Pramāṇa, Vol. 20, No. 3, March 1983, pp. 221–244. © Printed in India.

Weak neutral current effects in elastic electron deuteron scattering

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MS received 26 October 1982

Abstract. We study the effect of weak neutral currents in elastic electron deuteron scattering on both unpolarized and polarized deuteron targets. Theoretical expressions have been derived for the polarized electron asymmetry, polarized target asymmetry and recoil deuteron vector polarization within the framework of impulse approximation. We show that these polarization parameters can give vital information on the space-time and isospin structure of the hadronic weak neutral current. In particular, our numerical estimates show that a measurement of polarized target asymmetry is sensitive to the isoscalar axial vector piece in the hadronic neutral current which, though zero in the Weinberg-Salam model, is not completely ruled out by the data.

Keywords. Weak neutral current; electron deuteron scattering; Weinberg-Salam model; impulse approximation; polarized electron asymmetry; polarized target asymmetry; recoil deuterod veckor polarization.

1. Introduction

With the discovery of parity violation in polarized electron-deuteron inelastic scattering by the SLAC group (Prescott *et al* 1978, 1979), the SU(2) × U(1) model of Weinberg (1967) and Salam (1968), which successfully predicted the existence of weak neutral currents, has been established beyond doubt as the theory of unified electromagnetic and weak interactions. The predictions of the theory are in complete agreement with all the available experimental data (see for example Hung and Sakurai 1981) from neutral current interactions with neutrinos. The results from atomic parity violation also support the theory (Baird *et al* 1977; Lewis *et al* 1977; Barkov and Zolotorev 1978, 1979).

Theoretically, the weak neutral currents through the process of electron nucleon scattering has been studied earlier (Derman 1974, 1979; Reya and Schilcher 1974a, b; Wilson 1974; Berman and Primack 1974; Cahn and Gilman 1978; Gilman and Tsao 1978; Tsao 1980; Hoffman and Reya 1979). Recently Safin *et al* (1980) have calculated the lepton-nucleon scattering not only with polarized leptons, but with polarized nucleons as well. The fact that lepton nucleus scattering processes at intermediate energies can play an important role in studying the structure of the neutral currents has been emphasized by many authors (Feinberg 1975; Walecka 1977; Porrman and Gari 1977; Wolfenstein 1978; Ramachandran and Singh 1978; Donnelly and Peccei 1979; Marciano and Sanda 1979; Murthy *et al* 1977, 1979; Serot 1979; Tsao 1980; Hwang and Henley 1980; Fischer-Waetzmann 1981; Porrman 1981). Some experiments on electron-nucleus scattering have already been proposed (Otten *et al* 1978; Hughes 1981). These processes can be very helpful in deciding the spacetime and isospin structure of weak neutral currents. Within the framework of Weinberg-Salam (WS) model the weak neutral currents are linear combinations of vector (V) and axial vector (A) currents with the second class currents being completely absent (Wu 1978). The hadronic neutral currents are a combination of isovector ($I=1$) and isoscalar currents ($I=0$). Of particular interest in discussing the isospin structure of weak neutral currents is the presence or absence of the isoscalar axial vector piece in the hadronic neutral current (Wolfenstein 1978; Barnebeu and Eranhyan 1978; Hung 1978; Marciano and Sanda 1979). As such within the framework of WS model there is no isoscalar axial vector piece. It is therefore important to study $\triangle I=0$ transitions which could unambiguously establish the presence or absence of an isoscalar axial vector piece in the hadronic neutral current. An ambitious fit to all neutrino hadron data has been recently performed (Kim *et al* 1981, Hung and Sakurai 1981) which shows a possible small but non zero value for the isoscalar axial vector piece though in general the phenomenologically-determined parameters one in excellent agreement with the WS model predictions with $\sin^2\theta_w=0.23$ which is the free parameter of the theory. A final verdict on the presence or absence of such a piece can be given only when the data from electron-nucleon and electron-nucleus scattering experiments become available. The available data from atomic parity violation experiments and SLAC experiment are clearly insufficient for this purpose.

The process of elastic electron deuteron scattering plays a particularly important role in studying the spacetime and isospin structure of the hadronic neutral current. Since the deuteron is an isoscalar nucleus only $\triangle I=0$ currents contribute to this process. In this paper we discuss various parity violating effects in the elastic electron deuteron scattering caused by the presence of weak neutral currents. Calculations have been done at intermediate energies where impulse approximation can be used for the interaction of electrons with deuterons. We have calculated the parity violating effects due to the interference of the photons γ and the neutral weak boson Z exchanges. The parity violating effects can also come from abnormal parity admixture states in the two-nucleon system induced by the neutral as well as the charged currents (Ramachandran 1966; Fischbach and Tadic 1973; Gari 1973; Porrman and Gari 1977; Rekalo 1978; Porrman *et al* (1979). In a recent paper, Hwang and Henley (1980) and Henley and Hwang (1981) (see also Porrman 1981) have given an excellent treatment of the effects due to P-state admixtures on the polarized electron-deuteron elastic scattering. They find that at intermediate energies ($\sim$300 MeV) the polarized electron asymmetry would be dominated by Z° exchange contributions while the abnormal parity admixtures in deuteron are important only at low energies ($\sim$10 MeV). In fact the non-existence of a single nucleon-nucleon parity non-conserving potential that fits all of the existing data causes the analysis of the effects of abnormal parity admixtures really difficult (Serot 1979). As we are interested only in the intermediate energy region we neglect the effects due to abnormal parity states. We would like to come back to this problem in another communication.

In § 2, we discuss the non-relativistic reduction of the elementary electron-nucleon scattering amplitudes and develop the theory of electron-deuteron elastic scattering in § 3. In § 4 we discuss the longitudinally-polarized electron asymmetry $A(\theta)$ for 200 MeV$\leqslant E_e \leqslant$500 MeV. In §§ 5 and 6 we discuss the polarized target asymmetry and recoil deuteron polarization with unpolarized electrons. This section is now particularly relevant in view of the recent strides made in production and measurement of polarization (Ninikosky 1981; Meyer 1981). In fact there already exists a proposal for an experiment on electron scattering with polarized protons (Schüler 1981, Yale University proposal). Finally in § 7 we give a critical discussion of the topics presented in this paper.

2. Amplitudes

In order to calculate the spin flip and spin nonflip amplitudes, **K** and L, for the process $e + N \to e + N$, we consider the Feynman diagrams given in figure 1. The matrix elements for this process is written as

$$m = m^{\gamma} + m^{z}, \tag{1}$$

where
$$m^{\gamma} = \frac{e^2}{q^2} \bar{u}(k_2)\, \gamma_{\mu}\, u(k_1)\, \bar{u}(p_2) \left[F_1(q^2)\, \gamma_{\mu} + i\, F_2(q^2)\, \sigma_{\mu\nu} \frac{q_{\nu}}{2M} \right] u(p_1), \tag{2}$$

$$m^{z} = \frac{G}{\sqrt{2}} \bar{u}(k_2)\, \gamma_{\mu}\, (a - b\, \gamma_5)\, u(k_1)\, \bar{u}(p_2) \Big[f_v(q^2)\, \gamma_{\mu} + i f_w(q^2)\, \sigma_{\mu\nu} \frac{q_{\nu}}{2M}$$
$$+ f_A\, \gamma_{\mu}\, \gamma_5 + f_p\, q_{\mu}\, \gamma_5 \Big] u(p_1), \tag{3}$$

(we follow the convention of Bjorken and Drell (1964) through out this paper). Doing a non-relativistic reduction of the hadronic matrix element following the standard methods (Murthy *et al* 1977; Singh 1972, 1975), the matrix element m for an isoscalar nuclear target can be written as

$$m = i\vec{\sigma} \cdot \mathbf{K} + L \tag{4}$$

where
$$\mathbf{K} = \frac{e^2}{q^2} G_{MS}(q^2) \frac{\mathbf{l}^{\gamma} \times \mathbf{q}}{2M} \times \frac{G}{\sqrt{2}}$$
$$\left[F_{MS}(q^2) \frac{\mathbf{l}^{z} \times \mathbf{q}}{2M} + i\, F_{AS}(q^2) \left(\mathbf{l}^{z} - \mathbf{l}_0^{z} \frac{\mathbf{q}}{2M} \right) \right], \tag{5}$$

and
$$L = \frac{e^2}{q^2} G_{ES}(q^2) \left(l_0^{\gamma} - \frac{\mathbf{l}^{\gamma} \cdot \mathbf{q}}{2M} \right) + \frac{G}{\sqrt{2}} F_{ES}(q^2) \left(l_0^{z} - \frac{\mathbf{l}^{z} \cdot \mathbf{q}}{2M} \right),$$

$$l_{\mu}^{z} = \bar{u}(k_2)\, \gamma_{\mu}\, (a - b\, \gamma_5)\, u(k_1); \quad l_{\mu}^{\gamma} = l_{\mu}^{z}\, (a = 1, b = 0), \tag{6}$$

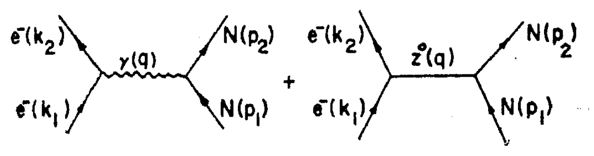

Figure 1. Feynman diagrams for elastic electron-nucleon scattering.

where the isoscalar electromagnetic and weak form factors are given by

$$G_{MS}(q^2) = \tfrac{1}{2}[G_{Mp}(q^2) + G_{Mn}(q^2)], \tag{7}$$

$$G_{ES}(q^2) = \tfrac{1}{2}[G_{Ep}(q^2) + G_{En}(q^2)], \tag{8}$$

$$F_{MS}(q^2) = \tfrac{1}{2}[F_{Mp}(q^2) + F_{Mn}(q^2)], \tag{9}$$

$$F_{ES}(q^2) = \tfrac{1}{2}[F_{Ep}(q^2) + F_{En}(q^2)], \tag{10}$$

$$F_{AS}(q^2) = \tfrac{1}{2}[F_{Ap}(q^2) + F_{An}(q^2)], \tag{11}$$

where p and n stand respectively for the proton and the neutron. The various nucleon form factors are given by

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2), \tag{12}$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2), \tag{13}$$

$$F_E(q^2) = f_v(q^2) + \frac{q^2}{4M^2} f_w(q^2), \tag{14}$$

$$F_M(q^2) = f_v(q^2) + f_w(q^2), \tag{15}$$

$$F_A(q^2) = f_A(q^2). \tag{16}$$

It should be noted that in equations (2) and (3), the initial and final momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ of the interacting (not spectator) nucleon are given in the rest frame of the deuteron by

$$\mathbf{p}_1 = \mathbf{p}, \quad \mathbf{p}_2 = \mathbf{p} + \mathbf{q}, \tag{17}$$

where $\mathbf{p}$ is the relative momentum of the nucleons inside the deuteron. Equations (5) and (6) were obtained by neglecting the Fermi momentum $\mathbf{p}$ of the nucleon inside the deuteron.

In the 'standard model' the weak form factors have the following values,

$$F_{AS}(q^2) = 0, \tag{18}$$

$$F_{ES}(q^2) = -\sin^2\theta_w/(1 + q^2/M_v^2)^2, \tag{19}$$

$$F_{MS}(q^2) = -\sin^2\theta_w(\mu_p + \mu_n)/(1 + q^2/M_v^2)^2, \tag{20}$$

along with

$$a = (1 - 4\sin^2\theta_w)/2, \tag{21}$$

$$b = -1/2, \tag{22}$$

where θ_w is the Weinberg angle. The electromagnetic form factors are given in the standard way

$$G_{ES}(q^2) = 1/2\,(1 + q^2/M_v^2)^2, \tag{23}$$

$$G_{MS}(q^2) = (\mu_p + \mu_n)/2\,(1 + q^2/M_v^2)^2. \tag{24}$$

3. Theory

3.1 *Matrix element in the impulse approximation*

We consider the process

$$e(k_1) + d(P_1) \rightarrow e(k_2) + d(P_2), \tag{25}$$

where the quantities in the brackets denote the four momenta. The momentum transfer q_μ is defined as

$$q_\mu = k_{1\mu} - k_{2\mu} = P_{2\mu} - P_{1\mu}. \tag{26}$$

The amplitude for the process (25), in impulse approximation, can be written as,

$$\mathbf{T} = \sum_{j=1,2} \exp(i\,\mathbf{q}\cdot\mathbf{r}_j)\,(i\,\vec{\sigma}_j\cdot\mathbf{K}_j + L_j), \tag{27}$$

where $\mathbf{r}_j$ denotes the position vector of jth nucleon and L_j and $\mathbf{K}_j$ denote the spin-independent and spin-dependent isoscalar amplitudes for the process $e + N \rightarrow e + N$ on a nucleon. The deuteron is described by the wave function

$$\psi(r) = \frac{\exp(i\,\mathbf{P}\cdot\mathbf{R})}{(2\pi)^3} \times$$

$$\int d^3p \exp(i\,\mathbf{p}\cdot\mathbf{r}) \sum_{L=0,2} \sum_{M=-L}^{L} C(L11;\,M\mu m)\,\Phi_L(p)\,Y_{LM}(\hat{p})\,\chi_{1\mu}, \tag{28}$$

where $\mathbf{R}$ and $\mathbf{r}$ describe centre of mass and relative coordinates. $\Phi_L(p)$ are the Fourier transforms of the $S(L = 0)$ and $D(L = 2)$ state wave functions of the deuteron and are explicitly given by

$$\Phi_0(p) = (2/\pi)^{1/2} \int u(r)\,j_0(pr)\,r\mathrm{d}r, \tag{29}$$

$$\Phi_2(p) = -(2/\pi)^{1/2} \int w(r)\,j_2(pr)\,r\mathrm{d}r, \tag{30}$$

where $u(r)$ and $w(r)$ are S and D state radial distributions. The matrix element for the process (25) is now written as

$$\langle 1m_f|\mathbf{T}|1m_i\rangle = \frac{1}{(2\pi)^6}\int d^3p\, d^3p'\, d^3R\, d^3r \exp[i(\mathbf{p}-\mathbf{p}^1)\cdot\mathbf{r}]\exp(-i\,\mathbf{q}\cdot\mathbf{R})$$

$$\sum_{L_i L_f M_i M_f} C(L_f\,11;\, M_f\,\mu_f\,m_f)\; C(L_i\,11;\, M_i\,\mu_i\,m_i)\; \Phi^*_{L_f}(p')$$

$$\times\, \Phi_{L_i}(p)\; Y^*_{L_f M_f}(\hat{p}')\; Y_{L_i M_i}(\hat{p})$$

$$\left\langle \chi_{1\mu_f}\left|\sum_j \exp(i\,\mathbf{q}\cdot\mathbf{r}_j)(i\vec{\sigma}_j\cdot\mathbf{K}_j + L_j)\right|\chi_{1\mu_i}\right\rangle \tag{31}$$

Substituting for $\mathbf{r}_1$ and $\mathbf{r}_2$ in terms of $\mathbf{r}$ and $\mathbf{R}$ and performing the integrations, we obtain

$$\langle 1m_f|\mathbf{T}|1m_i\rangle = \sum_{L_i L_f M_i M_f} C(L_f\,11;\, M_f\,\mu_f\,m_f)\; C(L_i\,11;\, M_i\,\mu_i\,m_i)$$

$$\int d^3p\, \langle \chi_{1\mu_f}|\,[\phi^*_{L_f}(p_+)\; Y^*_{L_f M_f}(\hat{p}_+)\,(i\,\mathbf{S}\cdot\mathbf{K}_1 + L_1) + \Phi^*_{L_f}(p_-)\; Y^*_{L_f M_f}(\hat{p}_-)$$

$$(i\,\mathbf{S}\cdot\mathbf{K}_2 + L_2)]\; \Phi_{L_i}(p)\; Y_{L_i M_i}(\hat{p})\,|\,\chi_{1\mu_i}\rangle, \tag{32}$$

where $\quad \mathbf{p}_\pm = \dfrac{\mathbf{q}}{2} \pm \mathbf{p}$ and $\mathbf{S} = \frac{1}{2}(\sigma_1 + \sigma_2)$.

In deriving (32) from (31), use has been made of the fact that the operator $(\vec{\sigma}_1 - \vec{\sigma}_2)$, being antisymmetric between neutron and proton, does not contribute to the process. Writing $i\,\mathbf{S}\cdot\mathbf{K} + L$ in spherical tensor notation as

$$i\,\mathbf{S}\cdot\mathbf{K} + L = \sum_{n=0}^{1}\sum_{\mu=-n}^{n} (i)^n(-1)^\mu S_{n\mu} K_{n-\mu};$$

$$S_0 = 1,\; S_1 = \mathbf{S};\; K_0 = L;\; K_1 = \mathbf{K} \tag{33}$$

and neglecting Fermi momentum (we shall return to this question later) the expression for matrix element simplifies to

$$\langle 1m_f|\mathbf{T}|1m_i\rangle = \sum_{Ln\lambda} (-1)^{m_\lambda}\, C(|\lambda|;\, m_i\, \lambda m_f)\,(Y_L(\hat{q})\otimes K_n)^\lambda_{-m_\lambda}\, B_{Ln\lambda}, \tag{34}$$

where $\quad B_{Ln\lambda} = 6\sqrt{4\pi}\,(-i)^{L+n}\,(-1)^\lambda \langle 1\,\|S_n\|\,1\rangle\,[(2L+1)(2\lambda+1)]^{1/2}.$

$$\cdot \sum_{L_i L_f} (2L_i + 1)^{1/2} f_{L_i L L_f} \begin{Bmatrix} L_i & 1 & 1 \\ L & n & \lambda \\ L_f & 1 & 1 \end{Bmatrix} C\,(L_i\, L\, L_f;\, 000), \qquad (35)$$

where the symbol $\{\ \}$ stands for a q_j coefficient. In the absence of Fermi momentum $K_1 = K_2 = \mathbf{K}$ and $L_1 = L_2 = L$. $\langle 1 \,\|\, S_n \,\|\, 1\rangle$ is the reduced matrix element of the operator S_n and

$$f_{L_i L L_f} = \int \mathrm{d}r\, u_{L_f}(r)\, j_L(qr/2)\, u_{L_i}(r) \qquad (36)$$

with $u(r) = u_0(r)$ and $w(r) = u_2(r)$.

3.2 *Final state density matrix for the deuteron*

The final state density matrix is given by

$$\rho^f_{m_f^1 m_f} = \sum_{m_i m_i^1} \langle 1\, m_f^1 \,|\, \mathbf{T} \,|\, 1\, m_i\rangle\, \rho^i_{m_i^1 m_i} \langle 1\, m_f \,|\, \mathbf{T} \,|\, 1\, m_i\rangle^*, \qquad (37)$$

where ρ^i denotes the initial density matrix of the deuteron. The density matrix ρ^i in terms of the spherical tensor parameters t^i_{kq} is given by (Ramachandran and Murthy 1978)

$$\rho^i_{m_i^1 m_i} = \frac{1}{3} \sum_{kq} (-1)^q\, C\,(1\, k\, 1;\, m_i - q\, m_i')\, (2\,k + 1)^{1/2}\, t^i_{kq}, \qquad (38)$$

where $(2k + 1)^{1/2}\, C\,(1\, k\, 1;\, m_i - q\, m_i')$ is simply the matrix element of the spherical tensor operator T_{kq} (S) following Madison Convention (Darden 1970). Substituting (38) in (37) and using (34) for the transition matrix element, the following expression is obtained for

$$\rho^f_{m_f' m_f} = \frac{1}{4\pi} \sum_{\substack{L n \lambda L' n' \lambda' \\ l N \Lambda\, k k'}} (-1)^{q'} (-1)^{l+n+k-\lambda-\lambda'-k'} [(2\lambda+1)(2\lambda'+1)(2\Lambda+1)(2l+1)$$

$$\cdot (2N+1)(2k'+1)(2k+1)(2L+1)(2L'+1)]^{1/2} B_{L' n' \lambda'} B^*_{L n \lambda} C\,(L'\, L\, l;\, 000)$$

$$\cdot\, C\,(1\, k'\, 1;\, m_f - q'\, m_f') \begin{Bmatrix} L' & n' & \lambda' \\ L & n & \lambda \\ l & N & \Lambda \end{Bmatrix} \begin{Bmatrix} 1 & \lambda & 1 \\ k & \Lambda & k' \\ 1 & \lambda' & 1 \end{Bmatrix}$$

$$(4\pi/2l+1)^{1/2}\, (t^i_k \otimes (Y_l(\hat{q}) \otimes (K_{n'} \otimes K^*_n)^N)\Lambda)^{k'}_{q'} \qquad (39)$$

4. Polarized electron asymmetry

The longitudinally polarized electron asymmetry in electron deuteron elastic scattering is defined as

$$A = \frac{(\mathrm{d}\sigma/\mathrm{d}\Omega)_+ - (\mathrm{d}\sigma/\mathrm{d}\Omega)_-}{(\mathrm{d}\sigma/\mathrm{d}\Omega)_+ + (\mathrm{d}\sigma/\mathrm{d}\Omega)_-}, \tag{40}$$

where $(\mathrm{d}\sigma/\mathrm{d}\Omega)_\pm$ are the differential cross-sections for the right-handed and left-handed electrons. Since $(\mathrm{d}\sigma/\mathrm{d}\Omega) \propto \mathrm{Tr}\,(\rho^f)$, the asymmetry can be written as

$$A = \frac{\mathrm{Tr}\,(\rho^f_+) - \mathrm{Tr}\,(\rho^f_-)}{\mathrm{Tr}\,(\rho^f_+) + \mathrm{Tr}\,(\rho^f_-)} \tag{41}$$

where $\mathrm{Tr}\,(\rho^f)$ is given by (39) (with unpolarized deuteron target, *i.e.*, $t^i_{00} = 1$; $t^i_{kq} = 0$). Written explicitly the asymmetry has the following form,

$$A = \frac{a_1(|L_+|^2-|L_-|^2)+a_2(|\mathbf{K}_+|^2-|\mathbf{K}_-|^2)+a_3(|\hat{q}\cdot\mathbf{K}_+|^2-|\hat{q}\cdot\mathbf{K}_-|^2)}{a_1(|L_+|^2+|L_-|^2)+a_2(|\mathbf{K}_+|^2+|\mathbf{K}_-|^2)+a_3(|\hat{q}\cdot\mathbf{K}_+|^2+|\hat{q}\cdot\mathbf{K}_-|^2)}, \tag{42}$$

where $\mathbf{K}_\pm$ and $L_\pm$ are the spin-dependent and spin-independent amplitudes for the right-handed and left-handed electrons and are given by

$$\mathbf{K}_\pm = \frac{e^2}{q^2} G_{MS}(q^2)\left(\mathbf{l}^\gamma_\pm \times \frac{\mathbf{q}}{2M}\right) + \frac{G}{\sqrt{2}}\left[F_{MS}(q^2)\left(\mathbf{l}^z_\pm \times \frac{\mathbf{q}}{2M}\right)\right.$$
$$\left. + i\, F_{AS}(q^2)\left(\mathbf{l}^z_\pm - \frac{\mathbf{q}}{2M} l_{0\pm}\right)\right], \tag{43}$$

$$L_\pm = \frac{e^2}{q^2} G_{ES}(q^2)\left(l^\gamma_{0\pm} - \mathbf{l}^\gamma_\pm \cdot \frac{\mathbf{q}}{2M}\right) + \frac{G}{\sqrt{2}} F_{ES}(q^2)\left(l^z_{0\pm} - \mathbf{l}^z_\pm \cdot \frac{\mathbf{q}}{2M}\right), \tag{44}$$

where
$$l^z_{\mu\pm} = (l^z_{0\pm},\ \mathbf{l}^z_\pm) = \bar{u}\,(k)\gamma_\mu\,(a - b^\gamma_5)\,\frac{(1 \pm \gamma_5)}{2}\,u\,(k_1), \tag{45}$$

$$l^\gamma_{\mu\pm} = l^z_{\mu\pm}\,(a = 1,\, b = 0). \tag{46}$$

The effect of the deuteron wave function is contained in the function a_1, a_2, a_3 which are given by (Ramachandran 1967),

$$a_1 = 4\,(F_1^2 + F_3^2),\ a_2 = \tfrac{1}{3}\,(2\sqrt{2}\,F_2 + F_4)^2,\ a_3 = (-4\sqrt{2}\,F_2\,F_4 + F_4^2) \tag{47}$$

with

$$F_1 = f_{000} + f_{202},\quad F_2 = f_{000} - \tfrac{1}{2} f_{202},$$

$$F_3 = 2 f_{022} - \frac{1}{\sqrt{2}} f_{222},\quad F_4 = 2 f_{022} + \sqrt{2}\, f_{222}, \tag{48}$$

where $f_{L_i L L_f}$ are defined by (36).

Substituting the values of $\mathbf{K}_\pm$ and $L_\pm$ as defined in (43) and (44), $\mathbf{K}_\pm \cdot \mathbf{K}_\pm$ and $L_\pm L_\pm$ are calculated. Relevant expressions are given in the appendix. We neglect $|\hat{q}\cdot \mathbf{K}_\pm|^2$ term as it is proportional to G^2. To first order in weak interaction coupling constant G, the asymmetry is given by

$$A = -\frac{GM^2}{2\sqrt{2}\pi\alpha}\cdot\frac{q^2}{M^2}\Big[b\,\mathscr{K}_1\frac{F_{ES}(q^2)}{G_{ES}(q^2)} + \eta\left\{a\,\mathscr{K}_3\frac{F_{AS}(q^2)}{G_{MS}(q^2)} + b\,\mathscr{K}_2\frac{F_{MS}(q^2)}{G_{MS}(q^2)}\right\}\Big]\times[\mathscr{K}_1 + \eta\,\mathscr{K}_3]^{-1}, \quad (49)$$

where $\eta = \frac{a_2}{a_1}\cdot\frac{G^2_{MS}(q^2)}{G^2_{ES}(q^2)}$,

$$\mathscr{K}_1 = 2E_1E_2\cos^2\theta/2\cdot\left(1+\frac{E_2-E_1}{2M}\right)^2 + O(1/M2),$$

$$\mathscr{K}_3 = -\frac{2E_1E_2}{M}(E_1+E_2)\sin^2\theta/2 + O(1/M^2),$$

$$\mathscr{K}_2 = \frac{E_1E_2}{M^2}\sin^2\theta/2\,(E_1^2+E_2^2+2E_1E_2\sin^2\theta/2). \quad (50)$$

The initial and final electron energies are denoted by E_1 and E_2.

Within the frame work of Weinberg-Salam Model, (49) reduces to a very simple expression, *viz*,

$$A = \frac{4GM^2}{\sqrt{2}\pi\alpha}\sin^2\theta_w\cdot\frac{E_1^2\sin^2\theta/2}{M^2\left(1+\frac{E_1}{M}\sin^2\theta/2\right)}, \quad (51)$$

where we have used the fact that

$$\frac{F_{ES}(q^2)}{G_{ES}(q^2)} = -2\sin^2\theta_w \quad (52)$$

and $b = -\frac{1}{2}$. Expression (51) for A is the same as the expression found in our earlier communication† (Murthy *et al* 1979).

The parity-violating asymmetry A in (51) arises due to the interference of the isoscalar vector (axial vector) currents at the hadronic vertex with the axial vector

† We wish to point out that there is an error in the corresponding expression in our earlier communication (Murthy *et al* 1979). The asymmetry A is positive and not negative as noted earlier. The values apart from the sign are the same.

(vector) currents at the electron vertex. The dominant contribution to the asymmetry comes from the interference of the isoscalar vector currents at the hadron vertex with the axial vector currents at the electron vertex, *i.e.* terms proportional to b in (51). However, as we shall see later, the dominant contribution at backward angles to A can come from the term proportional to $F_{AS}(q^2)$. This is purely an effect due to the deutron structure functions since the D-state contribution is sizable at large q^2 even though the term proportional to $F_{AS}(q^2)$ is suppressed by a factor $(1/M)$ compared to terms proportional to b. Even then the observation of this symmetry is not likely to yield any decisive information on the isoscalar axial vector piece unless $F_{AS}(0)$ is anomolously large. The indications from neutrino data (Hung and Sakurai 1981) are that $F_{AS}(0)$ is small even if it is present. The situation here is similar to the situation in atomic parity violation where the leading contributions come from terms involving the axial vector part of the leptonic current.

In order to obtain numerical values for the asymmetry A, we need to calculate the structure functions of the deuteron. We have done this using the Reid soft core wave functions (Reid 1969). It should be noted that within the framework of Weinberg-Salam Model, to the leading order, the asymmetry is independent of the nuclear structure functions (Hwang and Henley 1980, Porrman 1981). This however is not the case when we have a non zero F_{AS}. We have carried out our numerical calculations both in WS Model ($F_{AS} = 0$) and in the WP Model† (with $F_{AS} = 0{\cdot}186$) (see for example, Gounaris and Vergados 1978; Barger and Nanopoulos 1977). We have chosen, in particular, the values of 0·20 and 0·25 for $\sin^2\theta_w$ which are close to the experimentally determined value, *viz*, $\sin^2\theta_w = 0{\cdot}224 \pm 0{\cdot}12 \pm 0{\cdot}008$ (the first error is statistical and the second, systematic).

The asymmetry A for electron lab energies E_1 between 200 MeV and 500 MeV, the region where the impulse approximation works fairly well is shown in figures 2a and 2b as a function of the lab scattering angle θ of the outgoing electron. While the solid curves show the angular distribution of A for $\sin^2\theta_w=0{\cdot}20$ and 0·25 in WS model, the dashed curve shows the angular distribution of A in the WP model with $F_{AS}(0)=0{\cdot}186$ and $\sin^2\theta_w=0{\cdot}20$. The effect of a non-zero F_{AS} is felt only at backward angles as compared to the curves with $F_{AS}=0$. For $\sin^2\theta_w=0{\cdot}25$ both the models predict the same value for $A(\theta)$ since term proportional to F_{AS} is multiplied by the strength of the vector part of the leptonic current which is zero ($a=1-4\sin^2\theta_w$). We emphasize that these asymmetries are of the same order as those observed in Proscott's experiment (Prescott *et al* 1978, 1979) and should be observable experimentally. At intermediate energies, where impulse approximation can be applied with some confidence, the theoretical analysis of the results would be relatively clean. This is not the case with the experiments at high energies where one has to go beyond the valence quark-parton model and the effect of heavy quarks and antiquarks in the sea has to be taken into account at these energies.

† Recently Cochard and Mouafakov (1981), using the elastic scattering data at low energies and the high energy inclusive and semi-inclusive scattering data, show that the WP model can be rejected on the basis of the data analysis. We emphasize that we are only interested in the effects of a non-zero F_{AS} rather than in any particular model.

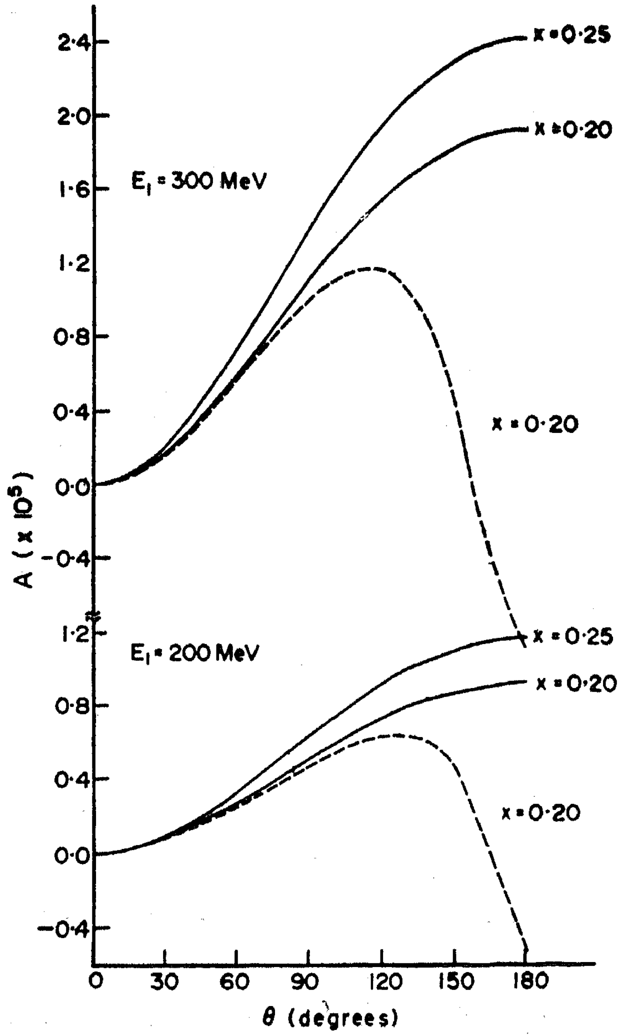

Figure 2a. Longitudinally polarized electron asymmetry A at incident electron energy $E_1 = 200$ MeV, 300 MeV as a function of lab scattering angle $\theta : x = \sin^2 \theta_w$: (————) corresponds to WS model: (- - - - -) corresponds to WP model ($F_{AS} = 0{\cdot}186$).

5. Polarized target asymmetry

With the increasing possibility of having polarized deuteion targets for scattering experiments (de Boer *et al* 1974, Ninikosky 1976, 1981), the elastic scattering of unpolarized electrons from polarized deuteron taiget can give valuable infoımation about the structure of neutral currents. We define the target asymmetry T by the following,

$$T = \frac{\left(\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\Omega}\right)_{\mathrm{P}} - \left(\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\Omega}\right)_{-\mathrm{P}}}{\left(\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\Omega}\right)_{\mathrm{P}} + \left(\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\Omega}\right)_{-\mathrm{P}}} \tag{53}$$

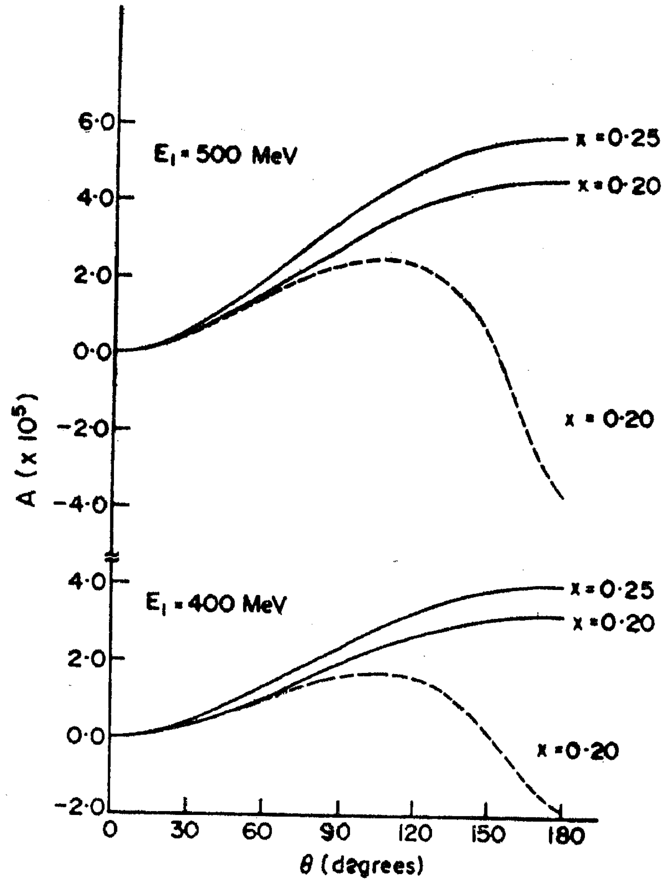

Figure 2b. Same as figure (2a). $E_1 = 400$ MeV, 500 MeV.

where **P** denotes vector polarization† of the deuteron and $(d\sigma/d\Omega)$ is proportional to Tr (ρ^f). Thus choosing

$$t^{i}_{00} = 1, \quad t^{i}_{1q} = \mathbf{P}_q, \quad t^{i}_{2q} = 0 \tag{54}$$

for purely vector-polarized deuterons and using (39), we obtain

$$\mathrm{Tr}\,(\rho^f) = a_1\, LL^* + a_2\, \mathbf{K}\cdot\mathbf{K}^* + a_3\,|\,\hat{\mathrm{q}}\cdot\mathbf{K}\,|^2 + t^{i}_{1}\cdot[b_1\, i\,(\mathbf{K}\,L^* - L\,\mathbf{K}^*)^1 - \frac{b_2}{\sqrt{2}}\, i\,(\mathbf{K}\times\mathbf{K}^*)^1 + b_3\,(4\pi/5)^{1/2}\, i\,(Y_2\,(\hat{q})\otimes(\mathbf{K}L^* - L\mathbf{K}^*)^1)^1 - b_4(4\pi/5)^{1/2}\, i\,(Y_2(\hat{q})\otimes(\mathbf{K}\times\mathbf{K}^*)^1)^1] \tag{55}$$

where a_i $(i = 1, \ldots, 3)$ are given by (47) and

$$b_1 = \sqrt{2/3}\,(4\,F_1\,F_2 + F_3\,F_4),$$

† Vector polarization $\mathbf{P} = t^{i}_{1} = \sqrt{3/2}\,\langle \mathbf{S}\rangle$ where **S** is the spin vector of the deuteron.

$$b_2 = 1/2\sqrt{3}\,(8F_2^2 - F_4^2),$$

$$b_3 = \sqrt{5/3}\,(2\sqrt{2}\,F_4 F_1 + 2\sqrt{2}\,F_2\,F_3 - F_3\,F_4),$$

$$b_4 = -\sqrt{5/6}\,(2\sqrt{2}\,F_2\,F_4 + F_4^2) \tag{56}$$

The target asymmetry is then calculated from (53) using (55),

$$T = t_1^i \cdot \Big[b_1\, i\,(\mathbf{K}\,L^* - L\,\mathbf{K}^*)^1 - \frac{b_2}{\sqrt{2}}\, i\,(\mathbf{K} \times \mathbf{K}^*)$$
$$+ b_3\,(4\pi/5)^{1/2}\, i\,(Y_2\,(\hat{q}) \otimes (\mathbf{K}L^* - L\,\mathbf{K}^*)^1)^1$$
$$-b_4 (4\pi/5)^{1/2} i (Y_2(\hat{q}) \otimes (\mathbf{K}L^* - L\mathbf{K}^*)^1)^1 \Big] \Big/ \, [a_1 LL^* + a_2 \mathbf{K}\cdot\mathbf{K}^* + a_3 |\hat{q}\cdot\mathbf{K}|^2]. \tag{57}$$

In order to put (57) in a simple form, we choose the Lakin (1955) transversity frame where the Z-axis is perpendicular to the reaction plane in the lab system. With such a choice $Y_{LM}(\hat{q})=0$ for odd values of M. The polar angle θ_q is equal to $\pi/2$ and the azimuthal angle ϕ_q is nothing but the angle made by the momentum transfer vector $\mathbf{q}$ with the incident electron direction, *i.e.* the x-axis in our choice. With this choice of the frame of reference we obtain

$$T = \mathbf{P} \cdot \mathbf{V}, \tag{58}$$

where
$$\mathrm{Tr}\,(\rho_0^f)\,\mathbf{V} = (Ib_1 - \chi b_3)\, i\,(\mathbf{K}L^* - L\mathbf{K}^*) - \frac{1}{\sqrt{2}}(Ib_2 - \chi b_4)\, i\,(\mathbf{K} \times \mathbf{K}^*), \tag{59}$$

where to simplify notation we have represented vectors by column matrices and I is a unit matrix while χ is a traceless symmetric matrix given by

$$\chi = \frac{1}{\sqrt{10}} \begin{bmatrix} (1 + 3\cos(2\phi_q))/2 & 3\sin(2\phi_q)/2 & 0 \\ 3\sin(2\phi_q)/2 & (1 - 3\cos(2\phi_q))/2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{60}$$

and
$$\mathrm{Tr}\,(\rho_0^f) = a_1\, LL^* + a_2\,\mathbf{K} \cdot \mathbf{K}^* + a_3\,|\,\hat{q} \cdot \mathbf{K}\,|^2. \tag{61}$$

Explicit expressions for $i(\mathbf{K}L^* - L\mathbf{K}^*)$, $i(\mathbf{K}\times\mathbf{K}^*)$, $\mathbf{K}\cdot\mathbf{K}^*$, LL^* are given in the appendix. We neglect $|\hat{q}\mathbf{K}|^2$ as it is proportional to G^2 which is very small. It is obvious from the expressions given in the appendix that $V_z=0$. This means that if the initial deuteron is polarized in the direction perpendicular to the reaction plane, then the target asymmetry is zero. A non-zero contribution to target asymmetry comes only if the target deuteron is polarized in the plane of the reaction. In order to discuss the target asymmetry when the target deuteron is polarized in the reaction plane, we define

$$T(x) = \mathbf{P}_x V_x \tag{62}$$

$$T(y) = \mathbf{P}_y V_y \tag{63}$$

where $T(x)$ ($T(y)$) denotes the target asymmetry if the polarization direction is $x(y)$ with $\mathbf{P}_x(\mathbf{P}_y)$ denoting the extent of polarization of the target along $x(y)$ direction. The quantities V_x and V_y can be evaluated using the expressions given in the appendix and they are given by (to the leading order),

$$V_x = -\frac{G M^2}{4\sqrt{2}\pi\alpha}\frac{q^2}{M^2}4a\frac{F_{AS}(q^2)}{G_{ES}(q^2)}\frac{1}{a_1}\left[\left\{b_1 - \frac{b_3}{2\sqrt{10}}[1 + 3\cos(2\phi_q)]\right\}\right.$$

$$\left. - \frac{3b_3}{2\sqrt{10}}\sin(2\phi_q)\tan\theta/2\right] + \text{higher order terms,} \tag{64}$$

$$V_y = -\frac{G M^2}{4\sqrt{2}\pi\alpha}\frac{q^2}{M^2}4a\frac{F_{AS}(q^2)}{G_{ES}(q^2)}\frac{1}{a_1}$$

$$\times\left[\left\{b_1 - \frac{b_3}{2\sqrt{10}}[(1 - 3(\cos(2\phi_q)]\right\}\tan\theta/2\right.$$

$$\left. - \frac{3b_3}{2\sqrt{10}}\cdot\sin(2\phi_q)\right] + \text{higher order terms,} \tag{65}$$

where θ as usual denotes the lab scattering angle of the electron. As can be seen from equations (64) and (65), the dominant contribution to the target asymmetry comes from the interference of the vector part of the leptonic current with the axial vector part of the hadronic current. The situation is complimentary to the case of polarized electron asymmetry discussed in § 4. Thus the experimental observation of the target asymmetry can be crucial in the determination of the existence of tho isoscalar axial vector piece in the hadronic current. These experiments, if done, would provide the valuable information about these pieces in the hadronic sector which is not available either from the SLAC type of experiments or the atomic physics experiments. Obviously in the standard model only the higher order terms contributes.

We present in figures 3a and 3b the target asymmetry $T(x)$ and $T(y)$ (with $\mathbf{P}_x = 1$) as a function of θ. While the solid curves correspond to the WS model with $F_{AS}=0$, the dashed curve corresponds to $F_{AS} \neq 0$. Unlike in the case of the polarized-electron asymmetry, these two cases are well distinguished over the angular range. In fact we find a considerable enhancement with a nonzero F_{AS}. In figures 4a and 4b

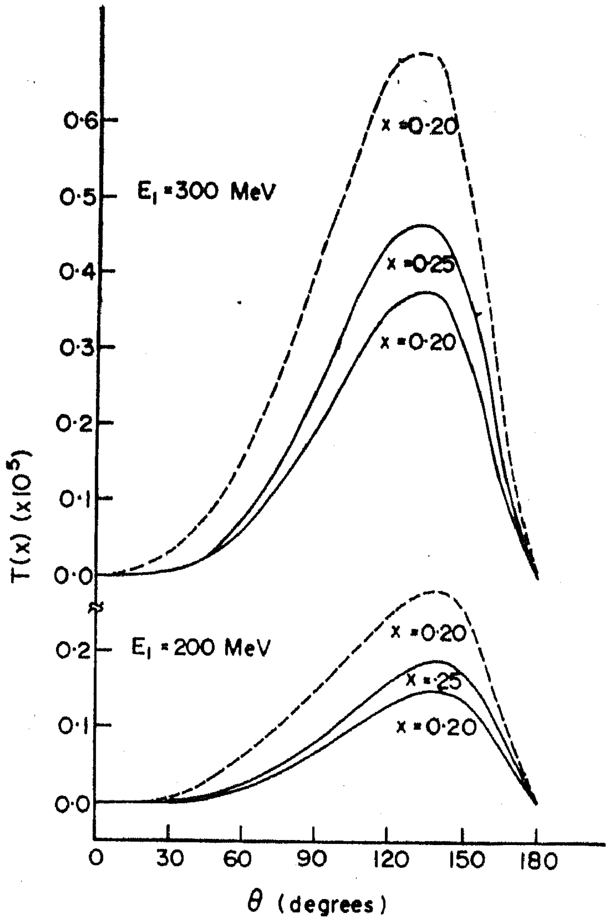

Figure 3a. Polarized target asymmetry $T(x)$ at $E_1 = 200$ MeV, 300 MeV as a function of lab scattering angle θ: $x = \sin^2 \theta_w$: (———) corresponds to WS model: (- - - - -) corresponds to WP model ($F_{AS} = 0{\cdot}186$).

the target asymmetry $T(y)$ is shown as a function of θ. The effect of F_{AS} is more or less the same for both $T(x)$ and $T(y)$. We have, as noted in § 4, confined ourselves to intermediate energies 200 MeV $\leqslant E_1 \leqslant$ 500 MeV where our theory is valid. The numerical estimates have been made using Reid soft core wave functions (Reid 1967) for the deuteron.

In view of the technological advances made in the field of dynamic nuclear polarization (Ninikosky 1976, 1981) it is possible to have vector and tensor-polarized nuclear targets with sizable polarization. Thus it may be possible in the near future to do these experiments and though they are difficult, they would be very interesting indeed.

6. Recoil deuteron vector polarization

The vector polarization of the recoil deuteron in the elastic electron deuteron scattering is zero in one-photon exchange approximation (Ramachandran 1969; Kamal and Moravcsik 1979). There have been attempts to observe this polarization which can be non-zero if certain p-conserving but T violating electromagnetic interactions or a possible phase difference between S and D states of the deuteron are entertained

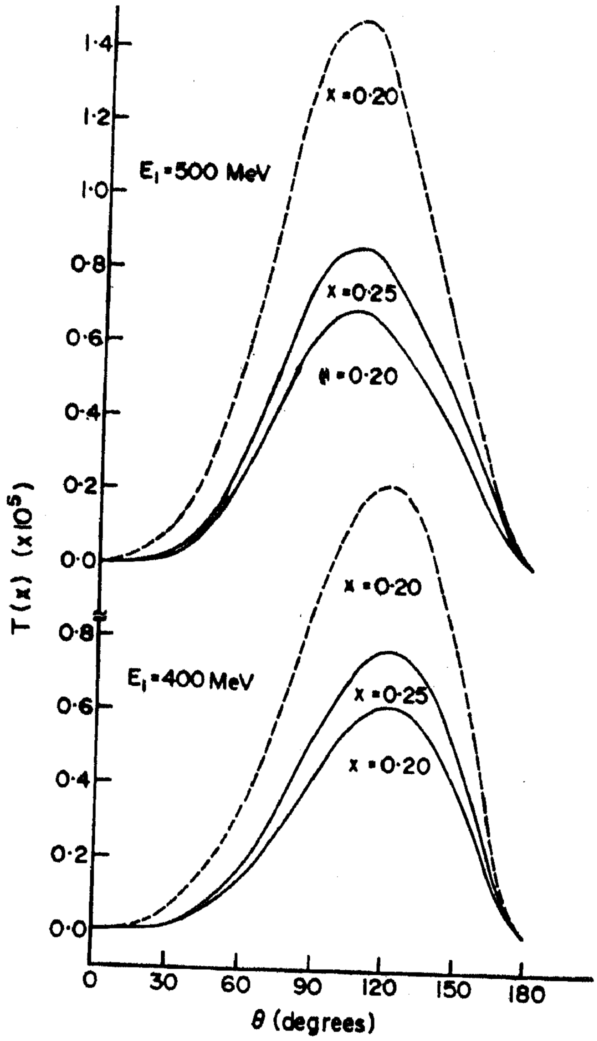

Figure 3b. Same as figure (3a). $E_1 = 400$ MeV, 500 MeV.

(Ramachandran 1969; Prepost *et al* 1965; Dubovik and Cheskov 1967). This can also arise from the interference of one-photon and two-photon exchange diagrams. However these polarizations lie along a direction perpendicular to the reaction plane. Non-zero recoil deuteron polarizations can also be due to the interference between the parity-conserving electromagnetic amplitude and the parity-violating weak neutral current amplitudes. These polarizations lie in the reaction plane and they can be easily distinguished from the polarizations coming from T-violating phases or S- and D state phase difference.

In order to calculate the vector polarization of the recoil deuteron, we note that the spherical tensor parameters t_{kq} characterizing the spin state of the recoil deuteron can be written as (Ramachandran and Umerjee 1964; Ramachandran and Murthy 1978)

$$t_{kq} = \mathrm{Tr}\,(\rho_0^f\, T_{kq}\,(\mathbf{S}))/\mathrm{Tr}\,(\rho_0^f), \tag{66}$$

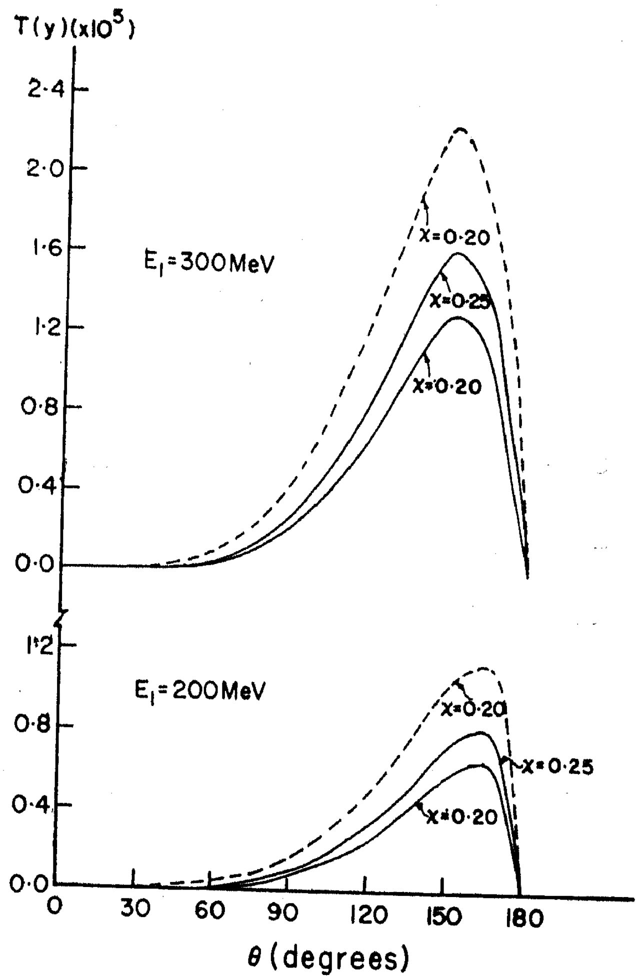

Figure 4a. Polarized target asymmetry $T(y)$ at $E_1 = 200$ MeV, 300 MeV as a function of lab scattering angle θ: $x = \sin^2\theta_w$: (———) corresponds to WS model: (- - - - -) corresponds to WP model ($F_{AS} = 0{\cdot}186$).

where ρ_0^f stands for the final state density matrix with unpolarized deuterons initially. Therefore,

$$\mathrm{Tr}\,(\rho_0^f)\, t_{kq} = \sum_{m_f m_f} C\,(1\, k\, 1;\, m_f\, q\, m_f')\,(2\,k+1)^{1/2}\,(\rho_0^f)_{m_f' m_f}. \tag{67}$$

Using (39) along with (67), we obtain for the vector polarization ($k = 1$) of the recoil deuteron,

$$\begin{aligned}\mathrm{Tr}\,(\rho_0^f)\, t_{1q} &= b_1\, i\,(\mathbf{K}\, L^* - L\,\mathbf{K}^*)^1_q + b_2 \frac{i}{\sqrt{2}}(\mathbf{K}\times\mathbf{K}^*)^1_q \\ &+ b_3\, i\left(\frac{4\pi}{5}\right)^{1/2}(y_2(\hat{q})\otimes(\mathbf{K}\,L^* - L\,\mathbf{K}^*)^1)^1_q \\ &+ b_4 \frac{i}{\sqrt{2}}\left(\frac{4\pi}{5}\right)^{1/2}(y_2(\hat{q})\otimes(\mathbf{K}\times\mathbf{K}^*)^1)^1_q,\end{aligned} \tag{68}$$

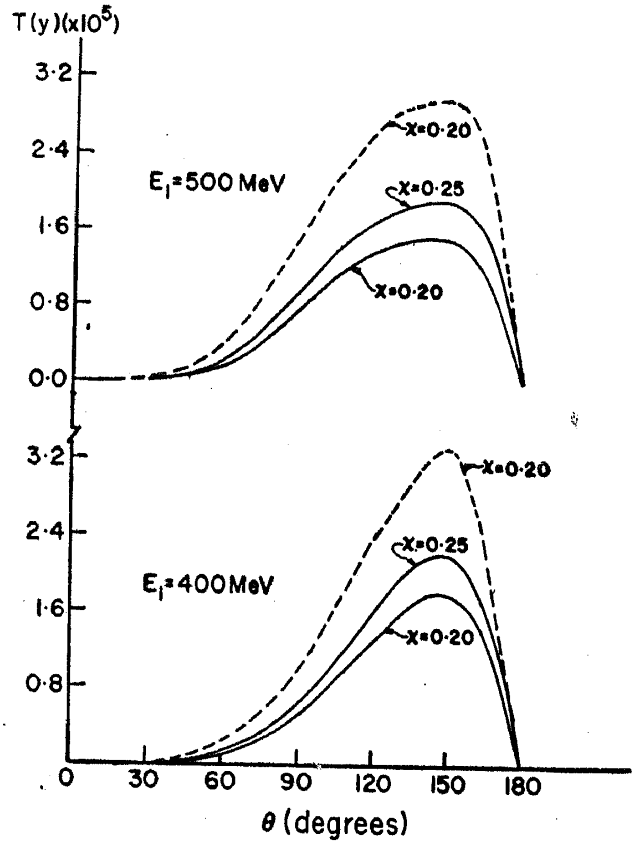

Figure 4b. Same as 4a. $E_1 = 400$ MeV, 500 MeV.

where Tr (ρ_0^f) is given by (61) and the structure functions b_i are defined by (56). Evaluating (68) in Lakin frame, discussed in § 5, we obtain

$$\mathrm{Tr}\,(\rho_0^f)\,\mathbf{t}_1 = (I\,b_1 - \chi\,b_3)\,i\,(\mathbf{K}\,L^* - L\,\mathbf{K}^*) + \frac{1}{\sqrt{2}}(Ib_2 - \chi\,b_4)\,i\,(\mathbf{K}\times\mathbf{K}^*). \tag{69}$$

with the matrix χ being the same as defined by (61) and I is the unit matrix. Explicit expressions for $\mathbf{K}\,.\,\mathbf{K}^*\,\mathbf{L}\,\mathbf{L}^*$, $i\,(\mathbf{K}\,L^* - L\,\mathbf{K}^*)$, $i\,(\mathbf{K}\times\mathbf{K}^*)$ are given in the appendix. It is obvious that in Lakin frame $t_{1z} = 0$ and the recoil deuteron vector polarization lies in the reaction plane. Though the expression for $\mathbf{t}_1$ resembles the expression for $\mathbf{V}$ ((60)) note the important sign difference.

Before giving numerical estimates we would like to make some comments on (69) The second term in (69), *i.e.* $\mathbf{K}\times\mathbf{K}^*$, is proportional to $F_{AS}(q)^2$ which is zero in Weinberg-Salam Model. Thus we find within the framework of the standard model,

$$t_{1x} = V_x, \quad t_{1y} = V_y, \tag{70}$$

where V_x and V_y are defined by (60) and (61). Thus within the framework of ws model all our conclusions about the target asymmetry also apply to the recoil deuteron vector polarization. In fact for arbitrary polarization $\mathbf{P}$ of the target we find

$$T = \mathbf{P} . \mathbf{t}_1. \tag{71}$$

This relation is valid for all energies up to about 1 GeV. On the other hand if we restrict ourselves to only leading order neglecting terms of order $O(1/M)$, the relation (71) is still valid since $\mathbf{K} \times \mathbf{K}^*$ terms is suppressed by a factor of $(1/M)$. Thus even though (71) is an interesting relation it does not give any vital information about the existence of the isoscalar axial vector currents since it is valid irrespective of whether $F_{AS}(q^2)$ is zero or non-zero. However this relation can be useful in planning experiments to detect the recoil deuteron vector polarization. If it turns out that this relation is satisfied at low energies ($E_1 \ll M$) but is violated at higher energies ($E_1 \leqslant M$), it would constitute a proof of the presence of isoscalar axial vector currents.

The numerical estimates for t_{1x} and t_{1y} need not be given separately in Weinberg-Salam model as they are the same as $T(x)$ and $T(y)$ shown in figures (3a, b) and (4a, b) by virtue of the relations (70) and (71). The estimates in WP model are shown in figures 5a and 5b for t_{1x} and t_{1y} and are compared with $T(x)$ and $T(y)$. The difference between recoil deuteron polarization t_{1x} and polarized target asymmetry $T(x)$ is more perceptible at backward angles while the difference between t_{1y} and $T(y)$ is negligible. Though there have been some attempts earlier to measure recoil deuteron vector polarization, we understand that these experiments are in general difficult to perform than the other experiments described in §§ (4) and (5).

7. Summary and conclusions

We have in this paper studied the parity-violating effects in elastic electron deuteron

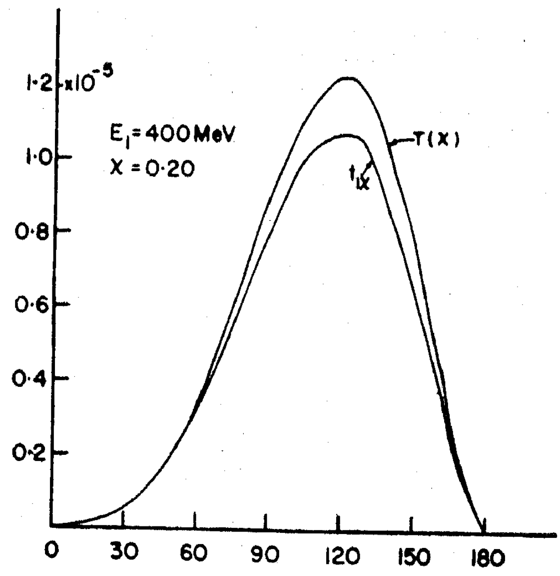

Figure 5a. Polarized target asymmetry $T(x)$ and recoil deuteron vector polarization t_{1x} at $E_1 = 400$ MeV as a function of lab scattering angle θ in WP model.

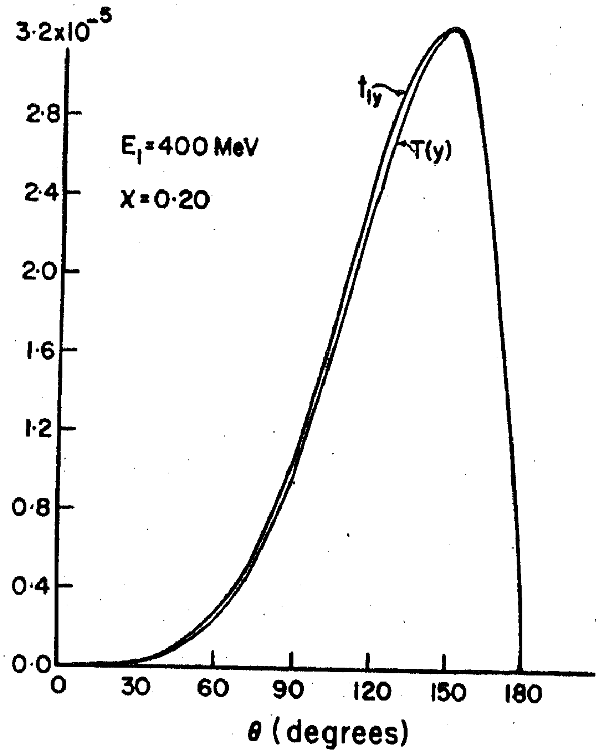

Figure 5b. Polarized target asymmetry $T(y)$ and recoil deuteron vector polarization t_{1y} at $E_1 = 400$ MeV as a function of lab scattering angle θ in WP model.

scattering which arise due to the interference of the parity-conserving electromagnetic amplitudes with the parity-violating weak amplitudes due to the presence of neutral currents.

In the following we summarise our main results:

(i) Polarized electron asymmetry with unpolarized deuterons can be as large as 10^{-4} for incident electron energies around 500 MeV. The asymmetry is the largest at $\theta = \pi$ where it could be observed experimentally. At these energies the theoretical analysis is relatively clean as impulse approximation can be applied with confidence. This is not the case at high energies where one has to worry about the contribution from the quark, antiquark sea in the nucleon. The effect of a non-zero value for $F_{AS}(q^2)$ is felt only at backward angles (particularly at $\theta = \pi$).

(ii) The polarized target asymmetry in the elastic scattering of unpolarized electrons on deuterons is found to increase rapidly with energy as shown in figures (3a, b). Though we have not shown explicitly, the D-state plays a dominant role at backward angles as can be seen from the figures. The estimates in WS model and in the WP model can be quite different. We can expect the target asymmetry to be much larger for a sizable admixture of the isoscalar axial vector piece. Within the standard model the asymmetry is typically between $10^{-5} - 10^{-4}$. At low energies, where we need not worry about higher order terms, the target asymmetry is proportional to $F_{AS}(q^2)$. Therefore, the target asymmetry seems to be the best observable to study the the existence of isoscalar axial vector currents.

(iii) The recoil deuteron vector polarization with completely-unpolarized initial state lies in the reaction plane and therefore can be distinguished from polarizations coming from other exotic mechanisms which give rise to a polarization perpendicular to the reaction plane. Our conclusions about the target asymmetry are also valid for recoil deuteron vector polarization. We however understand that these experiments though difficult are not impossible and some attempts have been made in the past to do them.

We obtain the interesting relation $T=\mathbf{P}\cdot\mathbf{t}_1$, which is generally true in the Weinberg-Salam model at intermediate energies and true in all models if we restrict to lower energies ($E_1 \ll M$). Therefore, if this relation is verified at low energies but found to be violated as the energy is increased, it would indicate definitely the presence of isoscalar axial vector currents.

The effect of Fermi motion, though not shown explicitly, is found to be small. In fact with a pure S-state for the deuteron, the Fermi motion corrections are found to be proportional to terms containing $S_1\, E_1/MS_0$ where

$$S_1 = \frac{4}{q}\int r\mathrm{d}r\; u(r)\, j_1\,(qr/2)\, \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{u(r)}{r}\right),$$

and $$S_0 = 2\int \mathrm{d}r\;\; u^2\,(r)\, j_0\,(qr/2).$$

It is found that S_1 is an order of magnitude smaller than S_0. At 500 MeV incident electron energy, maximum correction from Fermi motion is found to be about 5%. Hence our conclusions are not altered by including Fermi motion effects.

It is straightforward to extend these calculations to determine the effect of neutral currents on the tensor polarizations in the elastic electron-deuteron scattering. We have not discussed this, even though there has been some interest in studying tensor polarization in electron and proton scattering on deuteron (Levinger 1973; Moravscik and Ghosh 1974; Kamal and Moravscik 1979).

The calculations presented in this paper are not complete as they do not take into account the parity-violating effects coming from γNN vertex, which arise due to parity-violating effects in the nucleon-nucleon system (see for example, Fischbach and Tadic 1973; Hwang and Henley 1980). However these effects are shown to be small at intermediate energies while they compete with Z^0 exchange at very low energies. While our calculation shows the dependence on the deuteron structure functions explicity *via* the use of impulse approximation, Hwang and Henley resort to 'elementary-particle approach' to get over the deficiencies in the standard impulse approximation (Holstein 1974; Hwang 1980) related to gauge invariance.

The present calculations are in the spirit of calculations of Sommer *et al* (1979) in the case of inelastic scattering of electrons on deuterons and are meant to provide some guidelines to future experiments in this direction.

Appendix

We give below explicit expressions for various quantities used in the text. Using the expressions for $\mathbf{K}_\pm$ and $L_\pm$ given in (43) and (44) for right-handed and left-handed electrons, we derive,

$$L_\pm L_\pm^* = \frac{e^4}{q^4}\frac{G_{ES}^2(q^2)}{2m^2}\mathscr{K}_1\left[1+\frac{2G}{\sqrt{2}}\frac{q^2}{e^2}\frac{F_{ES}(q^2)}{G_{ES}(q^2)}(a\mp b)\right], \tag{A1}$$

$$\mathbf{K}_\pm \cdot \mathbf{K}_\pm^* = \frac{e^4}{q^4}\cdot\frac{G_{MS}^2(q^2)}{2m^2}\left[\mathscr{K}_2\left\{1+\frac{2G}{\sqrt{2}}\frac{q^2}{e^2}\frac{F_{MS}(q^2)}{G_{MS}(q^2)}(a\mp b)\right\}\right.$$

$$\left.+\frac{2G}{\sqrt{2}}\frac{q^2}{e^2}\cdot\frac{F_{AS}(q^2)}{G_{MS}(q^2)}(b\mp a)\mathscr{K}_3\right], \tag{A2}$$

where
$$\mathscr{K}_1 = 2E_1E_2\cos^2\theta/2\left[1+\frac{E_2-E_1}{2M}\right]^2, \tag{A3}$$

$$\mathscr{K}_2 = \frac{E_1E_2}{M^2}\sin^2\theta/2\,[E_1^2+E_2^2+2E_1E_2\sin^2\theta/2], \tag{A4}$$

$$\mathscr{K}_3 = -\frac{2E_1E_2}{M}(E_1+E_2)\sin^2\theta/2, \tag{A5}$$

For unpolarized electrons, initially,

$$\mathbf{K}\cdot\mathbf{K}^* = \tfrac{1}{2}(\mathbf{K}_+\cdot\mathbf{K}_+^* + \mathbf{K}_-\cdot\mathbf{K}_-^*), \tag{A6}$$

$$LL^* = \tfrac{1}{2}(L_+L_+^* + L_-L_-^*). \tag{A7}$$

Similarly, for unpolarized electrons initially

$$i(\mathbf{K}L^* - L\mathbf{K}^*) = \frac{e^2}{q^2}\cdot\frac{G}{\sqrt{2}}\frac{1}{m^2}\left[1+\frac{E_2-E_1}{2M}\right]$$

$$\times\left[-2b\{G_{MS}(q^2)F_{ES}(q^2)+F_{MS}(q^2)G_{ES}(q^2)\}\right.$$

$$\frac{\mathbf{q}\times(\mathbf{k}_2\times\mathbf{k}_1)}{2M} - 2aF_{AS}(q^2)G_{ES}(q^2)(E_1\mathbf{k}_2+E_2\mathbf{k}_1)$$

$$\left.+2aF_{AS}(q^2)G_{ES}(q^2)\frac{E_1E_2}{M}\cos^2\theta/2\,\mathbf{q}\right], \tag{A8}$$

$$i(\mathbf{K}\times\mathbf{K}^*) = \frac{e^2}{q^2}\cdot\frac{G}{\sqrt{2}}\cdot\frac{1}{m^2}\frac{F_{AS}(q^2)G_{MS}(q^2)}{2M}2a(\mathbf{k}_1+\mathbf{k}_2)\times(\mathbf{k}_1\times\mathbf{k}_2). \tag{A9}$$

Acknowledgements

One of us (MVN) wishes to thank A. Dhar for a careful reading of the manuscript. He also thanks his colleagues at the Dept. of Physics, University of Mysore and at the Centre for Theoretical Studies, Indian Institute of Science where parts of this work were done.

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