theory refers to a new way of compactifying Type-IIB theory in which the complex coupling λ of Type-IIB theory is allowed to vary over space. The coupling is given by $\lambda = \xi + ie^{-\phi}$ where ϕ is the dilaton from the NSNS sector and ξ is the RR scalar. Consider an elliptically fibered Calabi-Yau manifold K which is a fiber bundle over a base manifold B with a torus as a fiber whose complex structure parameter is τ. Even-though K is a smooth manifold, there will be points in the base manifolds where the fiber becomes singular, and the parameter τ can have a nontrivial $SL(2,Z)$ monodromy around these points. An F-theory compactification on K refers to a compactification of Type-IIB theory on B, where the coupling λ is identified with τ. The nontrivial monodromy of λ around the singular points then means that there are 7-branes at those points that are magnetically charged with respect to the scalar λ. Typically, the base manifold is not Ricci-flat and moreover, because λ is varying, there is a nonvanishing RR background. These backgrounds cannot, therefore, be described using conformal field theory. For special choices of the manifolds K, however, an F-theory compactification is equivalent to a perturbative Type-IIB orientifold. This follows from an observation due to Sen [20] that the element $-\mathbf{1}$ of $SL(2,Z)$ which is not an element of $PSL(2,Z)$ is a perturbative symmetry of Type-IIB. It flips the sign of the two 2-form fields B^1_{MN} and B^2_{MN}, but leaves all other massless fields, in particular, the coupling fields λ invariant. From its action on the massless fields it is easy to check that this element represents the action of $\Omega(-1)^{F_L}$ where Ω is orientation reversal on the worldsheet and F_L is the spacetime fermion number of the left-movers. In the example considered by Sen, K is a $K3$ surface that is a Z_2 orbifold of a four-tours; F-theory on this surface corresponds to a Type-IIB orientifold with the orientifold group $\{1, \Omega(-1)^{F_L}\sigma\}$ where σ is a specific Z_2 involution of $K3$, and is T-dual to Type-I theory. Such an identification of F-theory with an orientifold is very useful. For instance, it was used in [20]to establish the duality between F-theory on $K3$ and the heterotic string on T^2 by relating it to the duality between the Type-I and the heterotic string in ten dimensions.

In this note we analyze an orientifold of a $K3$ orbifold which gives $N = 1$ supersymmetry in six dimensions. Its T-dual has the same orientifold group as the Type-I orientifold analyzed by Gimon and Polchinski [3], but the orientation reversal symmetry Ω acts with an additional minus sign on the twisted sector states of the orbifold. One is familiar with an analogous situation in orbifold constructions. For a $Z_k \times Z_k$ orbifold symmetry, there are k inequivalent orbifolds which correspond to turning on discrete torsion[21,22]. These different orbifolds correspond to the k distinct choices of phases for the action of the generator of one Z_k subgroup on the sectors twisted by other generators.

This model illustrates interesting new features that are relevant to all the applications mentioned earlier: the unusual action of orientation reversal gives rise to multiple tensor multiplets, the 5-branes at the fixed points of the orbifold have orthogonal projection instead of the symplectic projection of a small instanton at a nonsingular point, and it is perturbatively equivalent to F-theory on a Calabi-Yau orbifold $T^6/\{Z_2 \times Z_2\}$ with Hodge numbers $(h^{11}, h^{21}) = (51, 3)$ [22]. Using the formulae in [18] we see that this F-theory compactification gives 17 tensor multiplets, four neutral hypermultiplets, $SO(8)^8$ gauge group, and no charged hypermultiplets. Our aim in the following is to see how the orientifold reproduces this spectrum.

Let us denote the complex coordinates of the six-torus by z_1, z_2, z_3 with identifications $z_l \equiv z_l + 1 \equiv z_l + i, l = 1, 2, 3$. The $Z_2 \times Z_2$ symmetry is generated by the elements α and β where

$$\begin{aligned} \alpha &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3), \\ \beta &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3). \end{aligned} \tag{1}$$

It is easy to work out the cohomology [23,24,22]. The untwisted sector contributes $(3, 3)$ to (h^{11}, h^{21}), and the sectors twisted by α, β, and $\alpha\beta$ each contribute $(16, 0)$, giving $(51, 3)$ altogether. To obtain the corresponding orientifold, we take z_3 as the coordinate of the fiber, and consider Type-IIB compactified on a four-torus with coordinates (z_1, z_2): $z_1 = X^6 + iX^7$, $z_2 = X^8 + iX^9$. Orbifolding with the symmetry α gives Type-IIB on $K3 = T^4/Z_2$. The element β can be written as $R_2 R_3$ where R_2 is a geometric symmetry $(z_1, z_2) \to (z_1, -z_2)$, and R_3, which reflects the fiber, is nothing but the element $-\mathbf{1}$ of $SL(2, Z)$ which corresponds to the operation $\Omega(-1)^{F_L}$ as explained in the preceding paragraph. We are thus led to consider an orientifold of Type-IIB on $K3$ with the orientifold group $\{1, \Omega(-1)^{F_L} R_2\}$ [1].

This orbifold is a special case of a large class of elliptic Calabi-Yau threefolds studied by Voisin [26] and Borcea [27] and discussed in [28,19]. One can take the base to be a $K3$ which admits an involution σ under which the holomorphic 2-form ω is odd, and construct the Calabi-Yau as an orbifold $K3 \times T^2/\{1, \sigma R_3\}$ where R_3 is the reflection of the torus.

[1] We would like to thank S. Mukhi for this observation which prompted this investigation [25].

It should be possible to generalize the considerations of this paper to this whole class of models.

The projection that we wish to perform is $\frac{1}{4}(1+\Omega(-1)^{F_L}R_2)(1+R)$ where $R = R_1R_2$. The projection $\frac{1}{2}(1+R)$ gives us Type-IIB theory on a $K3$ which has 21 tensor multiplets of $N = 2$ supersymmetry which is sum of a tensor multiplet and a hypermultiplet of $N = 1$ supersymmetry. Five of these multiplets come from the untwisted sector, and the remaining 16 come from the twisted sectors at the 16 fixed points of the orbifold. Now, from the arguments of [12,20], one would have expected, by T-duality in the 89 directions, that the operation $\Omega(-1)^{F_L}R_2$ is equivalent to the operation Ω. It seems, therefore, that we get an orientifold of T^4 with the orientifold group $\{1, R, \Omega, \Omega R\}$ which is nothing but a Type-I orientifold on $K3$ analyzed by [3]. The massless spectrum, however, is very different; for example, the closed-string spectrum of the model of [3] has only one tensor multiplet instead of 17, and 20 neutral hypermultiplets instead of four. The reason for this mismatch is that, even though the two projections are the same in the untwisted sector, they are different in the twisted sectors of the orbifold. This is clear if we look at the action of $\Omega(-1)^{F_L}R_2$ on the twisted sectors. The operation Ω that is dual to $\Omega(-1)^{F_L}R_2$ corresponds to $\Omega_0 T$, where Ω_0 is the operation considered in [3], and T is a symmetry of the orbifold that flips the sign of the twist fields at all fixed points. In untwisted sector in both theories give one tensor multiplet and four hypermultiplets. But in the twisted sector at each fixed point, Ω_0 projects out the tensor multiplet and keeps the hypermultiplet giving the closed string spectrum of Type-I on $K3$ whereas Ω keeps the tensor multiplet and projects out the hypermultiplet giving 17 tensor multiplets and four hypermultiplets altogether, as required.

Let us now turn to the open-string sector. We shall follow the notation of [3] in the T-dual picture so that we have 7-branes and 7'-branes instead of 9-branes and 5-branes respectively. The T-dual picture turns out to be easier because then the symmetry breaking is given by geometric separation between branes instead of by Wilson lines. The orientifold group in this case is $\{1, R, \Omega(-1)^{F_R}R_1, \Omega(-1)^{F_L}R_2\}$. Note that both R_1 and R_2, and similarly $\Omega(-1)^{F_L}$ and $\Omega(-1)^{F_R}$ all square to $(-1)^F$ but the elements of the orientifold group all square to $\mathbf{1}$ as they should. To simplify the notation, let us denote

$\Omega(-1)^{F_R} R_1$ and $\Omega(-1)^{F_L} R_2$ by Ω_1 and Ω_2 respectively. To determine the open-string sector we need to determine, as in [3], the number of branes of each type and the eight γ matrices that give the action of the four orientifold group elements on 7 and $7'$ branes.

Before discussing the details of the calculation let us present the results. Tadpole cancellation requires 32 branes of each kind; the 32 7-branes are located at the four fixed planes of R_1 in groups of eight, and the 32 7'-branes are located at the fixed planes of R_2 in groups of eight. Moreover, by a unitary change of basis, the various gamma matrices are given by

$$\begin{aligned} &\gamma_{1,7} = \mathbf{I}, \quad \gamma_{\Omega_1,7} = \mathbf{I}, \quad \gamma_{R,7} = \mathbf{I}, \quad \gamma_{\Omega_2,7} = \mathbf{I}; \\ &\gamma_{1,7'} = \mathbf{I}, \quad \gamma_{\Omega_2,7'} = \mathbf{I}, \quad \gamma_{R,7'} = -\mathbf{I}, \quad \gamma_{\Omega_1,7'} = -\mathbf{I}. \end{aligned} \tag{2}$$

Now consider the massless bosonic states coming from the 77 sector at the fixed point where eight 7-branes are located. The vectors are given by $\psi^{\mu}_{-1/2}|0, ij\rangle\lambda_{ji}, \mu = 1, 2, 3, 4$; $R = +1$ implies $\lambda = \lambda$, and $\Omega_1 = +1$ implies $\lambda = -\lambda^T$, which means that the vectors are in the adjoint of $SO(8)$. The scalars are given by $\psi^{\mu}_{-1/2}|0, ij\rangle\lambda_{ji}, \mu = 6, 7, 8, 9$; $R = +1$ implies $\lambda = -\lambda$, which means that they are all projected out. From four fixed planes of R_1 we get $SO(8)^4$, and similarly from the $7'7'$ sector we get another $SO(8)^4$. Thus, altogether we get $SO(8)^8$ with no charged hypermultiplets.

In the $77'$ sector there is a subtlety. In this case, we have to choose the oscillator vacuum of this sector to be odd under the action of R instead of even as in [3]. This is consistent with factorization because $77'$ and $7'7$ states can turn into 77 or $7'7'$ states, but we cannot have two 77 or two $7'7'$ states turning into a $77'$ state. So one can choose the $77'$ vacuum to be odd and the 77 and $7'7'$ vacua to be even. We shall explain two paragraphs later that this choice is indeed forced upon us by consistency. In this sector the fermions Ψ^m have integer modings, so the ground states are given by a representation of Clifford algebra generated by the zero modes. The total state after GSO projection is $|s_3, s_4, ij\rangle\lambda_{ji}, s_3 = -s_4$ where $s_3, s_4 = \pm\frac{1}{2}$. We choose R on these GSO-projected vacuum states to be -1 instead of $+1$. Thus, R=+1 on the total state implies $\lambda = -\lambda$ which projects out the massless states completely. To summarize, we get 17 tensor multiplets and four hypermultiplets from the closed-string sector, and $SO(8)^8$ gauge group with no charged hypermultiplets from the open-string sector, altogether in agreement with the F-theory spectrum.

This determination of the spectrum, however, poses the following puzzle. From arguments similar to those presented in [3], one would have expected that if $\gamma_{\Omega_1,7}$ is symmetric then $\gamma_{\Omega_1,7'}$ should be antisymmetric. How did we then obtain a solution in which both are symmetric? To see that this is a consistent choice, let us recall the argument of [3]. In the following we shall often switch between our model and its T-dual. In order to obtain a true representation (and not merely a projective representation) of the orientifold symmetry that we are gauging, we must have $\Omega^2 = \mathbf{I}$ in the full string Hilbert space, which is a direct product of the Fock space of string oscillators and the Chan-Paton index space. Now, because Ω^2 is -1 on the oscillator part of the massless states, it must be compensated by choosing -1 on the Chan-Paton part. This forces $\gamma_{\Omega,5}$ to be antisymmetric if $\gamma_{\Omega,9}$ is symmetric. In our case, however, because of our choice of $R = -\mathbf{I}$ on the GSO-projected vacuum states that we used in the previous paragraph, the massless states in the 59 are projected out. Moreover, it is easy to see that at the massive level, the oscillator part of the physical states that are left after the GSO and the R-projection all have $\Omega^2 = +1$. This is so because only the states at half-integer mass levels survive the projections. Now, $\Omega^2 = -1$ for the half integer oscillator modes, and moreover because $\Omega^2 = -1$ on the oscillator vacuum as noted by [3], the total oscillator state has $\Omega^2 = +1$. This in turn implies that in the Chan-Paton space we must choose $\Omega^2 = 1$ which means that if $\gamma_{\Omega,9}$ is symmetric then $\gamma_{\Omega,5}$ must also be symmetric. To put it differently, of the whole tower of states in the 59 sector, the states that are kept after the GSO and the R projection, all have $\Omega^2 = -1$ in [3], but have $\Omega^2 = +1$ in this paper. Thus, the choice of the projection R and the sign of the eigenvalue of Ω^2 are correlated. Under T-duality 59 sector corresponds to $7'7$ sector and the argument above can be repeated there.

Let us now show that the spectrum described above satisfies all consistency requirements, and is moreover uniquely determined. Tadpole calculation in this case is very similar to the T-dual of [3]. The Klein-Bottle and the Möbius strip amplitudes are identical, and for the cylinder amplitude, the only difference is the additional minus sign in the $77'$ and $7'7$ sector in calculating the trace of R. The tadpoles are thus given, in the notation of [3], by

$$
\begin{aligned}
&\frac{v_6 v_2}{16 v_2'}\left\{32^2 - 64\mathrm{Tr}(\gamma_{\Omega_1,7}^{-1}\gamma_{\Omega_1,7}^{T}) + (\mathrm{Tr}(\gamma_{1,7}))^2\right\}\\
&+\frac{v_6 v_2'}{16 v_2}\left\{32^2 - 64\mathrm{Tr}(\gamma_{\Omega_2,7'}^{-1}\gamma_{\Omega_2,7'}^{T}) + (\mathrm{Tr}(\gamma_{1,7'}))^2\right\}\\
&+\frac{v_6}{8}\left\{\mathrm{Tr}(\gamma_{R,7})\mathrm{Tr}(\gamma_{R,7'}) + 2\sum_{I=1}^{4}(\mathrm{Tr}(\gamma_{R,7}))^2 + 2\sum_{I'=1}^{4}(\mathrm{Tr}(\gamma_{R,7'}))^2\right\}.
\end{aligned}
\tag{3}
$$

Here v_6 is the regularized volume of the uncompactified dimensions, v_2 and v_2' is the the volume of the 2-tori in the 67 and in the 89 directions respectively; I and I' refer to the fixed points of R_1 and R_2 respectively.

The chain of reasoning that determines the solution is then as follows. To cancel the tadpoles of the 8-forms from the untwisted sector (the terms proportional to $\frac{v_6 v_2'}{v_2}$ and $\frac{v_6 v_2'}{v_2}$), we need 32 branes of each kind with $\gamma_{1,7}$ and $\gamma_{1,7'}$ equal to $\mathbf{I}$, and $\gamma_{\Omega_1,7}$ and $\gamma_{\Omega_2,7'}$ both symmetric, which can be chosen to be $\mathbf{I}$ with a unitary change of basis of Chan-Paton indices. One can then use the argument presented in [29] which considers the amplitude in which a closed-string twisted state turns into open string states. Conservation of Ω_1 and Ω_2 requires that $\gamma_{R,7}$ and $\gamma_{R,7'}$ both be symmetric, which in turn implies that $\gamma_{\Omega_2,7}$ and $\gamma_{\Omega_1,7'}$ must also be symmetric. This can be consistent only if we choose vacuum states in the $77'$ to have $R=-1$ so that all oscillator states with $\Omega^2=-1$ are projected out. Cancellation of the tadpoles of 6-forms from the twisted sector (the terms in (3)proportional to v_6) then determines that the branes are distributed in groups of eight at the fixed planes, with $\gamma_{R,7}=\mathbf{I}$ and $\gamma_{R,7'}=-\mathbf{I}$. This determines the solution completely.

The next simplest orientifold is when the $K3$ is given by Z_3 orbifold of a hexagonal lattice. In this case, $z_l \equiv z_l+1 \equiv e^{2\pi/3}z_l, l=1,2$. The element α in (1) is given by $\alpha : (z_1,z_2) \to (e^{2\pi/3}z_1, e^{-2\pi/3}z_2)$ and β is the same as in (1). We are thus interested in the projection $\frac{1}{6}(1+\alpha+\alpha^2)(1+\Omega(-1)^{F_L}R_2)$ Now, because $\Omega(-1)^{F_L}R_2$, in this case interchanges the sectors twisted by α with those twisted by α^2, one can easily see that this orientifold is T-dual to the Z_3 orientifold with the usual Ω projection discussed in [11,12]. This model has 10 tensor multiplets and 11 hypermultiplets, and 32 7-branes of one kind. If they are all located at the fixed point of R_2, that is also invariant under α, then the gauge group is $SO(16)\times U(8)$ with hypermultiplets in $(1,28)+(16,8)$.

To find a potential F-theory dual on a Voisin-Borcea orbifold, we consider the configuration in which there are eight 7-branes at each fixed point of R_2 so that the tadpoles are canceled locally. One fixed point of R_2 is invariant under α, and the remaining three form a triplet. The gauge group is $SO(8) \times SO(8)$ with one adjoint hypermultiplet under the first $SO(8)$ that comes from the fixed points that form a triplet under α. To identify the F-theory dual we need to find an elliptic Calabi-Yau X with the right Hodge numbers. The Hodge number can be calculated by compactifying further on a T^2 and computing the Type-IIA spectrum as in [19]. We then have

$$h^{11}(X) = r(V) + T + 2, \quad h^{21} = H^0 - 1, \tag{4}$$

where $r(V)$ is the rank of the gauge group, T is the number of tensor multiplets, and H^0 is the number of hypermultiplets that are uncharged with respect to the Cartan subalgebra of the gauge group. Thus, the candidate Calabi-Yau should have $h^{11} = 20$ and $h^{21} = 14$. Happily, there is a unique Voisin-Borcea with the above Hodge numbers which corresponds to $(r, a, \delta) = (11, 9, 1)$ in the notation of [19,27]. Indeed, this model has the same matter content as the orientifold configuration with local tadpole cancellation.

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