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## TEN DIMENSIONAL HETEROTIC STRING AS A SOLITON

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### ABSTRACT

It is shown that the heterotic string emerges as a soliton in the type I superstring theory in ten dimensions. The collective coordinates of the soliton are described by a smooth, chiral worldsheet theory. There are eight bosonic and eight right-moving fermionic zero modes that arise from the partially broken supertranslations. In addition, there are 496 charged bosonic zero modes of the gauge field that describe a left-moving WZNW model on a  $spin(32)/Z_2$  group manifold. Small, stable loops of the solitonic string furnish the massive states required by duality that transform as spinors of  $spin(32)$ .

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## 1. Introduction

It has recently been conjectured that in ten dimensions, the type I superstring and the heterotic string with gauge group  $SO(32)$  are dual to each other [1]. While the evidence for this duality has so far been only at the level of the low energy effective lagrangian, it is likely that the two string theories are exactly dual to each other. One requirement of such an exact duality is that the spectrum of BPS saturated states must match in the two theories. Of particular interest are the neutrally stable, macroscopic winding states that exist in the perturbative spectrum of the heterotic string [2,3]. In the type I theory such states do not occur perturbatively and hence must arise as solitons. In this paper, we show that the macroscopic heterotic string does indeed exist as a soliton in the type I theory. The collective coordinates of this soliton are described by a smooth worldsheet theory that is identical to the worldsheet of the perturbative heterotic string. Remarkably, the intricate chiral structure of the heterotic string emerges from a theory of unoriented strings.

A similar exact duality has been conjectured in six dimensions between a heterotic string compactified on a four-torus and the type IIA string compactified on a  $K3$  surface [4,1,5]. The corresponding heterotic soliton that arises in the type IIA theory has been described in [6,7]. At a generic point in the  $K3$  moduli space, the gauge symmetry is abelian and the zero modes of the soliton are chiral worldsheet bosons that live on an even, self-dual lattice with signature  $(20,4)$ . The current algebra is thus realized in the free boson representation. In ten dimensions, the situation is quite different. The spacetime coordinates and the right-moving fermions are still realized as free fields. The left-moving charge carrying current algebra, however, is realized as a chiral WZNW model on the soliton worldsheet. In six dimensions also, at points of enhanced, nonabelian symmetry in the  $K3$  moduli space, the charged current algebra will be realized in this manner.

An important requirement for obtaining the heterotic string is that the WZNW model be formulated not on the simply connected group manifold of  $spin(32)$  but rather on the group manifold of  $spin(32)/Z_2$ . For the perturbative heterotic string it follows from modular invariance. For the soliton, we can partially understand this from the fact that the zero modes naturally give rise to only left-moving currents. In order to describe them

it is necessary to obtain a WZNW model that factorizes holomorphically. This essentially determines the correct identification.

Another requirement for consistency is that we have a level-one current algebra. It is not apparent how to derive this condition for the soliton, but it may conceivably follow from some topological considerations. In what follows, we shall simply assume this for nonperturbative consistency.

In the next section we describe the heterotic string soliton and the structure of zero modes in its background. In the last section we briefly comment upon the spacetime structure of the soliton and the relation of our work to others in different dimensions.

## 2. Heterotic String as a Soliton in the Type I String Theory

The massless bosonic fields of the type I string theory consist of the dilaton  $\phi$  and the metric tensor  $g_{MN}$  from the NS-NS sector, an antisymmetric tensor  $B_{MN}$  from the R-R sector and an  $SO(32)$  gauge field  $A_M$  from the open string sector. The low-energy action for these fields [8,1] is

$$S = \frac{1}{\alpha'^4} \int d^{10}x \sqrt{-g} \left( e^{-2\phi} (R + 4(\nabla\phi)^2) - \frac{\alpha'}{30} e^{-\phi} \text{Tr} F^2 - \frac{1}{3} H^2 \right), \quad (2.1)$$

where  $F = dA + A \wedge A$  and the three-form antisymmetric tensor field strength is given by

$$H = dB + \alpha' \left( \omega_3^L(\Omega_-) - \frac{1}{30} \omega_3^{YM}(A) \right) + \dots \quad (2.2)$$

Here  $\omega_3$  is the Chern-Simons three-form and the connection  $\Omega_{\pm}$  is a non-Riemannian connection related to the usual spin connection  $\omega$  by

$$\Omega_{\pm M}^{AB} = \omega_M^{AB} \pm H_M^{AB} + \left( \frac{1}{2} e^{NA} e_M^B \partial_N \phi - A \leftrightarrow B \right). \quad (2.3)$$

When the action is written this way, the gauge fields have standard, dilaton independent transformation laws  $\delta A = D\epsilon$  and  $\delta B = d\lambda$ , but now the lagrangian no longer scales with an overall factor of  $e^{-2\phi}$ . This is what we expect for the type I theory where as usual  $e^{2\phi}$  is the loop counting parameter and the  $F^2$  term is higher order because it comes from the disc diagram. The duality transformation for these fields is  $\phi' = -\phi$ ,  $g' = e^{-\phi} g$ ,  $A' = A$

and  $B' = B$  where the unprimed fields are type I and the primed fields are heterotic. Under duality one obtains the heterotic string Lagrangian with the normalization used in [9].

For type I string theory compactified on a very large circle, we expect to find macroscopic string solitons that wrap around this circle. We can take  $\sigma^\mu$  ( $\mu = 0, 1$ ) as the coordinates on the worldsheet of the soliton and  $y^m$  ( $m = 2, \dots, 9$ ) as the transverse coordinates. The solitonic string solution is then given by

$$\begin{aligned}
g_{\mu\nu} &= e^{-\phi} \eta_{\mu\nu}. \\
g_{mn} &= e^{+\phi} \delta_{mn}. \\
H_{\mu\nu p} &= -\frac{1}{2} \epsilon_{\mu\nu} \partial_p e^{-2\phi}, \\
e^{2\phi} &= e^{2\phi_0} + \frac{Q}{y^6},
\end{aligned}
\tag{2.4}$$

where  $y$  is the radial distance in the transverse space,  $\eta_{\mu\nu}$  is the flat lorentzian metric on the worldsheet and  $\epsilon_{\mu\nu}$  is the antisymmetric tensor with  $\epsilon_{01} = -\eta_{00} = 1$ . The parameter  $Q$  is the axion charge per unit length:

$$Q = 2 \int_{S^7} e^{-\phi^*} H.
\tag{2.5}$$

It is this Ramond-Ramond charge that gives the soliton string its orientation. The original type I string is unoriented because the perturbative string worldsheet does not couple to the  $B_{MN}$  field from the RR sector. It is easy to see that the solution saturates a Bogomol'nyi bound [3] and the mass per unit length equals the charge  $Q$ . As a result, half the supersymmetries remain unbroken in the soliton background.

Let us now discuss the various zero modes. There are eight bosonic zero modes coming from the broken translational invariance that correspond to the location of the soliton. We denote these by  $X^m(\sigma)$  which are free fields on the worldsheet. Then there are eight right-moving, fermionic zero modes coming from the broken supersymmetries. The supersymmetry transformations for the fermions are

$$\begin{aligned}
\delta\lambda &= (\Gamma^M \partial_M \phi - \frac{1}{6} H_{MNP} \Gamma^{MNP}) \epsilon \\
\delta\psi_M &= (\partial_M + \frac{1}{4} \Omega_{-M}^{AB} \Gamma_{AB}) \epsilon,
\end{aligned}
\tag{2.6}$$

All spinors are Majorana-Weyl in ten dimensions. The gravitino  $\psi_M$  and the supersymmetry variation parameter  $\epsilon$  have positive chirality whereas the dilatino  $\lambda$  has negative chirality. Under the natural embedding  $SO(1,9) \rightarrow SO(1,1) \times SO(8)$  the spinor  $\epsilon$  decomposes as

$$\epsilon \rightarrow 8_+^+ + 8_-^-, \quad (2.7)$$

where the superscript and the subscript refer to  $SO(8)$  and  $SO(1,1)$  chiralities respectively. Half of these supersymmetries  $8_-^-$  are annihilated by the solution (2.4). The unbroken supersymmetries imply worldsheet supersymmetry [10]. The other half  $8_+^+$  gives rise to right-moving zero modes on the soliton worldsheet which we denote by  $S_+^a(\sigma)$ . These are free, right-moving Majorana fermions on the worldsheet where  $a$  is the  $SO(8)$  spinor index with positive chirality.

As in [3,11,12], the effective worldsheet action for these zero modes can be easily obtained. The bosonic part of the action is given by

$$\frac{N\alpha'}{\kappa} \int d^2\sigma e^{-\phi} (\eta^{\mu\nu} \partial_\mu X^m \partial_\nu X^n g_{mn}) + \epsilon^{\mu\nu} \partial_\mu X^m \partial_\nu X^n B_{mn}, \quad (2.8)$$

where  $N$  is a normalization factor,  $\kappa$  is the gravitational coupling and  $B$  is the RR, Kalb-Ramond field. Using heterotic variables and the fact that the heterotic string tension is  $\frac{\kappa}{\alpha'}$ , one obtains the usual action in static gauge ( $X^\mu \sim \sigma^\mu$ ). The action for the fermionic zero modes is determined by worldsheet supersymmetry [10] and is similar to the light-cone Green-Schwarz action for the field  $S_+^a$  [8].

We now turn to the charged zero modes that come from solving the equation of motion for the gauge fields. In the differential form notation, the relevant part of the action (2.1) is

$$-\frac{2}{\alpha'^4} \int \left( \frac{1}{3} H \wedge *H + \frac{\alpha'}{30} e^{-\phi} \text{Tr} F \wedge *F \right), \quad (2.9)$$

where  $*$  is the Hodge star operation with the conventions of [13]. In varying with respect to  $A$  we have to remember that  $H$  depends on  $A$  through the Yang-Mills Chern-Simons three form (2.2). Since  $\delta w_{YM}^3 = 2 \text{Tr} \delta A \wedge F + d(A \wedge \delta A)$ , and  $d * H = 0$  is one of the equations of motion, we get,

$$D * (e^{-\phi} F) = -2F \wedge *H. \quad (2.10)$$

We would like to solve this equation for small fluctuations of the gauge field in the background of the remaining fields given by solution (2.4). Now recall that  $H = -\frac{1}{2}\epsilon \wedge de^{-\phi}$  and  $*H = \frac{1}{2}e^{\phi}\hat{*}de^{-2\phi}$  where  $\hat{*}$  is the Hodge star operation either in the transverse space or along the worldsheet depending on whether the forms are cotangent to the transverse space or the worldsheet. The zero mode equation can then be solved by the ansatz  $A = a(\sigma)de^{-2\phi}$  if,

$$\begin{aligned} da &= \hat{*}da, \\ d\hat{*}da &= 0. \end{aligned} \tag{2.11}$$

Since  $A$  is a one-form valued in the Lie algebra of  $SO(32)$ , we obtain 496 bosonic left-moving zero modes. The chirality of the zero modes is correlated with the sign of the exponent in the ansatz. The right-moving zero modes are proportional to  $de^{2\phi}$  and are not normalizable. Note that  $A \wedge A = 0$  for our ansatz and for constant  $a$  the zero modes are pure gauge.

The dimension of the moduli space of the left-moving charge-carrying modes can also be seen by a slightly different argument. The Lagrangian in (2.1) is invariant under a global group  $SO(32)$  that is generated by gauge transformations with a gauge parameter that is constant at infinity. This group leaves the neutral solution (2.4) invariant and does not generate a new solution. However, one can use the solution-generating technique of Sen [14], to obtain a solution that carries charge under a  $U(1)$ . This is achieved by taking a  $U(1)$  inside  $SO(32)$  and embedding it inside the solution generating T duality group. Once we have a solution that is charged under a  $U(1)$  we can act with the  $SO(32)$  to obtain a 496 parameter family of solutions.

We can write  $a(\sigma) = \epsilon^a(\sigma)T_a$  where  $T_a$  are the (anti-Hermitian) generators in some representation. These zero modes then parametrize, up to global identifications, the group manifold of  $spin(32)$ . If we write  $g = e^{\epsilon^a T_a}$  then the field strength is given by,

$$F = d\sigma^- \partial_- gg^{-1} \wedge de^{-2\phi} \tag{2.12}$$

with  $\partial_+(\partial_- gg^{-1}) = 0$ . These equations of motion imply a chiral WZNW action [15] along the worldsheet.

We have taken the moduli to parametrize the universal covering group but there can be discrete moddings by the center of  $spin(32)$ . In fact, such discrete identifications are

necessary since we would like to define the chiral WZNW model as the holomorphic part of the full WZNW model. In general, the WZNW model does not completely factorize. For example, the level-one current algebra of  $spin(32)$  contains four representations: singlet, vector, spinor and conjugate spinor. The one-loop partition function picks the diagonal modular invariant and is a sum of four terms [16]. Each term is an absolute square of a holomorphic term but the sum itself is not an absolute square. However, the WZNW model on  $spin(32)/Z_2$  group manifold does give a partition function that factorizes holomorphically. Thus, the fact that we have only left-moving currents essentially determines the correct global identification on the group manifold. This argument requires some knowledge about higher loop quantization and it would be nice to obtain this identification completely from semiclassical reasoning, if possible. Under this  $Z_2$  the spinor and the adjoint are invariant whereas the vector and the conjugate spinor are not. As a result, the spectrum obtained from the quantization of collective coordinates will contain representations whose weights are in the conjugacy classes of the adjoint and the spinor and lie on an even, self-dual lattice of  $spin(32)/Z_2$ .

This completes the analysis of charged zero modes. Together with (2.8) we have recovered the entire structure of the ten-dimensional heterotic string on the soliton worldsheet.

### 3. Discussion

We would now like to comment upon the singularity structure of the soliton. All the zero modes that we have obtained are smooth and normalizable and thus describe a smooth worldsheet. The antisymmetric field-strengths  $H$  and  $F$  are also smooth everywhere. The dilaton and the metric, on the other hand, appear to diverge at  $y = 0$ . The line element for the solution (2.4) is

$$ds^2 = e^{-\phi}(-dt^2 + dz^2) + e^{\phi}(dy^2 + y^2 d\Omega^2), \quad (3.1)$$

where  $d\Omega^2$  is the volume element for the transverse seven spheres. At  $y = 0$ ,  $e^{\phi}$  diverges as  $\frac{1}{y^3}$ . If we change variables to  $\rho \sim \frac{1}{y^{\frac{1}{2}}}$  then the line element near  $y = 0$  becomes

$$ds^2 = \frac{A}{\rho^6}(-dt^2 + dz^2) + d\rho^2 + B\rho^2 d\Omega^2, \quad (3.2)$$

where  $A$  and  $B$  are numerical constants. Along the surface of constant  $t$  and  $z$  we see that singularity at  $y = 0$  has receded an infinite geodesic distance away to  $\rho = \infty$ . On this slice, the space has the geometry of a wormhole with an infinite throat much like the geometry of some extremal black holes [17]. One difference here is that as we go down the throat to  $\rho \rightarrow \infty$ , the circumference of the throat increases. The line element in  $t, z$  plane still seems singular and it is not clear if the singularity is an infinite distance away if we consider different approaches to  $y = 0$ . In particular, it is not obvious if the spacetime is null-geodesically complete. The situation is somewhat better in six dimensions [7,6], where the  $t, z$  plane is flat and the singularity is manifestly infinitely far away [18]. In both dimensions, however, the dilaton grows down the throat as we approach  $y = 0$  and the theory gets into strong coupling. This means that no matter how small the coupling at asymptotic infinity, the semiclassical approximation itself breaks down at a finite distance down the throat once the coupling becomes of order one. It appears, therefore, that both in ten and in six dimensions, we lack an adequate criterion for deciding which solutions should be regarded as nonsingular.

Another issue concerns the  $\alpha'$  corrections to the leading order solution. In six dimensions it was possible to argue that the  $\alpha'$  correction will vanish from (4,4) worldsheet supersymmetry [6,12]. In ten dimensions, the corrections appear to be nonzero. For example, the generalized spin connection does not equal the gauge connection for the charged solution. From the anomaly equation,

$$dH = \alpha'(trR \wedge R - \frac{1}{30}TrF \wedge F), \quad (3.3)$$

we see that  $H$  would have to be modified at the next order. We will have a left-right asymmetric solution and it seems unlikely that worldsheet supersymmetry will protect the lowest order solution.

Even though the solution will be modified at higher orders, it is reasonable to assume that the structure of zero modes will be essentially unaltered. This is because the zero modes are normalizable and in fact behave as  $y^6$  near  $y = 0$ . Thus, they are well localized near the throat, far away from the singularity. The  $\alpha'$  expansion is an expansion in  $\alpha'/y^2$ , therefore, the zero modes will not be substantially modified at large  $y$ . Unless the  $\alpha'$



corrections create new singularities near  $y = 0$  for these zero modes, they will remain normalizable. Further analysis of the corrected solution, its causal structure and the zero modes certainly deserves further investigation.

So far we have considered macroscopic string solitons wrapped around a large circle which are stable because of the winding. But one can also contemplate small, wiggling loops of the solitons. Some of these solitons, if they exist, will also be stable. In particular, a soliton that carries the smallest spinor charge of  $spin(32)$  will be absolutely stable. The states in the perturbative spectrum of the type I string carry only integer charges, therefore a soliton with half-integer charge cannot decay into them. These small, stable loops of solitons will provide the massive states with spinor charges that are required by duality.

It may seem puzzling that in [7,6] the number of charged zero modes equals the rank of the group, whereas our analysis in six dimensions will give as many zero modes as the dimension of the group. This happens because at a generic point in the  $K3$  moduli space of type IIA compactifications, where the gauge group is completely abelian, the rank equals the dimension of the group. At special points of enhanced symmetry, extra massless states appear and the low energy Lagrangian changes abruptly. With the new low-energy Lagrangian at these points, one will find that the number of zero modes also jumps abruptly in accordance with the structure that we have discussed. Thus, the emergence of a chiral, WZNW model on a group manifold with appropriate discrete identifications is a general property of these solitons applicable also in six dimensions at various points of nonabelian symmetry.

The results of this paper in ten dimensions, along with the results of [7,6] in six dimensions provide a satisfying picture of ‘exact’ string-string duality. This duality is analogous to the exact duality for  $N = 4$  supersymmetric gauge theories in four dimensions [19,20,21,22]. In field theory, the spectrum of BPS saturated soliton states is also important in the understanding of the ‘effective’ duality in  $N = 2$  theories [23]. We expect that the soliton strings that we have described will play a similar role in the understanding of effective string-string dualities.

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