TIFR/TH/02-11
hep-th/0203231

Dp Branes in PP-wave Background

Atish Dabholkar and Shahrokh Parvizi†

Department of Theoretical Physics

Tata Institute of Fundamental Research

Homi Bhabha Road, Mumbai, India 400005.

Email: atish, parvizi@theory.tifr.res.in

ABSTRACT

Dirichlet p-branes in the background of pp waves are constructed using the massive Green-Schwarz worldsheet action for open strings. These branes are localized at the origin and only for $p = 7, 5, 3$ preserve half the supersymmetries. The spectrum of the brane theory is analyzed and is found to be in agreement with the spectrum of the small fluctuations of the world-volume super Yang-Mills theory in this background. These branes are expected to correspond to objects that are nonperturbative in N in the dual gauge theory.

March 2002

† Address after April 2002: *Institute for Studies in Theoretical Physics and Mathematics, POB 19395-5531, Tehran, Iran*

1. Introduction

Recently it has been proposed that a particular subsector of the four-dimensional $SU(N)$ gauge theory with $\mathcal{N}=4$ supersymmetry is dual to Type-IIB superstring theory in a pp wave background [1]. This duality is deduced by taking a scaling limit of the usual correspondence [2,3,4,5] between $\mathcal{N}=4$ super Yang-Mills theory and Type-IIB superstring in $AdS_5 \times S^5$ background. On the Type-IIB side, the scaling limit is equivalent to the 'Penrose limit' [6,7] of the geometry near a null geodesic in $AdS_5 \times S^5$ carrying a large angular momentum J on the S^5 [8,1]. In the gauge theory, this amounts to picking a $U(1)_R$ subgroup of the $SU(4)$ R-symmetry and focusing on operators with a $U(1)_R$ charge J and conformal weight Δ which both scale as $\Delta, J \sim \sqrt{N}$ keeping the difference $\Delta - J$ finite in the large-N limit. The metric of the pp wave is given by

$$ds^2 = 2dx^+dx^- + \sum_{I=1}^{8} \left(dx^I dx^I - \mu^2 x^I x^I dx^+ dx^+\right), \tag{1.1}$$

and there is a constant R-R 5-form flux,

$$F_{+1234} = F_{+5678} = 2\mu, \tag{1.2}$$

where μ is a mass parameter characterizing the spacetime. Type-IIB string theory in this background has a number of remarkable properties. The background is maximally supersymmetric and thus preserves 32 supersymmetries [9,8]. Moreover, the world-sheet theory describing the string motion in this background is exactly solvable. In particular, in the light-cone gauge,$X^+ = p^+\tau$, the Green-Schwarz action [10] contains eight free massive bosons and fermions [11,1,12] and is given by,

$$S = \frac{1}{2\pi\alpha'}\int d\tau \int_0^\pi d\sigma \left[\frac{1}{2}\partial_+ X^I \partial_- X^I - \frac{1}{2}m^2 (X_I)^2 + i\bar{S}(\partial\!\!\!/ + m\Pi)S\right] \tag{1.3}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$ on the worldsheet and $m \equiv \mu p^+$. Here $X^I, I = 1, \ldots, 8$, are the transverse coordinates that transform as a vector $\mathbf{8_v}$ of the transverse $SO(8)$ whereas S is a Majorana spinor on the world-sheet that transforms as a positive chirality spinor $\mathbf{8_s}$ under the $SO(8)$. The appearance of $\Pi = \gamma^{1234}$ in the action breaks the $SO(8)$ symmetry to $SO(4) \times SO(4)$. In this paper we investigate the open string sector of this theory. In

§2 we find that Dirichlet p-branes of various orientations are possible and are localized at the origin of the pp wave. They are supersymmetric only for $p = 3, 5, 7$. In §3 we analyze the spectrum of the worldvolume theory and find it to be in agreement with the spectrum of low lying excitations of world-volume super Yang-Mills theory of these branes in the pp wave background. The Chern-Simons coupling of the gauge field to the 5-form field is crucial for obtaining this agreement. We conclude in §4 and discuss the supersymmetry algebra in the Appendix.

2. Massive Open Superstrings

The equations of the string that follow from (1.3) are given by

$$\begin{aligned} \partial_-\partial_+X^I + m^2X^I =& 0, \\ \partial_+S^{1a} - m\Pi^a_b S^{2b} =& 0, \\ \partial_-S^{2a} + m\Pi^a_b S^{1b} =& 0, \end{aligned} \tag{2.1}$$

where a, b are $\mathbf{8_s}$ spinor indices and the labels $1, 2$ refer to the two chiralities on the worldsheet. These equations are supplemented by open string boundary conditions. To describe a Dirichlet p-brane, we impose Neumann boundary conditions on $p-1$ coordinates and Dirichlet boundary conditions on the remaining transverse coordinates.

$$\begin{aligned} \partial_\sigma X^r =& 0, \quad r = 1, \cdots, p-1 \\ \partial_\tau X^{r'} =& 0, \quad r' = p, \cdots, 8. \end{aligned} \tag{2.2}$$

Notice that the light cone gauge $X^+ = p^+\tau$ implies that $\frac{\partial X^+}{\partial\sigma} = 0$. Thus, X^+ necessarily satisfies Neumann boundary condition. Moreover, solving the Virasoro constraints, X^- can be written as

$$\partial_\sigma X^- = \frac{-1}{p^+}\partial_\sigma X^I\partial_\tau X^I + \text{fermions}, \tag{2.3}$$

which implies that X^- too necessarily satisfies Neumann boundary condition. For the fermionic coordinates, the boundary condition is

$$S^1|_{\sigma=0,\pi} = \Omega S^2|_{\sigma=0,\pi}, \tag{2.4}$$

where, as in flat space, Ω is a matrix $\Pi_k \gamma^k$ acting the $\mathbf{8_s}$ representation with the product running over the index k that labels the Dirichlet directions. We are interested in BPS objects that preserve sixteen supesymmetries. The choice of allowed Ω is constrained further by the following two conditions:

$$[\Omega, \gamma] = 0 \tag{2.5a}$$

$$\Omega\Pi\Omega\Pi = -1 \tag{2.5b}$$

In the first condition (2.5a), γ is the chirality matrix of $SO(8)$ and it follows from the fact that both fermions have the same $SO(8)$ chirality in Type-IIB string. This condition allows only odd p branes in the theory as in flat space. It is easy to see from the equations of motion for the fermions that, because of the mass term, eight fermion zero modes exist only if condition (2.5b) is satisfied. Note that the 'zero' modes have zero worldsheet momentum ($n = 0$), but nonzero worldsheet energy because of the mass term. They satisfy a Clifford-like algebra and generate the supermultiplet of the unbroken supersymmetries. The allowed choices for Ω consistent with these two constraints are

$$\begin{aligned} &\mathrm{D7}: \ \gamma^i\gamma^j, \ \ \gamma^{i'}\gamma^{j'}, \\ &\mathrm{D5}: \ \gamma^{i'}\gamma^i\gamma^j\gamma^k, \ \ \gamma^i\gamma^{i'}\gamma^{j'}\gamma^{k'}, \\ &\mathrm{D3}: \ \gamma^{i'}\gamma^{j'}\Pi, \ \ \gamma^i\gamma^j\Pi, \end{aligned} \tag{2.6}$$

where $i, j, k = 1, \cdots, 4$ and $i', j', k' = 5, \cdots, 8$. Each choice for Ω, corresponds to a Dp-brane transverse to the indices appearing in the above γ matrices. Because the rotation symmetry is reduced to $SO(4) \times SO(4)$, different orientations of branes are physically distinct. Note that $D1$ and $D9$ branes are not allowed. The mode expansion of the bosonic coordinates satisfying the boundary conditions and the equations of motion is given by

$$\begin{aligned} X^r(\sigma,\tau) =& x_0^r \cos m\tau + \frac{1}{m} p_0^r \sin m\tau + i \sum_{n\neq 0} \frac{1}{\omega_n} \alpha_n^r \mathrm{e}^{-i\omega_n \tau} \cos n\sigma \\ X^{r'}(\sigma,\tau) =& \sum_{n\neq 0} \frac{1}{\omega_n} \alpha_n^{r'} \mathrm{e}^{-i\omega_n \tau} \sin n\sigma, \end{aligned} \tag{2.7}$$

with

$$\omega_n = \mathrm{sgn}(n)\sqrt{n^2 + m^2}. \tag{2.8}$$

In the mode expansion of the Dirichlet directions $X^{r'}$, there is no zero mode consistent with the supersymmetry algebra corresponding to the position of the brane. As a result, the Dp brane is stuck at the origin $X^{r'} = 0$. The zero mode of the Neumann directions can fluctuate but experiences a harmonic oscillator potential centered at the origin. The mode expansion of the fermions is

$$\begin{aligned} S^1 =& S_0 \cos m\tau + \tilde{S}_0 \sin m\tau + \sum_{n\neq 0} c_n \left(\frac{i}{m}(\omega_n - n)\Pi S_n \phi_n + \tilde{S}_n \tilde{\phi}_n \right) \\ S^2 =& -\Pi S_0 \sin m\tau + \Pi \tilde{S}_0 \cos m\tau + \sum_{n\neq 0} c_n \left(S_n \phi_n - \frac{i}{m}(\omega_n - n)\Pi \tilde{S}_n \tilde{\phi}_n \right), \end{aligned} \tag{2.9}$$

where,

$$\begin{aligned} \phi_n =& \exp -i(\omega_n \tau + n\sigma), \qquad \tilde{\phi}_n = \exp -i(\omega_n \tau - n\sigma) \\ c_n =& \left(1 + \frac{(\omega_n - n)^2}{m^2} \right)^{-1/2}, \end{aligned} \tag{2.10}$$

The boundary condition on fermions with a general Ω subject to conditions (2.5)gives us,

$$\begin{aligned} \tilde{S}_0 =& -\Omega \Pi S_0 \\ \tilde{S}_n =& \Omega S_n, \quad n \neq 0. \end{aligned} \tag{2.11}$$

For canonical quantization we introduce canonical momenta

$$\begin{aligned} P^I =& \dot{X}^I, \quad I = 1, \cdots, 8 \\ \mathcal{P}^{\mathcal{I}a} =& S^{\mathcal{I}a}, \quad \mathcal{I} = 1, 2. \end{aligned} \tag{2.12}$$

The phase space for the fermions is constrained because the momenta in (2.12) are proportional to the coordinates. Also a further constraints come from the boundary conditions (2.11). As usual, these constraints can be consistently incorporated by using Dirac Brackets. We regard the S_n oscillators as independent variables and solve for $\tilde{S}_n$ using (2.11). The Dirac brackets for the independent variables are then given by,

$$\begin{aligned} \{\alpha_n^I, \alpha_l^J\}_{DB} =& \frac{i\omega_n}{2} \delta_{n+l,0} \delta^{IJ} \\ \{S_0^a, S_{0b}\}_{DB} =& \frac{i}{4} \delta_b^a \\ \{S_n^a, S_{lb}\}_{DB} =& \frac{i}{4} \delta_b^a \delta_{n+l,0}. \end{aligned} \tag{2.13}$$

Canonical quantization proceeds by replacing the classical brackets with (anti-)commutators including a factor of i. We rescale the oscillators properly to find

$$\begin{aligned} [\bar{a}^I_n, a^J_l] &= \delta_{n+l,0}\delta^{IJ}, \quad [\bar{a}^i_0, a^j_0] = \delta^{IJ}, \\ \{S^a_0, S_{0b}\} &= \frac{1}{4}\delta^a_b, \quad \{S^a_n, S_{lb}\} = \frac{1}{4}\delta_{n+l,0}\delta^a_b, \end{aligned} \tag{2.14}$$

in which

$$a^r_0 = \frac{1}{\sqrt{2m}}(p^r_0 + imx^r_0), \qquad \bar{a}^r_0 = \frac{1}{\sqrt{2m}}(p^r_0 - imx^r_0), \qquad a^I_n = \sqrt{\frac{2}{|\omega_n|}}\alpha^I_n. \tag{2.15}$$

Note that for the allowed choices of $p = 3, 5, 7$, there are eight independent real fermionic zero modes. They are proportional to the supercharges and satisfy a Clifford-like algebra. As we describe in §3, the representation of this algebra gives a short 16-dimensional supermultiplet. The Hamiltonian can now be written as

$$H = \frac{1}{\pi p^+}\int_0^\pi \left(\frac{1}{2}(\mathcal{P}_I^2 + X'^2_I + \mu^2 X_I^2) + i(S^1\dot{S}^1 + S^2\dot{S}^2)\right) d\sigma, \tag{2.16}$$

in which we have used the equations of motion for fermions. In terms of the quantized oscillators, the Hamiltonian takes the form

$$\begin{aligned} H =& E_0 + E_{\mathcal{N}}, \\ E_0 =& \frac{m}{p^+}\left(\sum_{r=1}^{p-1} \bar{a}^r_0 a^r_0 - 2iS_0\Omega\Pi S_0 + e_0\right), \\ E_{\mathcal{N}} =& \frac{1}{p^+}\left(\sum_{n\neq 0} a^I_n a^I_{-n} + i\sum_{n\neq 0}\omega_n S_n S_{-n}\right). \end{aligned} \tag{2.17}$$

The zero point energy $e_0 = \frac{p-1}{2}$ is a result of normal ordering $p-1$ harmonic oscillator bosonic zero modes. In the next section we consider $p = 7$ and describe in detail the worldvolume spectrum of the D7-brane.

3. World-volume Spectrum

Let us consider a D7-brane extending along the $(+-123456)$ directions so that $\Omega = \gamma^{78}$. The $SO(4) \times \widetilde{SO}(4)$ symmetry of the pp wave is broken by the boundary conditions to the longitudinal $SO(4) \times SO(2)$ times a transverse $SO(2)'$. Writing $SO(4)$ factor as $SU(2)_L \times SU(2)_R$, we have the embedding

$$\begin{aligned} SO(8) &\supset SU(2)_L \times SU(2)_R \times \widetilde{SU}_L(2) \times \widetilde{SU}(2)_R \\ &\supset SU(2)_L \times SU(2)_R \times SO(2) \times SO(2)'. \end{aligned} \tag{3.1}$$

The $SO(2) \times SO(2)'$ are rotations in the 56 and 78 and are generated by T_{56} and T_{78} respectively. In terms of the generators of $\widetilde{SU}(2)_L \times \widetilde{SU}(2)_R$, they are given by

$$\begin{aligned} T_{56} =& \tilde{J}_{3L} + \tilde{J}_{3R} \\ T_{78} =& \tilde{J}_{3L} - \tilde{J}_{3R}. \end{aligned} \tag{3.2}$$

Under this embedding the spinor decomposes as

$$\mathbf{8_s} \sim (\mathbf{2},\mathbf{1})^{(\frac{1}{2},\frac{1}{2})} \oplus (\bar{\mathbf{2}},\mathbf{1})^{(-\frac{1}{2},-\frac{1}{2})} \oplus (\mathbf{1},\mathbf{2})^{(\frac{1}{2},-\frac{1}{2})} \oplus (\mathbf{1},\bar{\mathbf{2}})^{(-\frac{1}{2},\frac{1}{2})}, \tag{3.3}$$

where the superscripts denote $SO(2) \times SO(2)'$ charges. Using this decomposition, we can now organize the zero modes of the spinor in terms of fermionic creation and annihilation operators:

$$\begin{aligned} \bar{\lambda}_\alpha \equiv S_{0\alpha}^{(\frac{1}{2},\frac{1}{2})}, \quad \lambda_\alpha \equiv S_{0\alpha}^{(-\frac{1}{2},-\frac{1}{2})}, \\ \bar{\lambda}_{\dot{\alpha}} \equiv S_{0\dot{\alpha}}^{(-\frac{1}{2},\frac{1}{2})}, \quad \lambda_{\dot{\alpha}} \equiv S_{0\dot{\alpha}}^{(\frac{1}{2},-\frac{1}{2})}. \end{aligned} \tag{3.4}$$

where α and $\dot{\alpha}$ are the doublet indices of $SU(2)_L$ and $SU(2)_R$ respectively. Let us now look at the Hamiltonian (2.17) for the fermionic zero modes. The matrix $\Omega\Pi$ is nothing but γ^{56}, which is the generator of T_{56} in the spinor representation. We have chosen the basis (3.4) such that the Hamiltonian is diagonal and can be written as

$$E_0 = \mu \left(\text{bosons} + \bar{\lambda}_\alpha \lambda^\alpha - \bar{\lambda}_{\dot{\alpha}} \lambda^{\dot{\alpha}} + e_0\right) \tag{3.5}$$

with zero-point energy $e_0 = 3$. The commutation relations in this basis are given by

$$\begin{aligned} \{\bar{\lambda}_\alpha, \lambda^\beta\} =& \delta_\alpha^\beta, \\ \{\bar{\lambda}_{\dot{\alpha}}, \lambda^{\dot{\beta}}\} =& \delta_{\dot{\alpha}}^{\dot{\beta}}. \end{aligned} \tag{3.6}$$

We choose the Fock vacuum that is $SO(4) \times SO(2)$ invariant with T_{78} charge -1:

$$a_0|0,-1\rangle = 0, \qquad \lambda^\alpha|0,-1\rangle = 0, \qquad \lambda^{\dot{\alpha}}|0,-1\rangle = 0. \tag{3.7}$$

The labels of the ket denote T_{56} and T_{78} charges respectively. The low lying multiplet of ground states is constructed on top of this vacuum by successively acting with the zero-mode creation operators $\bar{\lambda}_\alpha$ and $\bar{\lambda}_{\dot{\alpha}}$. This ground state multiplet is summarized in the table below. The entire Hilbert space of the 7-brane world volume theory is obtained by the action of various creation operators of the bosonic and fermionic modes on the ground state multiplet. Note that as described in the Appendix, the supersymmetry generators in this background do not commute with the light-cone Hamiltonian. However, the entire superalgebra closes. Thus the supersymmetry algebra is not a symmetry of the Hamiltonian but rather a spectrum-generating algebra.

State	Representation	Energy	Field
$\|0,-1\rangle$	$(\mathbf{1},\mathbf{1})^{(0,-1)}$	3	ϕ
$\bar{\lambda}_\alpha\|0,-1\rangle$	$(\mathbf{2},\mathbf{1})^{(\frac{1}{2},-\frac{1}{2})}$	$3+1$	ψ_α
$\bar{\lambda}_{\dot{\alpha}}\|0,-1\rangle$	$(\mathbf{1},\mathbf{2})^{(-\frac{1}{2},-\frac{1}{2})}$	$3-1$	$\psi_{\dot{\alpha}}$
$\bar{\lambda}_\alpha\bar{\lambda}_\beta\|0,-1\rangle$	$(\mathbf{1},\mathbf{1})^{(1,0)}$	$3+2$	A
$\bar{\lambda}_\alpha\bar{\lambda}_{\dot{\alpha}}\|0,-1\rangle$	$(\mathbf{2},\mathbf{2})^{(0,0)}$	3	A_i
$\bar{\lambda}_{\dot{\alpha}}\bar{\lambda}_{\dot{\beta}}\|0,-1\rangle$	$(\mathbf{1},\mathbf{1})^{(-1,0)}$	$3-2$	$\bar{A}$
$\bar{\lambda}_\alpha\bar{\lambda}_{\dot{\alpha}}\bar{\lambda}_{\dot{\beta}}\|0,-1\rangle$	$(\mathbf{2},\mathbf{1})^{(-\frac{1}{2},\frac{1}{2})}$	$3-1$	$\bar{\psi}_\alpha$
$\bar{\lambda}_\alpha\bar{\lambda}_\beta\bar{\lambda}_{\dot{\alpha}}\|0,-1\rangle$	$(\mathbf{1},\mathbf{2})^{(\frac{1}{2},\frac{1}{2})}$	$3+1$	$\bar{\psi}_{\dot{\alpha}}$
$\bar{\lambda}_\alpha\bar{\lambda}_\beta\bar{\lambda}_{\dot{\alpha}}\bar{\lambda}_{\dot{\beta}}\|0,-1\rangle$	$(\mathbf{1},\mathbf{1})^{(0,1)}$	3	$\bar{\phi}$

For a given open string state in the first column, its $SU(2)_L \times SU(2)_R \times SO(2) \times SO(2)'$ representation is listed in the second column and the energy in units of μ is listed in third column. The last column lists the corresponding mode of the world-brane supergauge field. The entries in this column follow from an analysis of the spectrum of small fluctuations of the world brane theory which we now describe. As we will see, the Chern-Simons coupling of the gauge field to the five-form field plays a crucial role.

The low effective theory on a 7-brane contains a gauge field A_M, a complex scalar ϕ and gauginos. The scalar field does not couple to the five-form field and hence the small fluctuation of the scalar field is a solution of the equation

$$\Box\phi = 0, \qquad \Box \equiv \frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N) = 2\partial{+}\partial{-} + \mu^2 x_I^2 \partial_-^2 + \partial_I^2, \tag{3.8}$$

where $M = +, -, 1, 2, 3, 4, 5, 6$ and $I = 1, \ldots, 6$ are the worldvolume indices. Let us now Fourier transform in x^-, x^I

$$\phi(x^+, x^-, x^I) = \int dp^+ d^8 p \, e^{\mathrm{i}(p^+ x^- + p^I x^I)} \, \tilde{\phi}(x^+, p^+, p^I). \tag{3.9}$$

Then the equation (3.8) becomes

$$i\partial_+ \tilde{\phi} = H\phi \equiv \frac{1}{2p^+}(p_I^2 - m^2 \partial_{p^I}^2)\phi. \tag{3.10}$$

Introducing the standard creation and annihilation operators, the normal ordered Hamiltonian become

$$H = \mu(\bar{a}^I a^I + 3). \tag{3.11}$$

Note that the energy of the Fock space ground state $|0\rangle$ annihilated by the creation operators is 3 because there are 6 transverse directions on the 7-brane worldvolume. The gauge field dynamics is governed by the Yang-Mills action in the pp-wave background:

$$S_B = \int_\Sigma \frac{1}{2} F \wedge *F + C^{(4)} \wedge F \wedge F \tag{3.12}$$

where $C^{(4)}$ is the 4-form potential of the background Ramond-Ramond gauge field. The equation of motion that follows from this action is

$$d * F = F^{(5)} \wedge F. \tag{3.13}$$

In the light-cone gauge, $A_- = 0$, the component A_+ is determined in terms of the physical transverse modes $A_r, r = 1, \ldots 6$. To find the gauge field fluctuations we note that, under the worldvolume rotation symmetry $SO(4) \times SO(2)$, A_m decomposes as $A_i(\mathbf{4}^0) \oplus A(\mathbf{1}^1) \oplus \bar{A}(\mathbf{1}^{-1})$. The CS coupling gives a term proportional to $\mu\epsilon^{+-1234pq}$. The ϵ tensor effectively acts like the rotation matrix T_{56} in the 56 plane and as a result the transverse modes A_m satisfy the equation

$$(\Box + 4i\mu T_{56} \partial_-) A_r = 0. \tag{3.14}$$

The normal ordered Hamiltonian now has an extra piece compared to (3.11) and is given by

$$H = \mu(\bar{a}^I a^I + 3 + 2T_{56}). \tag{3.15}$$

This explains the splittings listed in column four above. The A_i components are neutral under T_{56} and hence their energy is 3 whereas the combinations $A_5 \pm iA_6$ have energy 3 ± 2. The equations of motion for the gaugino are related to (3.13) by supersymmetry and also contain a term proportional to the 5-form field. The second order equation for the gaugino that follows leads to the same Hamiltonian as in (3.15). Now, under the above decompositions all fermions have charge $\pm\frac{1}{2}$ under T_{56} and hence their energies are given by 3 ± 1. Thus the energies and representations of all low-lying modes of the worldvolume theory are in agreement with the quantized spectrum of the massive worldsheet theory.

4. Conclusions

Open strings usually describe nonperturbative Dirichlet p-branes in the theory [13]. We have seen that a variety of Dp branes are possible in the pp wave background. As explained earlier, the light cone directions of the open string are always Neumann. Thus, one leg of the brane always wraps the direction in S^5 used in the Penrose limit. In this respect, these D-branes resemble the giant gravitons in that they are brane configurations that wrap the S^5 and preserve half the supersymmetries [14]. We expect that these branes correspond to some objects in the gauge theory that are nonperturbative in N such as the baryons [15,16]. It would be interesting to find precise gauge theory interpretation of these states. There are a number of interesting papers studying various aspects of the pp-wave background [17,18,19,20,21,22,23,24,25,26,27,28,29]. Considerations in this paper would be relevant in these other contexts also.

There are a number of open questions about the D-branes such as their interactions with the closed string modes which we hope to return to in future. At present, various aspects that are well-known about the massless theory such as the open-closed channel duality, determination of the critical dimension, incorporation of interactions etc. are not fully understood for the massive theory and are in need of further elucidation.

Note Added: In a related work [30] that appeared during the course of this investigation, various allowed boundary states in the closed string sector of the theory have been constructed. These boundary states would naively correspond, in the open string channel, to Dirichlet boundary conditions for the light-cone coordinates. Thus, they do not seem to be directly related to the p-branes considered in this paper which are Neumann in the lightcone coordinates. Noncommutative Dp branes in the pp-wave background have been considered in [31] with some overlap with this paper.

Acknowledgments

We would like to thank Sunil Mukhi and Sandip Trivedi and especially Anindya Biswas for useful discussions and M. Sheikh-Jabbari for comments on the draft. A. D. would like to acknowledge the hospitality of the theory group at SLAC and Stanford University where part of this work was completed.

Appendix: Supersymmetry Algebra

We will now describe the supersymmetry algebra and exhibit the supersymmetry of the spectrum. We will use the Super-Noether charges for the closed string introduced by Metsaev [11]:

$$\begin{aligned}
\mathbf{P}^+ = p^+, \qquad \mathbf{P}^I &= \frac{1}{\pi}\int d\sigma (P^I \cos m\tau + X^I m \sin m\tau) \\
\mathbf{J}^{+I} &= \frac{ip^+}{\pi}\int d\sigma (\frac{1}{m}P^I \sin m\tau - X^I \cos m\tau) \\
Q^+ &= \frac{\sqrt{2p^+}}{\pi}\int d\sigma \mathrm{e}^{im\tau\Pi}(S^1 + iS^2), \\
\bar{Q}^+ &= \frac{\sqrt{2p^+}}{\pi}\int d\sigma \mathrm{e}^{-im\tau\Pi}(S^1 - iS^2) \\
\mathbf{J}^{ij} &= \frac{1}{\pi}\int d\sigma \left(x^i P^j - x^j P^i - i(S^1 - iS^2)\gamma^{ij}(S^1 + iS^2)\right), \\
\mathbf{J}^{i'j'} &= \frac{1}{\pi}\int d\sigma \left(x^{i'} P^{j'} - x^{j'} P^{i'} - i(S^1 - iS^2)\gamma^{i'j'}(S^1 + iS^2)\right), \\
Q^{-1} &= \frac{2}{\pi\sqrt{p^+}}\int d\sigma [(P^I - \acute{X}^I)\gamma^I S^1 - m X^I \gamma^I \Pi S^2], \\
Q^{-2} &= \frac{2}{\pi\sqrt{p^+}}\int d\sigma [(P^I - \acute{X}^I)\gamma^I S^2 + m X^I \gamma^I \Pi S^1], \\
\mathbf{P}^- &= -H,
\end{aligned} \tag{4.1}$$

in which $(I, J = 1, \cdots, 8)$, $(i, j = 1, \cdots, 4)$, $(i', j' = 5, \cdots, 8)$, and $Q^-, \bar{Q}^- = (Q^{-1} \pm iQ^{-2})/\sqrt{2}$. We use the explicit solutions (2.7) and (2.9) to derive the supercharges in terms of open string modes. To do this, one can use a proper doubling of the interval $[0, \pi]$ to $[0, 2\pi]$, such that all the classical solutions satisfy the open string boundary conditions for interval $[0, \pi]$ and periodic boundary conditions for $[0, 2\pi]$. The resulting charges will be as follows,

$$\mathbf{P}^+ = p^+, \qquad \mathbf{P}^r = p_0^r, \qquad \mathbf{P}^{r'} = 0, \tag{4.2}$$

$$\mathbf{J}^{+r} = -ip^+ x_0^r, \qquad \mathbf{J}^{+r'} = 0, \tag{4.3}$$

$$J^{rs} = a_0^r \bar{a}_0^s - a_0^s \bar{a}_0^r + \frac{1}{2} S_0 \gamma^{rs} S_0 + \sum_{n\neq 0}\left[a_n^r a_{-n}^s - a_n^s a_{-n}^r + \frac{1}{4} S_{-n}\gamma^{rs} S_n\right], \tag{4.4}$$

$$J^{r's'} = \frac{1}{2} S_0 \gamma^{r's'} S_0 + \sum_{n\neq 0}\left[a_n^{r'} a_{-n}^{s'} - a_n^{s'} a_{-n}^{r'} + \frac{1}{4} S_{-n}\gamma^{r's'} S_n\right] \tag{4.5}$$

$$J^{r's} = 0 \tag{4.6}$$

$$Q^+ = \sqrt{p^+}(1 + i\Omega^T)S_0, \qquad \bar{Q}^+ = \sqrt{p^+}(1 - i\Omega^T)S_0 \tag{4.7}$$

$$\begin{aligned}\sqrt{2p^+}Q^{-1} =& 2p_0^I\gamma^I S_0 - 2mx_0^r\gamma^r\Pi\Omega^T S_0 \\ &+ \sum_{n=1}^{\infty}\left[\sqrt{2\omega_n}c_n a_n^I\gamma^I\Omega S_n + \frac{im}{\sqrt{2\omega_n}c_n}(a_n^r\gamma^r - a_n^{r'}\gamma^{r'})\Pi S_n + h.c.\right]\end{aligned} \tag{4.8}$$

$$\begin{aligned}\sqrt{2p^+}Q^{-2} =& 2p_0^I\gamma^I\Omega^T S_0 + 2mx_0^r\gamma^r\Pi S_0 \\ &+ \sum_{n=1}^{\infty}\left[\sqrt{2\omega_n}c_n(a_n^r\gamma^r - a_n^{r'}\gamma^{r'})S_n - \frac{im}{\sqrt{2\omega_n}c_n}a_n^I\gamma^I\Pi\Omega S_n + h.c.\right].\end{aligned} \tag{4.9}$$

We have also used the following identities in the expressions for rotation generators, J^{IJ},

$$\begin{aligned}\gamma^{rs} + \Omega\gamma^{rs}\Omega^T =& 2\gamma^{rs}, \\ \gamma^{r's'} + \Omega\gamma^{r's'}\Omega^T =& 2\gamma^{r's'}, \\ \gamma^{r's} + \Omega\gamma^{r's}\Omega^T =& 0,\end{aligned} \tag{4.10}$$

Note that in (4.7), the components Q_α^+ and $\bar{Q}_\beta^+$ are not independent and as a result half the linear combinations of the supercharges are zero. This means that the D-brane breaks half the supersymmetry as expected. To see the superalgebra of the unbroken supersymmetries explicitly, consider the following combinations,

$$\begin{aligned}(Q^+ + \bar{Q}^+) + i\Omega(Q^+ - \bar{Q}^+) =& 0 \\ Q^{-1} - \Omega Q^{-2} =& 0.\end{aligned} \tag{4.11}$$

Then we can use the following independent supercharges,

$$\begin{aligned}q^+ =& \frac{1}{2}((Q^+ + \bar{Q}^+) - i\Omega(Q^+ - \bar{Q}^+)) = 2\sqrt{p^+}S_0 \\ q^- =& \frac{1}{2\sqrt{2}}(Q^{-1} + \Omega Q^{-2}) = \frac{1}{\sqrt{2}}Q^{-1}.\end{aligned} \tag{4.12}$$

The supersymmetry algebra the becomes

$$\begin{aligned}[P^-, P^I] =& \mu^2 J^{+I}, \qquad [P^I, J^{+J}] = -\delta^{IJ}P^+, \qquad [P^-, J^{+I}] = P^I, \\ [P^r, J^{st}] =& \delta^{sr}P^t - \delta^{tr}P^s, \qquad [J^{+r}, J^{st}] = \delta^{sr}J^{+t} - \delta^{tr}J^{+s}, \\ [J^{rs}, J^{tu}] =& \delta^{st}J^{ru} + 3 \text{ terms}, \qquad [J^{r's'}, J^{t'u'}] = \delta^{s't'}J^{r'u'} + 3 \text{ terms},\end{aligned} \tag{4.13}$$

$$
\begin{aligned}
[J^{IJ}, q^{\pm}_{\alpha}] =& \frac{1}{2} q^{\pm}_{\beta} (\gamma^{IJ})^{\beta}_{\alpha}, \\
[J^{+I}, q^{-}_{\alpha}] =& \frac{1}{2} q^{+}_{\beta} (\gamma^{+I})^{\beta}_{\alpha}, \qquad [P^{-}, q^{-}_{\alpha}] = 0 \\
[P^{r}, q^{-}_{\alpha}] =& \frac{1}{2} \mu q^{+}_{\beta} (\Pi\Omega^{T}\gamma^{+I})^{\beta}_{\alpha}, \qquad [P^{-}, q^{+}_{\alpha}] = \mu q^{+}_{\beta} (\Pi\Omega^{T})^{\beta}_{\alpha},
\end{aligned}
\tag{4.14}
$$

$$
\begin{aligned}
\{q^{+}_{\alpha}, q^{+}_{\beta}\} =& 2p^{+}\delta_{\alpha\beta} \\
\{q^{+}_{\alpha}, q^{-}_{\beta}\} =& (\gamma^{r})_{\alpha\beta} P^{r} + i\mu(\gamma^{r}\Pi)_{\alpha\beta} J^{+r} \\
\{q^{-}_{\alpha}, q^{-}_{\beta}\} =& \delta_{\alpha\beta} P^{-} + \mu(\gamma^{rs}\Pi)_{\alpha\beta} J^{rs}.
\end{aligned}
\tag{4.15}
$$

Thus, the supersymmetry algebra closes. The worldvolume states are organized in a representation of this superalgebra. Note that in the limit $\mu = 0$ we recover the flat space superalgebra.

References

[1] D. Berenstein, J. Maldacena and H. Nastase, "Strings in flat space and pp waves from $\mathcal{N} = 4$ super Yang Mills," [arXiv:hep-th/0202021].

[2] J. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998) 231, [arXiv:hep-th/9711200].

[3] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**(1998) 105, [arXiv:hep-th/9802109].

[4] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2** (1998) 253, [arXiv:hep-th/9802150].

[5] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323** (2000) 183, [arXiv:hep-th/9905111].

[6] R. Penrose, "Any space-time has a plane wave as a limit," in *Differential geometry and relativity*, pp. 271-275, Reidel, Dordrecht, 1976.

[7] R. Güven, "Plane wave limits and T-duality," *Phys. Lett.* B482 (2000) 255, [arXiv: hep-th/0005061].

[8] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, "Penrose limits and maximal supersymmetry," [arXiv:hep-th/0201081].

[9] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, "A new maximally supersymmetric background of IIB superstring theory," JHEP **0201**, 047 (2002) [arXiv:hep-th/0110242].

[10] M. B. Green and J. H. Schwarz, "Covariant description of superstrings," Phys. Lett. B **136** (1984) 367.

[11] R. R. Metsaev, "Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background," [arXiv:hep-th/0112044].

[12] R. R. Metsaev and A. A. Tseytlin, "Exactly solvable model of superstring in plane wave Ramond-Ramond background," [arXiv:hep-th/0202109].

[13] J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," Phys. Rev. Lett. **75**, 4724 (1995) [arXiv:hep-th/9510017].

[14] J. McGreevy, L. Susskind and N. Toumbas, "Invasion of the giant gravitons from anti-de Sitter space," JHEP **0006**, 008 (2000) [arXiv:hep-th/0003075].

[15] V. Balasubramanian, M. Berkooz, A. Naqvi and M. J. Strassler, "Giant gravitons in conformal field theory," [arXiv:hep-th/0107119].

[16] D. Berenstein, C. P. Herzog and I. R. Klebanov, "Baryon spectra and AdS/CFT correspondence," [arXiv:hep-th/0202150].

[17] N. Itzhaki, I. R. Klebanov and S. Mukhi, "PP wave limit and enhanced supersymmetry in gauge theories," [arXiv:hep-th/0202153].

[18] J. Gomis and H. Ooguri, "Penrose limit of $N = 1$ gauge theories," [arXiv:hep-th/0202157].

[19] L. A. Zayas and J. Sonnenschein, "On Penrose limits and gauge theories," [arXiv:hep-th/0202186].

[20] M. Hatsuda, K. Kamimura and M. Sakaguchi, "From super-AdS(5) x S**5 algebra to super-pp-wave algebra," [arXiv:hep-th/0202190.]

[21] M. Alishahiha and M. M. Sheikh-Jabbari, "The PP-wave limits of orbifolded AdS(5) x S**5," [arXiv:hep-th/0203018.]

[22] N. w. Kim, A. Pankiewicz, S. J. Rey and S. Theisen, "Superstring on pp-wave orbifold from large-N quiver gauge theory," [arXiv:hep-th/0203080.]

[23] T. Takayanagi and S. Terashima, "Strings on orbifolded pp-waves," [arXiv:hep-th/0203093.]

[24] U. Gursoy, C. Nunez and M. Schvellinger, "RG flows from Spin(7), CY 4-fold and HK manifolds to AdS, Penrose limits and pp waves," [arXiv:hep-th/0203124.]

[25] E. Floratos and A. Kehagias, "Penrose limits of orbifolds and orientifolds," [arXiv:hep-th/0203134.]

[26] S. R. Das, C. Gomez and S. J. Rey, "Penrose limit, spontaneous symmetry breaking and holography in pp-wave background," [arXiv:hep-th/0203164].

[27] J. Michelson, "(Twisted) toroidal compactification of pp-waves," [arXiv:hep-th/0203140.]

[28] M. Cvetic, H. Lu and C. N. Pope, "Penrose limits, pp-waves and deformed M2-branes," [arXiv:hep-th/0203082.]

[29] M. Cvetic, H. Lu and C. N. Pope, "M-theory pp-waves, Penrose limits and supernumerary supersymmetries," [arXiv:hep-th/0203229].

[30] M. Billo' and I. Pesando, "Boundary states for GS superstrings in an Hpp wave background," [arXiv:hep-th/0203028].

[31] C. S. Chu and P. M. Ho, "Noncommutative D-brane and open string in pp-wave background with B-field," [arXiv:hep-th/0203186].