Performance of Differential Chaos Shift Keying over Multipath Fading Channels

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Abstract—**We analyze the performance of Differential Chaos Shift Keying (DCSK) over communication channels exhibiting multipath fading. Formulas are derived for the bit error rate (BER) performance in the single user case and compared with simulation results.**

Keywords—**chaos, communication, fading, DCSK**

I. Introduction

THE inherently broadband nature of chaotic signals has made them promising for spread spectrum communication systems [1], [2], [3]. DCSK and other schemes have been proposed and their performance analyzed [4], [5], [6], [7], [8] for this application. In this paper, we extend DCSK performance analysis to a variety of communication channels encountered in practical systems.

DCSK is a non-coherent method for transmitting binary information using a chaotic signal. A reference chaotic waveform $x(t)$ is transmitted during the first half of each data bit. If the bit is a '1', $x(t)$ is transmitted again during the second half. If the bit is a '0', $-x(t)$ is transmitted. At the receiver, the signal is delayed by half a bit period and correlated with the undelayed signal to get the decision variable for producing the output data stream. The structure of a typical DCSK transmitter and receiver is shown in Fig. 1.

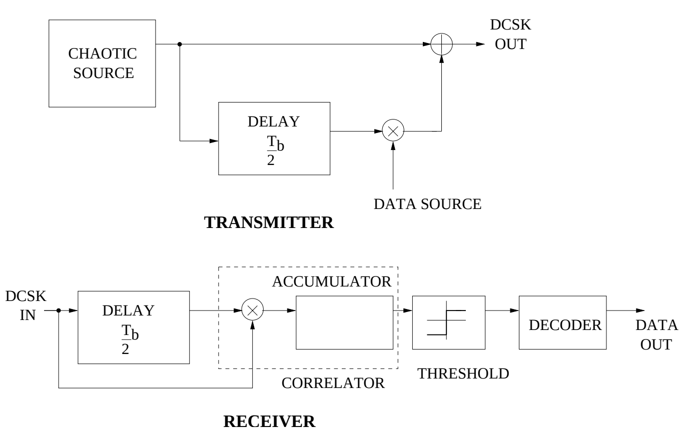

Fig. 1. DCSK transmitter and receiver structures.

Using chaotic signals for spreading introduces a non-zero variance in the transmitted energy per bit, which degrades the BER performance of DCSK. In [9], [10], FM was used to reduce this variance. Alternatively, one can get constant bit energy by using binary valued chaotic sequences for spreading. In this paper, we have used binary chaotic sequences obtained by quantizing the output of a wide variety of chaotic maps (iterated at the chip rate) for spreading. Binary chaotic sequences can be designed to have purely noise-like correlation properties and efficiently spread the spectrum of digital data [11].

This paper is organized as follows. Section II investigates DCSK performance in additive white Gaussian noise (AWGN) channels. Rayleigh fading channels are considered in Section III. Use of multipath diversity to improve performance is discussed in Section IV . Ricean fading channels are considered in Section V. Conclusions and suggestions for further work are presented in Section VI.

II. AWGN Channels

In our case, the transmitted energy per bit is constant and the BER of DCSK in AWGN channels takes the form

$$\mathrm{BER} = \mathrm{Q}\left(\sqrt{\frac{E_b}{2N_0}\left(1+\frac{MN_0}{2E_b}\right)^{-1}}\right) \qquad (1)$$

where E_b is the energy per bit, N_0 is the channel noise power spectral density, $2M$ chips are transmitted per bit period and $\mathrm{Q}(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$. This type of BER expression is characteristic of the correlation despreading of transmitted reference signals like DCSK. The increasing effect of the noise cross-correlation in the decision variable causes the BER to increase as $2M$ (the spreading factor) increases (Fig. 2). The BER values predicted by (1) have also been verified against numerical simulations.

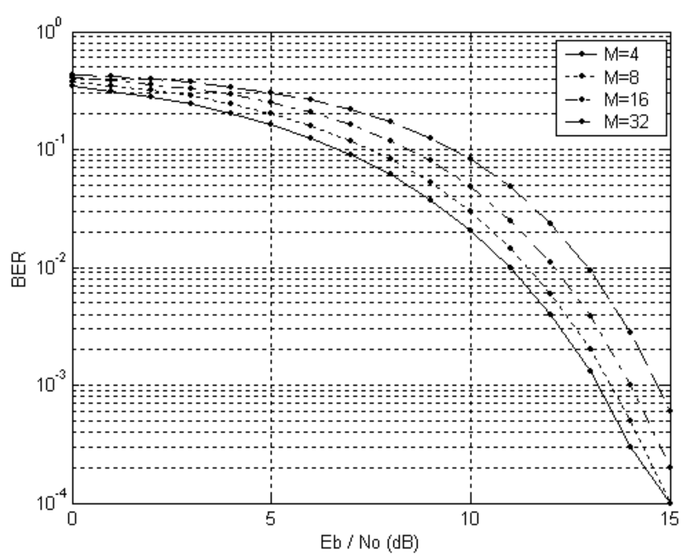

Fig. 2. Theoretical DCSK performance in AWGN channels.

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III. Rayleigh Fading Channels

Consider a transmitted signal $s(t) = A\cos(2\pi f_c t + \theta(t))$ through a channel exhibiting multipath fading. In general, the received signal is given by

$$y(t) = A\alpha(t)\cos(2\pi f_c t + \theta\prime(t)) + n(t) \tag{2}$$

where $n(t)$ is assumed AWGN. It can be shown [12] that if N, the number of received multipath components, is large, the in-phase and quadrature phase components of $y(t)$ become independent zero-mean Gaussian random variables. Then the amplitude $\alpha(t)$ has a Rayleigh pdf given by

$$f_\alpha(r) = \frac{r}{2\sigma^2}e^{\frac{-r^2}{2\sigma^2}}, \quad r > 0 \tag{3}$$

Such a fading process is called Rayleigh fading. In practical communication systems, phase distortion can be overcome using differential modulation, but the amplitude distortion $\alpha(t)$ is a problem. In this work, we assume slow fading, i.e., that $\alpha(t)$ stays constant for at least one signal period. We can then represent the fading phenomenon using a random variable α whose value is fixed during one signal period. The instantaneous SNR per bit γ_b is now a random variable given by

$$\gamma_b = \alpha^2\frac{E_b}{N_0} \tag{4}$$

where E_b is the energy per bit and $\frac{1}{2}N_0$ is the average noise power spectral density in the channel (W/Hz). Assuming ideal coherent detection of the received signal, the BER for DCSK as a function of the received instantaneous SNR per bit γ_b is

$$P_2(\gamma_b) = \mathrm{Q}\left(\sqrt{\frac{\gamma_b}{2}\left(1+\frac{M}{2\gamma_b}\right)^{-1}}\right) \tag{5}$$

To obtain the average BER when α is random, we average $P_2(\gamma_b)$ over the pdf of γ_b, to get

$$P_2 = \int_0^\infty P_2(\gamma_b)p(\gamma_b)\mathrm{d}\gamma_b \tag{6}$$

where $p(\gamma_b)$ is the pdf of γ_b when α is random. Since α is Rayleigh-distributed, α^2 has a chi-squared pdf with two degrees of freedom. Consequently, γ_b is also chi-square distributed

$$p(\gamma_b) = \frac{1}{\overline{\gamma_b}}e^{-\frac{\gamma_b}{\overline{\gamma_b}}}, \quad \gamma_b \geq 0 \tag{7}$$

where $\overline{\gamma_b} = \frac{E_b}{N_0}E\left\{\alpha^2\right\}$ is the average SNR per bit. Now we can substitute (7) and (5) into (6) to get the average BER as

$$P_2 = \int_0^\infty \mathrm{Q}\left(\sqrt{\frac{\gamma_b}{2}\left(1+\frac{M}{2\gamma_b}\right)^{-1}}\right)\frac{1}{\overline{\gamma_b}}e^{-\frac{\gamma_b}{\overline{\gamma_b}}}\mathrm{d}\gamma_b \tag{8}$$

This integral may be evaluated numerically. Like in AWGN channels, increasing the value of M increases the BER. Approximate formulas for the BER can also be found analytically. For example, when $\frac{M}{2\gamma_b} \ll 1$, we get

$$P_2 = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma_b}}{4+\overline{\gamma_b}}}\right) \tag{9}$$

A better approximation, valid when $\frac{2\beta}{\gamma_b} \ll 1$, where $\beta = \frac{M}{16}$ and $\overline{\gamma_b'} = \frac{\overline{\gamma_b}}{4}$ is [13]

$$P_2 = \frac{1}{2}\left[1-\frac{2}{\sqrt{\mu}(\sqrt{\mu}+\gamma)\overline{\gamma_b}}\exp\left[-2\left(\beta\gamma+\beta\sqrt{\mu}\right)\right]\right] \tag{10}$$

where $\gamma = -1$ and $\mu = 1+\frac{1}{\gamma_b'}$. The exact value obtained by numerically integrating (8), the first (9) and second (10) analytical approximations are compared with simulation results in Fig. 3 for DCSK with $M = 8$. Simulation results and exact theoretical values match closely. The second analytical approximation (10) provides a closer fit than the first one.

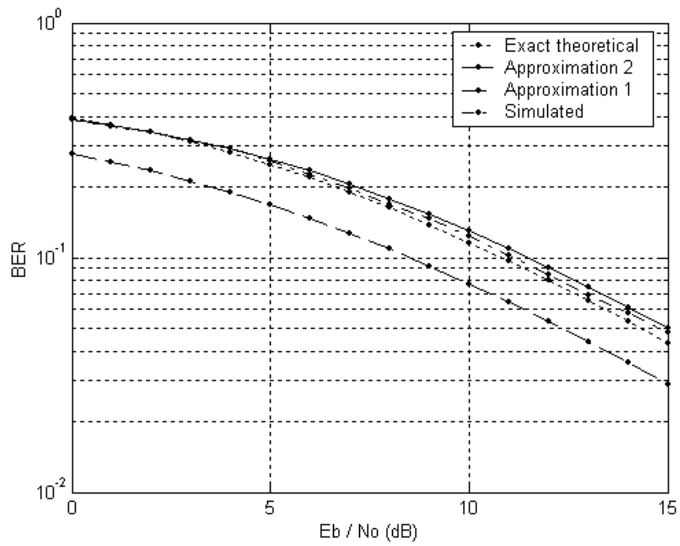

Fig. 3. Comparing different analytical BER expressions with simulated values for DCSK in Rayleigh fading channels (M=8)

IV. Diversity

Multiple independent signal paths can be used to combat fading. When one signal path undergoes a deep fade, the others may have strong signals. Using L independent paths to transmit the same information provides L-fold diversity. To improve system performance, the receiver has to efficiently process the received signals and then reach a decision about the received bit based on all of them. The most common diversity reception method is known as Maximal Ratio Combining (MRC).

In MRC, the signal from all the branches are co-phased and individually weighed to provide the optimal SNR at the output. It can be shown that the output SNR is maximized when the signals in each of the diversity branches are weighed by their own received envelopes. In case of a L-fold diversity scheme, the combining equation is given by

$$Z_i = \sum_{k=1}^{L} r_{ki}Z_{ki} \tag{11}$$

where r_{ki} represents the instantaneous envelope of the signal Z_{ki} received at the k-th diversity branch. For Rayleigh fading channels when L-fold diversity is used, the received signals for the L channels are

$$Z_{lk}(t) = \alpha_k e^{-\jmath\phi_k} s_{km}(t) + n_k(t) \tag{12}$$

where $k = 1, 2, ..., L$; $m = 1, 2$ (binary transmission) and $\{\alpha_k e^{-\jmath\phi_k}\}$ represents the attenuation factors and phase shifts for the L channels, $s_{km}(t)$ denotes the m-th signal transmitted on the k-th channel, and $n_k(t)$ denotes the AWGN on the k-th channel. All signals in the set $\{s_{km}(t)\}$ have the same energy.

The optimal MRC is based on the assumption that the channel attenuations $\{\alpha_k\}$ and phase shifts $\{\phi_k\}$ are known perfectly. The SNR per bit at the output of the MRC (γ_b) can then be written as

$$\gamma_b = \sum_{k=1}^{L} \gamma_k = \frac{E_b}{N_0} \sum_{k=1}^{L} \alpha_k^2 \tag{13}$$

where $\gamma_k = \alpha_k^2 \frac{E_b}{N_0}$ is the instantaneous SNR of the k-th diversity branch. The pdf of the output SNR $p(\gamma_b)$ is that of a chi-square-distributed random variable with $2L$ degrees of freedom [12]. It can thus be written as

$$p(\gamma_b) = \frac{1}{\overline{\gamma_c}^L (L-1)!} \gamma_b^{L-1} e^{-\frac{\gamma_b}{\overline{\gamma_c}}} \tag{14}$$

where $\overline{\gamma_c} = E\{\gamma_k\}$ is the average SNR per channel. To use (6), we need the functional form of $P_2(\gamma_b)$. This depends on the modulation scheme being used. For L-fold diversity and a fixed set of channel attenuations $\{\alpha_k\}$, the decision variable U, i.e., the MRC output, for DCSK is Gaussian with mean and variance given by

$$E(U) = \frac{E_b}{2} \sum_{k=1}^{L} \alpha_k^2 \tag{15}$$

$$\sigma_U^2 = \left(E_b \frac{N_0}{2} + M \frac{N_0^2}{4}\right) \sum_{k=1}^{L} \alpha_k^2 \tag{16}$$

Thus, we may write $P_2(\gamma_b)$ for DCSK as

$$P_2(\gamma_b) = \mathrm{Q}\left(\sqrt{\frac{\gamma_b}{2\left(1 + \frac{ML}{2\gamma_b} E\left(\{\alpha_k^2\}\right)\right)}}\right) \tag{17}$$

where $E\left(\{\alpha_k^2\}\right)$ is the expected value of the squares of the channel attenuation factors $\{\alpha_k^2\}$. Thus the average BER is

$$P_2 = \frac{1}{\overline{\gamma_c}^L (L-1)!} \int_0^\infty P_2(\gamma_b) \gamma_b^{L-1} e^{-\frac{\gamma_b}{\overline{\gamma_c}}} \mathrm{d}\gamma_b \tag{18}$$

This integral has to be evaluated numerically. However, under the condition that $\frac{ML}{2\gamma_b} E\left(\{\alpha_k^2\}\right) \ll 1$ we may approximate the integral as

$$P_2 \approx \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^k \tag{19}$$

where $\mu = \sqrt{\frac{\overline{\gamma_c}}{4+\overline{\gamma_c}}}$. For large values of $\overline{\gamma_c}$ (greater than 10 dB), (19) we have

$$P_2 \approx \left(\frac{1}{\overline{\gamma_c}}\right)^L \binom{2L-1}{L} \tag{20}$$

Numerically evaluated BER curves for various amounts of diversity are shown in Fig. 4 for $L = 1, 2, 3$ and compared with those obtained by numerical integration of (18). Increasing L causes the BER to decrease, but L cannot be increased indefinitely because of bandwidth limitations.

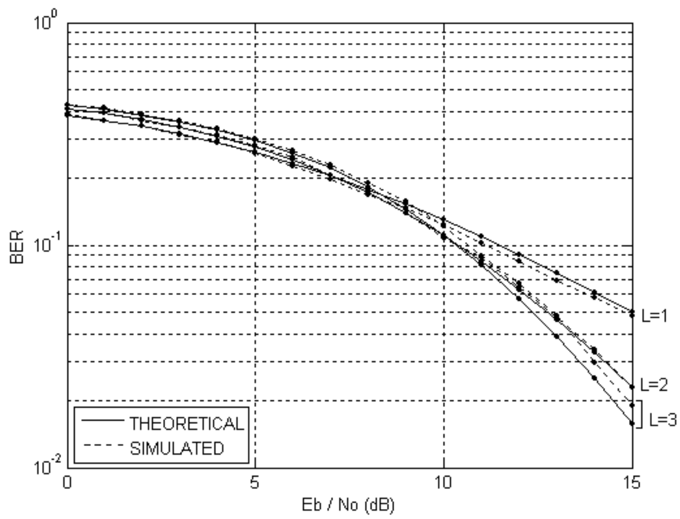

Fig. 4. Comparison of theoretical and simulated performances of DCSK in Rayleigh fading channels with diversity (M=8)

The accuracy of the approximate analytical expression in (19) is evaluated by comparing it with the numerically integrated expression from (18). The results are shown in Fig. 5. The approximate expression leads to an almost constant error of about 2-3 dB, mainly because of the inaccuracy involved when neglecting $\frac{ML}{2\gamma_b} E\left(\{\alpha_k^2\}\right)$ as compared to 1.

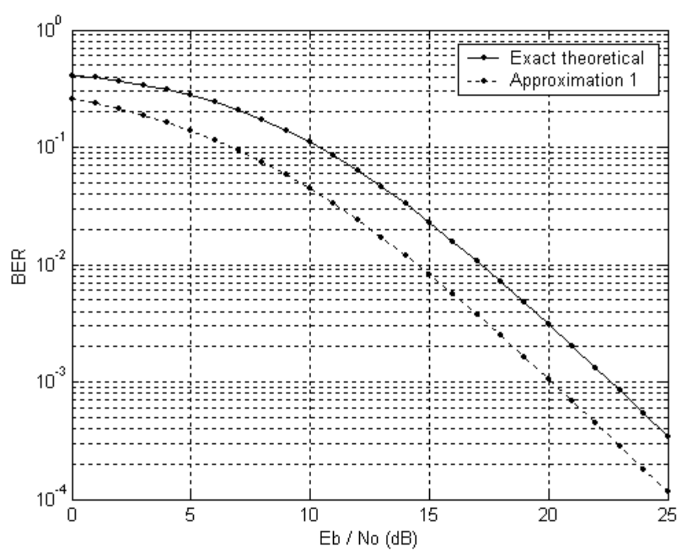

Fig. 5. Verification of the approximate analytical BER formula for DCSK in Rayleigh fading channels with 2-fold diversity

V. Ricean Fading Channels

There are many cases when the channel statistics are far from being Rayleigh. If the set of received waves is dominated by one strong component, Ricean fading is a more appropriate model. In Ricean fading, the in-phase and quadrature phase components of the received signal $I(t)$ and $Q(t)$ remain independently distributed jointly Gaussian random variables but are no longer zero mean. The pdf of the received amplitude $\rho = \sqrt{I^2 + Q^2}$ is given by

$$f_\rho(\rho) = \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2 + c_0^2}{2\sigma^2}\right) \mathrm{I}_0\left(\frac{c_0\rho}{\sigma^2}\right) \qquad (21)$$

where σ^2 is the local-mean scattered power, $\frac{1}{2}c_0^2$ is the power of the dominant component and I_0 is the modified Bessel function of the first kind and zero order. The Ricean channel becomes Rayleigh when $c_0 \to 0$ and AWGN when $c_0 \to \infty$. The Ricean K-factor is defined as the ratio of signal power in dominant component over the (local-mean) scattered power

$$K = \frac{c_0^2}{2\sigma^2} \qquad (22)$$

The pdf of the received power p is

$$f_p(p) = \frac{(1+K)e^{-K}}{\overline{p}} \exp\left(-\frac{1+K}{\overline{p}}p\right) \mathrm{I}_0\left(\sqrt{\frac{4K(1+K)}{\overline{p}}p}\right) \qquad (23)$$

In terms of the instantaneous SNR per bit γ_b and the average SNR of the channel $\overline{\gamma_b}$ we get

$$p(\gamma_b) = \frac{(1+K)e^{-K}}{\overline{\gamma_b}} \exp\left(-(1+K)\frac{\gamma_b}{\overline{\gamma_b}}\right) G(\gamma_b) \qquad (24)$$

where $G(\gamma_b) = \mathrm{I}_0\left(\sqrt{\frac{4K(1+K)\gamma_b}{\overline{\gamma_b}}}\right)$. Using (6) the average BER for DCSK is

$$P_2 = \int_0^\infty F(\gamma_b)\, G(\gamma_b) \frac{(1+K)e^{-K}}{\overline{\gamma_b}} e^{-\frac{(1+K)\gamma_b}{\overline{\gamma_b}}} \mathrm{d}\gamma_b \qquad (25)$$

where $F(\gamma_b) = \mathrm{Q}\left(\sqrt{\frac{\gamma_b}{2\left(1+\frac{M}{2\gamma_b}\right)}}\right)$. Theoretical BER values for DCSK in slow Ricean fading channels are obtained from (25) by numerical integration. These theoretical values are compared with values obtained from direct numerical simulations in Fig. 6 for various values of K and a 2-ray channel model.

As expected, the BER performance improves as K increases. Overall, DCSK performs better in Ricean than in Rayleigh fading channels. We also note that the theoretical and numerical values match closely with each other. This validates the theoretical BER calculations.

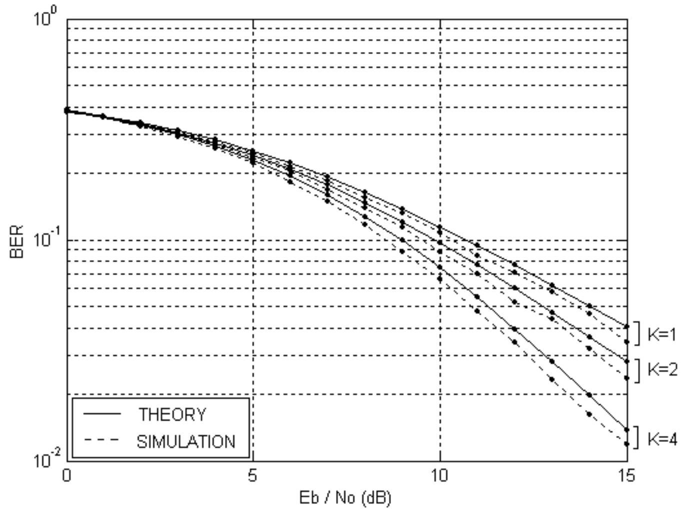

Fig. 6. Comparing simulated and theoretical performances of DCSK over 2-ray Ricean fading channels (M=8)

VI. Conclusions and Further Work

The performance of DCSK has been evaluated for a single user in various multipath fading environments. Analytical results have been compared against numerical simulations. We consider our work necessary for the introduction of chaos-based communication schemes in practical systems.

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