

AN ORIENTIFOLD OF TYPE-IIB THEORY ON $K3$ ATISH DABHOLKAR¹ AND JAEMO PARK²*Lauritsen Laboratory of High Energy Physics**California Institute of Technology**Pasadena, CA 91125, USA*

ABSTRACT

A new orientifold of Type-IIB theory on $K3$ is constructed that has $N = 1$ supersymmetry in six dimensions. The orientifold symmetry consists of a Z_2 involution of $K3$ combined with orientation-reversal on the worldsheet. The closed-string sector in the resulting theory contains nine tensor multiplets and twelve neutral hypermultiplets in addition to the gravity multiplet, and is anomaly-free by itself. The open-string sector contains only 5-branes and gives rise to maximal gauge groups $SO(16)$ or $U(8) \times U(8)$ at different points in the moduli space. Anomalies are canceled by a generalization of the Green-Schwarz mechanism that involves more than one tensor multiplets.

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1. Introduction

Theories of unoriented strings can be viewed as orientifolds [1,2,3] of oriented closed strings. Orientifolds are a generalization of orbifolds in which the orbifold symmetry includes orientation reversal on the worldsheet. For example, Type-I strings can be viewed as an orientifold of Type-IIB strings. It is obvious that the closed-string sector of unoriented strings can be obtained by projecting the spectrum of oriented strings onto states that are invariant under the orientifold symmetry. It is more difficult to see how and when the open string sector might arise, and in particular how to obtain the Chan-Paton factors. A proper understanding of this question has become possible only after the remarkable recent work on D-branes[4].

A D-brane is a submanifold where strings are allowed to end which corresponds to open strings that satisfy mixed Dirichlet and Neumann boundary conditions. In Type-II theories, D-branes represent non-perturbative extended states that are charged with respect to the R-R fields in the theory. D-branes provide a geometric understanding of how Chan-Paton factors arise: a Chan-Paton label is simply the label of the D-brane that an open string ends on.

One can now understand the open-string sector of an orientifold as follows. Orientifolding introduces unoriented surfaces in the closed-string perturbation theory. The unoriented surfaces such as the Klein bottle can have tadpoles of R-R fields in the closed string tree channel. The tadpoles can be canceled by including the right number of D-branes that couple to these R-R fields. This introduces the open string sector with appropriate boundary conditions and Chan-Paton factors.

With this enhanced understanding of orientifolds, one can contemplate more general constructions. In this paper we construct a simple orientifold of Type-IIB theory compactified on a $K3$ surface that has $N = 1$ supersymmetry in six dimensions. The orientifold symmetry group is $\{1, \Omega S\}$ where S is a Z_2 involution of $K3$ and Ω is orientation reversal on the worldsheet. The resulting closed string sector contains the gravity multiplet, nine tensor multiplets, and twelve neutral hypermultiplets. The maximal gauge group arising from the open string sector is $SO(16)$ with an adjoint hypermultiplet, or $U(8) \times U(8)$ with two hypermultiplets that transform as $(8, 8)$.

There are a number of motivations for considering this example. First, the requirement of anomaly cancellation in six dimensions is fairly restrictive and provides useful constraints on the construction of the worldsheet theory. In fact, this work was motivated in part by the observation [5] that anomalies cancel in a large class of supersymmetric models in six dimensions. The orientifold that we consider realizes one of these models as a string theory. Second, we obtain a massless spectrum that is markedly different from the only known string compactification to six dimensions with $N = 1$ supersymmetry *viz.* the heterotic string theory on $K3$, which has only one tensor multiplet. We thus have a new compactification with a moduli space that apparently is disconnected from the known compactifications. Finally, this orientifold is a useful practice case for various generalizations to different dimensions using other orientifold groups [6].

The organization of the paper is as follows. In section two we motivate the orientifold group from considerations of anomaly cancellation and describe the closed string sector. The open string sector is discussed in section three. Consistency requires inclusion of 32 Diriclet 5-branes but *no* 9-branes, with additional constraints on the Chan-Paton factors that determine the gauge group and matter representations completely.

2. Gravitational Anomalies and the Orientifold group

The massless representations of the $N = 1$ supersymmetry algebra in $d = 6$ are chiral; consequently their coupling to gravity is potentially anomalous. We would like to see what constraints are placed on the massless spectrum so that these anomalies cancel. We shall then use this information to see how such a spectrum may follow from a string compactification.

The massless states are labeled by the representations of the little group in six dimensions which is $SO(4) = SU(2) \times SU(2)$. The massless $N = 1$ supermultiplets are then as follows.

1. The gravity multiplet:

a graviton $(3, 3)$, a gravitino $2(2, 3)$, a self-dual two-form $(1, 3)$.

2. The vector multiplet:

a gauge boson $(2, 2)$, a gaugino $2(1, 2)$.

3. The tensor multiplet:

an anti-self-dual two-form $(3, 1)$, a fermion $2(2, 1)$, a scalar $(1, 1)$.

4. The hypermultiplet:

four scalars $4(1, 1)$, a fermion $2(2, 1)$.

The gravitino and the gaugino are right-handed whereas the fermions in the other two multiplets are left-handed. Up to overall normalization the gravitational anomalies are given by [7,8]

$$\begin{aligned} I_{3/2} &= -\frac{43}{288}(trR^2)^2 + \frac{245}{360}trR^4, \\ I_{1/2} &= +\frac{1}{288}(trR^2)^2 + \frac{1}{360}trR^4, \\ I_A &= -\frac{8}{288}(trR^2)^2 + \frac{28}{360}trR^4. \end{aligned} \tag{2.1}$$

Here $I_{3/2}$, $I_{1/2}$, and I_A refer to the anomalies for the gravitino, a right-handed fermion, and a self-dual two-form $(1, 3)$ respectively.

Consider n_V vector multiplets, n_H hypermultiplets and $n_T + 1$ tensor multiplets. Then the (trR^4) term cancels if the following condition is satisfied:

$$n_H - n_V = 244 - 29n_T. \tag{2.2}$$

The $(trR^2)^2$ term is in general nonzero, and needs to be canceled by the Green-Schwarz mechanism [9]. There are many solutions of (2.2). We would now like to see which can be realized as a string theory.

There are not many possibilities for string vacua with $N = 1$ supersymmetry in six dimensions. For the heterotic string, we must compactify on a $K3$ to obtain $N = 1$ supersymmetry. This leads to $n_T = 0$ and $n_H = n_V + 244$. For Type-II strings, usual Calabi-Yau compactification on a $K3$ leads to $N = 2$ supersymmetry. One way to reduce supersymmetry further is to take an orientifold so that only one combination of the left-moving and the right-moving supercharges that is preserved by the orientation-reversal survives. By considering different orientifold groups one may obtain different spectra, and in particular different number of tensor multiplets.

The model that we consider in this paper has $n_T = 8$ and $n_H - n_V = 12$ which clearly satisfies (2.2). The special thing that happens with this matter content is that the entire

anomaly polynomial including the $(tr R^2)^2$ term vanishes. We thus have anomaly cancellation without the need for the Green-Schwarz mechanism, analogous to what happens in the Type-IIB theory in ten dimensions [7], or in the chiral $N = 2$ theory obtained by compactifying Type-IIB theory on $K3$ [10].

If we wish to obtain a large number of tensor multiplets, a natural starting point for orientifolding is the Type-IIB theory compactified on $K3$, which has 21 ($N = 2$) tensor multiplets in the massless spectrum in addition to the gravity multiplet. The gravity multiplet contains 5 self-dual two-forms whereas the tensor multiplets contain one anti-self-dual two-form each. Let us recall how these two-forms arise. In ten dimensions the Type-IIB theory contains a two-form B_{MN}^1 from the R-R sector, a two-form B_{MN}^2 from the NS-NS sector and a four-form A_{MNPQ} from the R-R sector with self-dual field strength. Zero modes of these fields correspond to harmonic forms on $K3$ and give rise to massless fields in six dimensions [8]. The nonzero Betti numbers for $K3$ are $b_0 = b_4 = 1$, $b_2^+ = 3$, and $b_2^- = 19$ where b_2^+ are the self-dual two-forms and b_2^- are the anti-self-dual two-forms. From the two B_{MN} fields we get b_0 two-forms each, which means altogether 2 self-dual and 2 anti-self-dual two-forms. Similarly, from the zero modes of the A_{MNPQ} we get 3 self-dual and 19 anti-self-dual two-forms in six dimensions after imposing self-duality of field strength in ten dimensions.

The orientifold group can now be deduced as follows. In order to obtain $N = 1$ supersymmetry we need an orientation reversal Ω which takes σ to $\pi - \sigma$. A projection $(1 + \Omega)/2$ alone would give us the spectrum identical to the closed-string sector of Type-I theory on $K3$, eliminating A_{MNPQ} and B_{MN}^2 completely from the spectrum. Now consider a Z_2 involution S of $K3$ such that eight anti-self-dual harmonic forms are odd under S and all other 16 forms are even. It is clear that under the projection $(1 + \Omega S)/2$, eight zero-modes of A_{MNPQ} will now survive, giving us 8 anti-self-dual two-forms. Moreover, we shall also get eight scalars from the zero modes of B_{MN}^2 so that we have the complete bosonic content of eight tensor multiplets. We still have one zero mode of B_{MN}^1 giving one self-dual and one anti-self-dual two-form. The self-dual two-form is needed for the gravity multiplet; the anti-self-dual two-form combines with the zero mode of the dilaton to form an additional tensor multiplet. Altogether, we obtain the nine tensor multiplets that we were after.

Let us see if we get the rest of the spectrum right. There are no vector multiplets because there are no odd cycles on $K3$, and starting with even forms and the metric in ten dimensions we can never get a one-form as a zero mode. The scalars arise from zero modes of the metric tensor and the B_{MN}^1 field that are invariant under ΩS . Their zero modes can be found from the Dolbeault cohomology of $K3$ [8], so we need to know which (p, q) forms are left invariant by S . The main point for our purpose will be that the eight two-forms that are eliminated by S are $(1, 1)$ forms¹. We are thus left with 12 $(1, 1)$ forms and 1 each of $(0, 2)$, $(2, 0)$, $(0, 0)$, $(2, 2)$ forms. The zero modes of g_{MN} give 34 scalars [8]. The number of zero modes of B_{MN}^1 equals the number of harmonic two-forms which is 14. Altogether we have 48 scalars which make up 12 hypermultiplets. This construction ensures that the closed-string sector is anomaly free. We also get a constraint in the open-string sector that the number of vector multiplets must equal the number of hypermultiplets for canceling gravitational anomalies.

To proceed further we need to know the spectrum in the open-string sector and check that all tadpoles vanish. For computing the tadpoles we need a realization of the $K3$ as an explicit worldsheet conformal field theory. Furthermore, we need to know how the involution S acts in this conformal field theory. This can be easily done for a particular $K3$ represented as a T^4/Z_2 orbifold. Let (z_1, z_2) be complex coordinates on the torus T^4 defined by periodic identifications $z_1 \sim z_1 + 1$, $z_1 \sim z_1 + i$, and similarly for z_2 . The two Z_2 transformations of interest are generated by

$$\begin{aligned} R: (z_1, z_2) &\rightarrow (-z_1, -z_2) \\ S: (z_1, z_2) &\rightarrow (-z_1 + \frac{1}{2}, -z_2 + \frac{1}{2}). \end{aligned} \tag{2.3}$$

That S is the desired symmetry can be seen as follows. The $K3$ orbifold is obtained by dividing the torus by $Z_2^R \equiv \{1, R\}$. The Type-IIB theory on this orbifold has 5 self-dual and 5 anti-self-dual two-forms coming from the untwisted sector. In the twisted sector, there are 16 anti-self-dual forms from the 16 fixed points of R . Notice that S is the same as R acting on shifted coordinates $(z_1 - \frac{1}{4}, z_2 - \frac{1}{4})$. Now, S leaves all forms in the untwisted

¹ For a smooth $K3$ defined by a quartic polynomial in CP^3 , it is easy to construct an example of the involution S and verify this assertion [11].

sector invariant, but takes 8 fixed points of R into the other 8. Thus of the anti-self-dual two-forms coming from the twisted sector, 8 are even under S , and 8 are odd. This is precisely the structure we wanted. Note that S has 16 fixed points on the torus, but on the orbifold they are identified under R leaving only 8 as required by the Lefschetz fixed-point theorem [12].

3. Open String Sector

3.1. Tadpoles

Tree-channel tadpoles can be evaluated by factorizing the partition function in the loop channel. For closed strings, the one-loop amplitude for the orientifold is obtained by projecting onto the closed string states of the Type-IIB theory on $K3$ that are invariant under the symmetry ΩS . The partition function now receives a contribution from the Klein bottle in addition to the torus. The torus has no closed-string tree channel and is modular invariant by itself, so we need to consider only the Klein bottle. To determine the open string sector we first require closure of operator product expansion so that the S-matrix factorizes properly. This implies that we can consistently add only 5-branes and 9-branes [13]. We then have 55, 99, 59, 95 sectors for open strings from strings that begin and end on the two kinds of branes. The one-loop partition function is given by the cylinder and the Möbius strip diagram.

In this section we shall follow the general framework of Gimon and Polchinski [13] quite closely. The total projection that we wish to perform is $(\frac{1+R}{2})(\frac{1+\Omega S}{2})$. The orientifold group G is $\{1, R, \Omega S, \Omega R S\}$ which we can write as $G = G_1 + \Omega G_2$ with $G_1 = \{1, R\}$ and $G_2 = \{S, RS\}$. An open string can begin on a D-brane labeled by i and end on one labeled by j . The label of the D-brane is the Chan-Paton factor at each end. Let us denote a general state in the open string sector by $|\psi, ij\rangle$. An element of G_1 then acts on this state as

$$g : |\psi, ij\rangle \rightarrow (\gamma_g)_{ii'} |g \cdot \psi, i'j'\rangle (\gamma_g^{-1})_{j'j}, \quad (3.1)$$

for some unitary matrix γ_g corresponding to g . Similarly, an element of ΩG_2 acts as

$$\Omega h : |\psi, ij\rangle \rightarrow (\gamma_{\Omega h})_{ii'} |\Omega h \cdot \psi, j'i'\rangle (\gamma_{\Omega h}^{-1})_{j'j}. \quad (3.2)$$

The relevant partition sums for the Klein bottle, the Möbius strip, and the cylinder are respectively $\int_0^\infty dt/2t$ times

$$\begin{aligned}
\text{KB} : & \quad \text{Tr}_{\text{NSNS}+\text{RR}}^{\text{U}+\text{T}} \left\{ \frac{\Omega S}{2} \frac{1+R}{2} \frac{1+(-1)^F}{2} e^{-2\pi t(L_0+\tilde{L}_0)} \right\} \\
\text{MS} : & \quad \text{Tr}_{\text{NS}-\text{R}}^{99+55} \left\{ \frac{\Omega S}{2} \frac{1+R}{2} \frac{1+(-1)^F}{2} e^{-2\pi t L_0} \right\} \\
\text{C} : & \quad \text{Tr}_{\text{NS}-\text{R}}^{99+95+59+55} \left\{ \frac{1}{2} \frac{1+R}{2} \frac{1+(-1)^F}{2} e^{-2\pi t L_0} \right\}.
\end{aligned} \tag{3.3}$$

Here F is the worldsheet fermion number, and as usual $\frac{1+(-1)^F}{2}$ performs the GSO projection. The Klein bottle includes contributions both from the untwisted sector and the sector twisted by R of the original orbifold.

For evaluating the traces we need to know the action of various operators on the oscillator modes and the zero modes of the fields. Let us take X^m , $m = 6, 7, 8, 9$ to be the coordinates of the torus so that $2\pi r z_1 = X^6 + iX^7$ and $2\pi r z_2 = X^8 + iX^9$, where the radius r defines the overall size of the torus. Let X^i , $i = 1, 2, 3, 4$ be the transverse coordinates in the six-dimensional Minkowski space. Let ψ^m and ψ^i be the corresponding fermionic coordinates of the NSR string. The action of R on oscillator modes is obvious. For the ground states $|p_m, L^m\rangle$ without oscillations, but with quantized momentum $p_m \equiv k_m/R$ in the compact direction and winding $L^m \equiv X^m(2\pi) - X^m(0)$, R has the action

$$R|p_m, L^m\rangle = |-p_m, -L^m\rangle. \tag{3.4}$$

Note that S is $U(\frac{r}{4})RU^\dagger(\frac{r}{4})$ where $U(\frac{r}{4})$ performs translation along both X^6 and X^8 by $r/4$. Therefore, S has the same action on the oscillators as R but for the ground states there is a crucial difference of phase

$$S|p_m, L^m\rangle = (-1)^{k_6} (-1)^{k_8} |-p_m, -L^m\rangle. \tag{3.5}$$

The action of Ω depends on the sectors; Ω takes a field $\phi(\sigma)$ to $\phi(\pi - \sigma)$ and has obvious action on the modes.

The traces can be readily evaluated. Following [13] we define

$$\begin{aligned}
f_1(q) &= q^{1/12} \prod_{n=1}^{\infty} (1 - q^{2n}), & f_2(q) &= q^{1/12} \sqrt{2} \prod_{n=1}^{\infty} (1 + q^{2n}) \\
f_3(q) &= q^{-1/24} \prod_{n=1}^{\infty} (1 + q^{2n-1}), & f_4(q) &= q^{-1/24} \prod_{n=1}^{\infty} (1 - q^{2n-1}),
\end{aligned} \tag{3.6}$$

which satisfy the Jacobi identity

$$f_3^8(q) = f_2^8(q) + f_4^8(q) \tag{3.7}$$

and have the modular transformations

$$f_1(e^{-\pi/s}) = \sqrt{s} f_1(e^{-\pi s}), \quad f_3(e^{-\pi/s}) = f_3(e^{-\pi s}), \quad f_2(e^{-\pi/s}) = f_4(e^{-\pi s}). \tag{3.8}$$

The relevant amplitudes are then given by $(1-1) \frac{v_6}{128} \int_0^\infty \frac{dt}{t^4}$ times

$$\begin{aligned}
\text{KB : } & 8 \frac{f_4^8(e^{-2\pi t})}{f_1^8(e^{-2\pi t})} \left\{ \left(\sum_{n=-\infty}^{\infty} (-1)^n e^{-\pi t n^2 / \rho} \right)^2 \left(\sum_{n=-\infty}^{\infty} e^{-\pi t n^2 / \rho} \right)^2 + \left(\sum_{w=-\infty}^{\infty} e^{-\pi t \rho w^2} \right)^4 \right\} \\
\text{MS : } & - \frac{f_2^8(e^{-2\pi t}) f_4^8(e^{-2\pi t})}{f_1^8(e^{-2\pi t}) f_3^8(e^{-2\pi t})} \left\{ \text{Tr}(\gamma_{\Omega S, 5}^{-1} \gamma_{\Omega S, 5}^T) \left(\sum_{w=-\infty}^{\infty} e^{-2\pi t \rho w^2} \right)^4 \right. \\
& \left. + \text{Tr}(\gamma_{\Omega RS, 9}^{-1} \gamma_{\Omega RS, 9}^T) \left(\sum_{n=-\infty}^{\infty} (-1)^n e^{-2\pi t n^2 / \rho} \right)^2 \left(\sum_{n=-\infty}^{\infty} e^{-2\pi t n^2 / \rho} \right)^2 \right\} \\
\text{C : } & \frac{f_4^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \left\{ (\text{Tr}(\gamma_{1,9}))^2 \left(\sum_{n=-\infty}^{\infty} e^{-2\pi t n^2 / \rho} \right)^4 \right. \\
& \left. + \sum_{i,j \in 5} (\gamma_{1,5})_{ii} (\gamma_{1,5})_{jj} \prod_{m=6}^9 \sum_{w=-\infty}^{\infty} e^{-t(2\pi w r + X_i^m - X_j^m)^2 / 2\pi \alpha'} \right\} \\
& - 2 \frac{f_2^4(e^{-\pi t}) f_4^4(e^{-\pi t})}{f_1^4(e^{-\pi t}) f_3^4(e^{-\pi t})} \text{Tr}(\gamma_{R,5}) \text{Tr}(\gamma_{R,9}) \\
& + 4 \frac{f_3^4(e^{-\pi t}) f_4^4(e^{-\pi t})}{f_1^4(e^{-\pi t}) f_2^4(e^{-\pi t})} \left\{ (\text{Tr}(\gamma_{R,9}))^2 + \sum_{I=1}^{16} (\text{Tr}(\gamma_{R,I}))^2 \right\}.
\end{aligned} \tag{3.9}$$

We have defined $v_6 = V_6 / (4\pi^2 \alpha')^3$ where V_6 is the (regulated) volume of the non-compact dimensions, and $\rho = r^2 / \alpha'$. For the cylinder amplitude, as in [13], the sum i, j comes

from strings that begin and end at 5-branes i and j with arbitrary windings; the sum I is over 5-branes placed at the fixed points of R . Note that for the Klein bottle and the Möbius strip diagrams, in evaluating $\text{Tr}(\Omega RS)$ or $\text{Tr}(\Omega S)$, the sum over momenta contains a crucial factor of $(-1)^n$ for the 6 and 8 directions, but no such factor for the 7 and 9 directions.

To factorize in tree channel we use the modular transformations (3.8) and the Poisson resummation formula

$$\sum_{n=-\infty}^{\infty} e^{-\pi(n-b)^2/a} = \sqrt{a} \sum_{s=-\infty}^{\infty} e^{-\pi a s^2 + 2\pi i s b}. \quad (3.10)$$

An important fact for our purpose will be that

$$\sum_{n=-\infty}^{\infty} (-1)^n e^{-\pi t n^2/\rho} = \sqrt{\rho/t} \sum_{s=-\infty}^{\infty} e^{-\pi \rho (s-\frac{1}{2})^2/t}. \quad (3.11)$$

Tadpoles correspond to long tubes ($t \rightarrow 0$) in the tree channel. In this limit it is easy to see that the total amplitude is proportional to $(1-1) \int_0^\infty dl$ times

$$\begin{aligned} & \frac{v_6 v_4}{16} \{(\text{Tr}(\gamma_{1,9}))^2\} + \frac{v_6}{16 v_4} \left\{ 32^2 - 64 \text{Tr}(\gamma_{\Omega S,5}^{-1} \gamma_{\Omega S,5}^T) + (\text{Tr}(\gamma_{1,5}))^2 \right\} \\ & + \frac{v_6}{64} \sum_{I=1}^{16} (\text{Tr}(\gamma_{R,9}) - 4 \text{Tr}(\gamma_{R,I}))^2. \end{aligned} \quad (3.12)$$

Here l is the length of the tube, which is inversely proportional to the loop modulus t ; $v_4 = \rho^2 = V_4/(4\pi^2 \alpha')^2$ with V_4 the volume of the internal torus before orbifolding.

The $(1-1)$ above represents the contributions of NSNS and RR exchange respectively, which must vanish separately for consistency [14,15]. Using these requirements we determine the spectrum in the next section.

3.2. Gauge Group and Spectrum

We see from (3.12) that to cancel the tadpole proportional to $v_6 v_4$ corresponding to the 10-form exchange, we must have $\text{Tr}(\gamma_{1,9}) = 0$. Now $\text{Tr}(\gamma_{1,9})$ equals the number n_9 of 9-branes, so we conclude that there are no 9-branes. We are left with only the 55 sector so

from now on we drop the subscript 5 for the γ matrices. Vanishing of the term proportional to v_6/v_4 corresponding to the exchange of untwisted 6-forms gives

$$n_5 = 32, \quad \gamma_{\Omega S} = \gamma_{\Omega S}^T. \quad (3.13)$$

Finally, vanishing of the term proportional to v_6 corresponding to the exchange of twisted sector 6-forms gives $\text{Tr}(\gamma_{R,I}) = 0$. By a unitary change of basis $\gamma_{\Omega S} \rightarrow U \gamma_{\Omega S} U^T$ we can take

$$\gamma_{\Omega S} = \mathbf{1}. \quad (3.14)$$

We have additional constraints on the algebra of the γ matrices so that we obtain a representation of the orientifold group in the Hilbert space:

$$\begin{aligned} \gamma_{\Omega RS} &= \gamma_{\Omega S} \gamma_R \\ (\gamma_R)^2 &= \mathbf{1} \\ \gamma_{\Omega RS}^T &= \pm \gamma_{\Omega RS}. \end{aligned} \quad (3.15)$$

We have the choice of taking $\gamma_{\Omega RS}$ either symmetric or antisymmetric, but it turns out that both choices lead to the same spectrum.

Let us now discuss the massless bosonic spectrum coming from the NS sector. The states

$$\psi_{-1/2}^\mu |0, ij\rangle \lambda_{ji}, \quad \mu = 1, 2, 3, 4, \quad (3.16)$$

belong to the vector multiplets whereas the states

$$\psi_{-1/2}^m |0, ij\rangle \lambda_{ji}, \quad m = 6, 7, 8, 9, \quad (3.17)$$

belong to the hypermultiplets. We have to keep only the states that are invariant under R and ΩS ; this constrains the possible forms of the Chan-Paton wave functions λ_{ij} .

The conditions for the Chan-Paton factors depend crucially on where the 5-branes are placed. There are a number of ways one can distribute the 32 5-branes to obtain various gauge groups. We discuss only two distinct configurations that lead to maximal symmetry.

1. The first choice is to take 16 five-branes to lie at a fixed point x of S and the remaining 16 to lie at the image of x under R . In this case, the projection under R simply

relates the states at x to those at Rx and leads to no additional constraints on λ . $\Omega S = +1$ implies

$$\lambda = -\gamma_{\Omega S} \lambda^T \gamma_{\Omega S}^{-1} \quad (3.18)$$

for both scalars and vectors. This can be seen as follows. ψ^m satisfy Dirichlet boundary conditions on both ends and have the same mode expansion as ψ^μ which satisfy Neumann boundary conditions. Now $\psi_{-\frac{1}{2}}^\mu$ is odd under Ω as in Type-I theory in ten dimensions. But $\psi_{-\frac{1}{2}}^m$ is even because of the additional phase due to the Dirichlet boundary condition. Moreover, under S , ψ^m is odd and ψ^μ is even. Using (3.14) we conclude that $\lambda = -\lambda^T$, obtaining an adjoint representation of $SO(16)$ for both vectors and scalars, and the corresponding supermultiplets.

2. We can place 16 five-branes at a fixed point y of R and 16 at the image of y under S . This time we only need to impose the condition $R = +1$ on the states. For the matrix γ_R we had two choices. Let us first choose $\gamma_{\Omega RS}$ to be symmetric. Then from (3.15), γ_R is also a symmetric matrix that squares to one and is traceless. In transforming $\gamma_{\Omega S}$ to identity we already made a unitary change of basis, but we can still make an orthogonal change of basis to put γ_R in the form

$$\gamma_R = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}. \quad (3.19)$$

Now $R = 1$ implies

$$\lambda = \gamma_R \lambda \gamma_R^{-1}$$

for vectors and

$$\lambda = -\gamma_R \lambda \gamma_R^{-1}$$

for scalars. The condition for vectors means that we have a subgroup of $U(16)$ that commutes with γ_R *i.e.*, $U(8) \times U(8)$. The condition for scalars means that they transform as $(8, \bar{8})$ and $(\bar{8}, 8)$ under the $U(8) \times U(8)$. Another way to see this is to note that the Chan-Paton label transforms as $(1, 8) + (8, 1)$ at one end and as the complex conjugate at the other. The projection keeps $(8 \times \bar{8}, 1) + (1, 8 \times \bar{8})$ for the vectors, and $(8, \bar{8})$ and the complex conjugate for the scalars. If we chose $\gamma_{\Omega RS}$ antisymmetric, we would get $\gamma_R = \begin{pmatrix} 0 & -i\mathbf{1} \\ i\mathbf{1} & 0 \end{pmatrix}$ instead of (3.19), but the identical spectrum.

Notice that the rank of the gauge group is different in the two cases which correspond to two branches of the moduli spaces that are connected. With the group $SO(16)$ we have adjoint matter, so we cannot change the rank. We can break it to a $U(8)$ or all the way to $U(1)^8$. For the $U(8) \times U(8)$, the condensation of charged hypermultiplets can change the rank and we can also break it to the diagonal $U(8)$, for example. The two branches are thus connected.

The symmetry breaking can be seen geometrically. If we place a 5-brane away from the fixed points of R and S , then we need three more 5-branes at the image points. We can thus divide the 32 branes in four copies of 8. In this case, there will be no restrictions on the Chan-Paton matrices at a given point, except that they are hermitian. If all branes are placed at generic points and their images, we get $U(1)^8$. When they coincide at a point other than the fixed points, we get $U(8)$ with an adjoint hypermultiplet.

3.3. Anomaly Cancellation

The number of vector multiplets equals the number of hypermultiplets at all points of the moduli space discussed in the previous subsection, so the gravitational anomalies cancel. In fact, at a generic point in the moduli space where the symmetry is $U(1)^8$, or also when it is $SO(16)$, the entire anomaly vanishes. These theories are thus anomaly-free without the need for the Green-Schwarz mechanism.

Anomaly cancellation is more subtle when the gauge group is $U(8) \times U(8)$. We can factorize the group as $SU(8) \times SU(8) \times U(1) \times U(1)$. The states are neutral under the diagonal $U(1)$. So we need to consider only $SU(8)_1 \times SU(8)_2 \times U(1)$ under which the hypermultiplets transform as $(8, \bar{8})_+$ and $(\bar{8}, 8)_-$, where the subscript denotes the $U(1)$ charge. Let us denote the field strengths as F_1, F_2 , and f respectively.

The $U(1)$ factor is at first sight troublesome. The anomaly involving this factor has terms that are of the form $f(d_1 \text{tr} F_1^3 + d_2 \text{tr} F_2^3)$ where d_1, d_2 are constants. Such terms would seem problematic because they do not have the usual factorized form $f^2 \text{tr} F^2$. However, these can be canceled by a local counterterm of the form $\int b \text{Tr} F^3$ for some scalar b that has inhomogeneous gauge transformations. Let a be the gauge potential, $da = f$. Under the gauge transformation $\delta a = d\epsilon$, b must have the inhomogeneous transformation $\delta b = \epsilon$

to cancel the anomaly. The gauge invariant combination is $A = db - a$ which is nothing but the gauge-invariant form of the massive gauge boson associated with a . Now the kinetic term for b is of the form A^2 which can be viewed as the mass term for the massive gauge field A .

One is familiar with an analogous situation in four dimensions [16]. The scalar b is very similar to the axion in four dimensions which is the Goldstone boson of a global Peccei-Quinn symmetry. The fermionic current for the Peccei-Quinn symmetry is anomalous, but so is the axion current. Now, if we gauge this symmetry, then naively we would find that the gauge coupling to the fermions is anomalous. However, one can always define a linear combination of the fermionic current and the axionic current which is anomaly-free. The axion then is the would-be Goldstone boson associated with this anomaly-free current. The corresponding gauge-boson becomes massive after eating the axion.

Because the $U(1)$ gauge boson will always be massive, we shall discuss only the remaining factors $SU(8)_1 \times SU(8)_2$. Let us denote the field strengths for the two groups by F_1 and F_2 respectively, and define $\mathcal{F}_\alpha \equiv \text{tr}F_\alpha^2$, $\alpha = 1, 2$. The anomaly polynomial is then of the form

$$X = \mathcal{F}_1^2 + \mathcal{F}_2^2 - 2\mathcal{F}_1\mathcal{F}_2. \quad (3.20)$$

To cancel this anomaly one needs a generalization of the Green-Schwarz mechanism proposed by Sagnotti [17] which we now review briefly.

If we have n tensor multiplets, then there is a natural $SO(1, n)$ symmetry in the low-energy supergravity action [18]. Altogether there are $n + 1$ tensors H_r , $r = 0, \dots, n$ that transform as a vector of $SO(1, n)$; the time-like component is self-dual whereas the spacelike components are anti-self-dual. The scalars coming from the tensor multiplets parametrize the coset space $SO(1, n)/SO(n)$. We take η_{rs} to be the Minkowski metric with signature $(1, n)$. Let v be the time-like vector, $v \cdot v = 1$, so that $v \cdot H$ is self-dual. The scalar product is with respect to the metric η : for example, $v \cdot H \equiv v_r H_s \eta_{rs}$. Now consider the case when the gauge group has m nonabelian factors with field strengths F_α , $\alpha = 1, \dots, m$, and denote $\text{tr}F_\alpha^2$ by \mathcal{F}_α . In this case, anomaly cancellation can be achieved by a generalization of the Green-Schwarz mechanism if the anomaly polynomial is of the general form

$$X = - \sum_{\alpha\beta} (c_\alpha \cdot c_\beta) \mathcal{F}_\alpha \mathcal{F}_\beta, \quad (3.21)$$

where $c_\alpha, \alpha = 1, \dots, m$ are constant vectors of $SO(1, n)$. It is clear that the anomaly associated with X can be canceled by a local counterterm of the form

$$\Delta\mathcal{L} = \sum_{\alpha} \mathcal{F}_{\alpha}(c_{\alpha} \cdot B), \quad (3.22)$$

provided the fields B_r have appropriate gauge transformations. If ω_{α} are the Chern-Simons three-forms for the various gauge groups and $\delta\omega_{\alpha} = d\omega_{\alpha}^1$, then the required gauge transformations are $\delta B_r = c_{\alpha r} \omega_{\alpha}^1$. The modified gauge-invariant field strengths H_r are then given by

$$H_r = dB_r - c_{\alpha r} \omega_{\alpha}. \quad (3.23)$$

An important fact that follows from supersymmetry is that the coefficients $c_{\alpha r}$ that enter into (3.23) and the modified Bianchi identity are related to the kinetic term for the gauge field F_{α} , which is given by $v \cdot c_{\alpha}$ [17]. Given an anomaly polynomial, the vectors c_{α} must be chosen such that the kinetic terms for all gauge fields are positive-definite.

In our case, the gauge group has only two factors, *i.e.*, $m = 2$. We have ten tensors ($n = 9$), but it turns out that only three tensors are involved in the anomaly cancellation. This is because when all branes are localized at a given fixed point of R (and its image under S), the tensors coming from the twisted sectors localized at other fixed points that are far away, cannot be relevant. Therefore we restrict ourselves to a three dimensional subspace taking $n = 2$. We have one self-dual and one anti-self-dual tensor from the untwisted sector, and one anti-self-dual tensor from the twisted sector.

For simplicity, let us pick a special point in the tensor-multiplet moduli space so that $v = (\cosh \phi, \sinh \phi, 0)$. The anomaly polynomial (3.20) can be written in the form (3.21) by choosing $c_1 = (1, 1, 1)$ and $c_2 = (1, 1, -1)$. There is some freedom in choosing these vectors because of the $SO(1, n)$ symmetry and the freedom in choosing the signs of the tensor fields. With the above choice the field ϕ can be identified with the dilaton so that the coefficient of the gauge kinetic term, which comes from the disk diagram, goes as $e^{-\phi}$. Moreover, the kinetic terms are positive-definite for both the gauge groups because $v \cdot c_1$ and $v \cdot c_2$ are both positive-definite. Thus, the anomalies can be canceled by the generalized Green-Schwarz mechanism explained in the preceding paragraphs.

Worldsheet considerations are consistent with this spacetime reasoning. To obtain a counter-term like (3.22) we would require a coupling of the kind $B_2(\mathcal{F}_1 - \mathcal{F}_2)$, where B_2 is the tensor coming from the twisted sector. Such a term can be obtained by computing a disk diagram with two vertex operators for the gauge bosons on the boundary of the disk, and the vertex operator for the tensor at the center of the disk. The vertex operator at the center introduces a branch-cut corresponding to a twist by R . The twist acts on the Chan-Paton indices by the matrix γ_R which is $+1$ for \mathcal{F}_1 but -1 for \mathcal{F}_2 . This is in accordance with the relative minus sign between the third components of the two vectors c_1 and c_2 . By contrast, the vertex operators for the two tensors B_0 and B_1 coming from the untwisted sector of the orbifold introduce no branch cuts. These tensors therefore have identical couplings to the two gauge groups; correspondingly, c_1 and c_2 are identical in the $0, 1$ subspace.

We have not worked out the detailed couplings from a worldsheet calculation, but our tadpole calculation assures us that anomaly must cancel in this way. If gauge invariance were anomalous, then the longitudinal mode of the gauge boson would not decouple. This would lead to a tadpole, but we have already made certain that there are no tadpoles.

So far we have chosen to work at a special point in the moduli space, where one could ensure that the kinetic terms for both gauge groups are positive-definite. However, as we move around the tensor-multiplet moduli space, we eventually come across a boundary where the kinetic term for one of the gauge fields changes sign, and is no longer positive-definite. For example, we can take a more general form for the vector v , $v = (\cosh \phi, \sinh \phi \cos \psi, \sinh \phi \sin \psi)$ where ϕ and ψ are the moduli. It is easy to see that there is a range of values for ϕ and ψ where either $v \cdot c_1$ or $v \cdot c_2$ is negative. This phenomenon is similar to the one observed in [19] which is possibly an indication of some ‘phase transition’ at the boundary.

We can also contemplate more complicated possibilities. For example, if y_1 and y_2 are two fixed points of R that are not related by S , then we can place eight 5-branes at y_1 and eight at y_2 . The remaining 16 branes have to be placed at the images of these two points under S . In this case one would obtain $U(4) \times U(4)$ gauge group with two copies of $(4, \bar{4})$ from each of the fixed points. Now the anti-self-dual tensors coming from twisted sectors at both y_1 and y_2 will be needed for anomaly cancellation.

4. Discussion

We have constructed a string theory that does not seem to be connected to the known string vacua because we have a different number of tensor multiplets. It cannot be viewed as a compactification of Type-I theory because the orientifold symmetry mixes nontrivially with internal symmetries of the $K3$. We have discussed here only the simplest example but quite clearly there is a whole class of models one can consider at different points in this moduli space. Work on some of these models is in progress and will be reported elsewhere [6]. Models with multiple tensor multiplets have been considered before in [17,20] although from a somewhat different point of view.

By analogy with [21] one can ask if these theories are connected to other theories by a phase transition. In six dimensions, infrared dynamics is trivial, so it would seem impossible to change the number of anti-self-dual tensors because one can simply count the states in the infrared. Such a transition can occur only if there is non-trivial infrared dynamics at special points in the moduli space analogous to the situation considered in [22]. Perhaps the boundary in the tensor-multiplet moduli space where the kinetic term for the gauge fields changes sign is related to such a phase transition.

Finally, one can ask about the duals of the theories that we have constructed. A. Sen has informed us that at a generic point in the moduli space with $U(1)^8$ gauge symmetry, one can obtain identical spectrum by considering an orbifold of M-theory compactified on $K3 \times S^1$ [23]. In this theory the vector multiplets arise from the untwisted sector whereas the tensor multiplets arise from the addition of 5-branes of M-theory by a reasoning similar to [24,25]. This is complementary to our construction where the tensor multiplets arise from the untwisted sector (on a smooth $K3$) and the vector multiplets arise from the addition of 5-branes. In a recent paper that appeared after this work was completed, C. Vafa has obtained identical spectrum by a compactification of ‘F-theory’ [26]. It is plausible that these three models can be related to one another by duality.

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