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Group velocity of neutrino waves

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Abstract

We follow up on the analysis of Mecozzi and Bellini (arXiv:1110:1253v1) where they showed, in principle, the possibility of superluminal propagation of neutrinos, as indicated by the recent OPERA result. We refine the analysis by introducing wave packets for the superposition of energy eigenstates and discuss the implications of their results with realistic values for the mixing and mass parameters in a full three neutrino mixing scenario. Our analysis shows the possibility of superluminal propagation of neutrino flavour in a very narrow range of neutrino parameter space. Simultaneously this reduces the number of observable events drastically. Therefore, the OPERA result cannot be explained in this frame-work.

We dedicate this paper to the memory of Raju Raghavan who has made fundamental contributions in the area of neutrino physics, and who passed away while we were writing this paper.

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I. INTRODUCTION

The recent announcement of the OPERA result [1] indicating possible superluminal propagation of neutrinos has excited considerable interest. Various aspects of the experiment, the analysis of the data and their interpretation, must be subjected to a thorough examination since the result has important repercussions on fundamental physics. Furthermore, independent confirmation or refutation by other experiments is absolutely essential.

If the result stands, one must first see whether it can be understood within the usual framework of physics before giving up cherished notions such as Lorentz invariance. In fact, this is possible, in principle, as was shown by Mecozzi and Bellini [2]. By considering the interference between the different mass eigenstates of the neutrinos they showed that superluminal propagation is possible. We extend their analysis by explicitly including the effect of the finite width of wave packets and provide a numerical estimate of the effect for realistic neutrino parameters with three generation mixing and matter effects taken into account.

Our numbers indicate that there is a very narrow region in the allowed parameter space with three neutrino flavour mixing in which superluminal propagation is possible in principle. However, the survival probability for neutrinos with superluminal velocities is almost vanishing rendering them unobservable in practice. Furthermore, this depends crucially on the ratio of the distance of propagation and the energy of the neutrinos. At distances and energies corresponding to the OPERA experiment, superluminal propagation is not possible with the present limits imposed by the neutrino parameter space. If the OPERA result is confirmed it would require new physics. Therefore, we would like to stress the importance of further studies along the present lines since the OPERA experiment has opened up a new window on neutrino physics, which may be called neutrino optics and which should be pursued by future experiments.

In Sec. II we outline the group velocity calculation in the wave packet formalism. In the context of neutrino oscillation, this formalism has been discussed in detail in Ref. [3]. We derive results for short and very long base-line propagation of neutrinos, based on which we present a realistic numerical analysis in the framework of 3 neutrino mixing in Sec. III. Other issues related to group velocity measurements will be discussed in Sec. IV while Sec. V concludes the paper with some remarks on the implications of OPERA-type experiments in future.

II. CALCULATION OF THE GROUP VELOCITY

The neutrino flavour states $|\nu_\alpha\rangle,\ \alpha = e, \mu, \tau$ are related to mass eigenstates $|\nu_i\rangle,\ i = 1, 2, 3$ by

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle, \tag{1}$$

where U is a unitary matrix. For a neutrino that starts as a flavour state α at $t = 0$, the state vector at time t is

$$|\psi(t)\rangle = N \int \ dp \ \sum_i U_{\alpha i} \ e^{ipx - iE_i t} \ e^{-(p-p_0)^2/a^2} |\nu_i\rangle, \tag{2}$$

where we have superposed the three energy eigenstates with the same momentum p and then superposed different p with an amplitude of gaussian form $e^{-(p-p_0)^2/a^2}$ to form a wave packet. We have set $\hbar = c = 1$. The normalisation constant is determined by the condition $\langle\psi(t)|\psi(t)\rangle = 1$.

The probability amplitude for detecting $|\nu_\beta\rangle$ at time t is

$$A_{\beta\alpha} = \langle\nu_\beta|\psi(t)\rangle = N\int dp \sum_i U_{\alpha i}U^*_{\beta i}\; e^{ipx-iE_it}\; e^{-(p-p_0)^2/a^2} \tag{3}$$

$$= N\int dp \sum_i C_i^{\beta\alpha}\; e^{ipx-iE_it}\; e^{-(p-p_0)^2/a^2}, \tag{4}$$

where we have used

$$\langle\nu_\beta|\nu_i\rangle = U^*_{\beta i}; \quad C_i^{\beta\alpha} = U^*_{\beta i}U_{\alpha i}\ .$$

We expand the energy E around the peak p_0 of the gaussian and keep upto the quadratic terms (assuming the width of the wave packet in momentum space to be small enough):

$$E \approx E_0 + \left.\frac{dE}{dp}\right|_{p_0}(p-p_0) + \frac{1}{2}\left.\frac{d^2E}{dp^2}\right|_{p_0}(p-p_0)^2 + \cdots\ . \tag{5}$$

With this approximation, the p-integration in eq. (4) can be done to yield the result (absorbing the constant factors into N):

$$A_{\beta\alpha} = N\sum_i C_i^{\beta\alpha}\; e^{ip_0x-i\widetilde{E}_{0i}t}\; e^{-(x-x_0^i(t))^2a^2/4}; \quad \widetilde{E}_{0i} = E_{0i} + \frac{a^2}{4}\left.\frac{d^2E_i}{dp^2}\right|_{p_0}, \tag{6}$$

where $x_0^i(t) = dE_i/dp\,|_{p_0}\,t$. Thus we have a superposition of 3 gaussians in x-space with their centres travelling with 3 separate group velocities,

$$v^i \equiv \frac{dx_0^i(t)}{dt} = \left.\frac{dE_i}{dp}\right|_{p_0}\ . \tag{7}$$

However, if the x-space gaussians are broad, that is, if a is small, then these 3 gaussians will interfere. To study this, let us define

$$\langle x\rangle_{\beta\alpha} = \frac{\int\ x\ |A_{\beta\alpha}|^2dx}{\int |A_{\beta\alpha}|^2dx}\ . \tag{8}$$

The integrations are straightforward and the result is

$$\langle x\rangle_{\beta\alpha} = \frac{\sum_i |C_i^{\beta\alpha}|^2v_it + \sum_{i>j} Re(C_i^{\beta\alpha}C_j^{\beta\alpha *}(v_i+v_j)t\ e^{-i\Delta E_{ij}t})\ e^{-(v_i-v_j)^2t^2a^2/8}}{\sum_i |C_i^{\beta\alpha}|^2 + 2\ \sum_{i>j} Re(C_i^{\beta\alpha}C_j^{\beta\alpha *}\ e^{-i\Delta E_{ij}t})\ e^{-(v_i-v_j)^2t^2a^2/8}}, \tag{9}$$

where

$$\Delta E_{ij} = E_{0i} - E_{0j} + \frac{a^2}{4}\left.\frac{d^2(E_i-E_j)}{dp^2}\right|_{p_0}\ .$$

Defining the overall-group velocity of neutrinos generated as ν_α at time $t = 0$ and detected as ν_β at time t as $v_{\beta\alpha}$ as was done by Mecozzi and Bellini [2], we get

$$v_{\beta\alpha} = \frac{\langle x \rangle_{\beta\alpha}}{t} = \frac{\sum_i |C_i^{\beta\alpha}|^2 v_i + \sum_{i>j} Re(C_i^{\beta\alpha} C_j^{\beta\alpha *} (v_i + v_j)\ e^{-i\Delta E_{ij} t})\ e^{-(v_i - v_j)^2 t^2 a^2/8}}{\sum_i |C_i^{\beta\alpha}|^2 + 2\ \sum_{i>j} Re(C_i^{\beta\alpha} C_j^{\beta\alpha *}\ e^{-i\Delta E_{ij} t})\ e^{-(v_i - v_j)^2 t^2 a^2/8}} \ . \quad (10)$$

This is the main result of the paper.

We now consider the special case of two generation mixing with μ, τ as the two neutrino flavours. Substituting $\alpha = \beta = \mu$, and denoting $C_1^{\mu\mu} = \cos^2\theta, C_2^{\mu\mu} = \sin^2\theta$, we have

$$v_{\mu\mu} = \frac{v_1 \cos^4\theta + v_2 \sin^4\theta + (v_1 + v_2) \sin^2\theta \cos^2\theta\ \cos(\Delta E_{12} t)\ e^{-(v_1 - v_2)^2 t^2 a^2/8}}{\cos^4\theta + \sin^4\theta + 2 \sin^2\theta \cos^2\theta\ \cos(\Delta E_{12} t)\ e^{-(v_1 - v_2)^2 t^2 a^2/8}} \ . \quad (11)$$

Let us consider the two extreme limits from the above equation. First consider the case when the width a^{-1} of the wave packets in the x-space is large compared to the distance of separation between the two centres of the wave packets, that is $a(v_1 - v_2)t \ll 1$. In this limit eq. (11) becomes

$$\begin{aligned} v_{\mu\mu} &= \frac{v_1 \cos^4\theta + v_2 \sin^4\theta + (v_1 + v_2) \sin^2\theta \cos^2\theta\ \cos(\Delta E_{12} t)}{\cos^4\theta + \sin^4\theta + 2 \sin^2\theta \cos^2\theta\ \cos(\Delta E_{12} t)} \\ &= \frac{1}{2} \left[(v_1 + v_2) + (v_1 - v_2) \frac{\cos 2\theta}{1 - \sin^2 2\theta\ \sin^2(\Delta E_{12} t/2)} \right] \ . \end{aligned} \quad (12)$$

This agrees with the result of Mecozzi and Bellini[2].

However as the distance or time of propagation increases, the width of the wave packets in x-space becomes small compared to the distance of separation between the two centres of the wave packets, $a(v_1 - v_2)t \gg 1$, and we get

$$v_{\mu\mu} = \frac{v_1 \cos^4\theta + v_2 \sin^4\theta}{\cos^4\theta + \sin^4\theta} = \frac{1}{2} \left[(v_1 + v_2) + (v_1 - v_2) \frac{\cos 2\theta}{1 - \frac{1}{2} \sin^2 2\theta} \right] \ , \quad (13)$$

which is the weighted average of the group velocities of the two wave packets. Interestingly, the effect of the gaussian suppression is precisely the same as taking the average over energy and distance in the denominator of eq. (12). This makes sense since, for instance, at astrophysical distances, the neutrino wave-length is small compared to the distance of propagation.

Thus the result in eq. (10) generalises the result of Mecozzi and Bellini to wave packets of finite width. Because of the approximation made in eq. (6), for large a the damping factor $\exp(-(v_1 - v_2)^2 t^2 a^2/8)$ is only approximate, although its exact replacement also will damp the oscillatory factor $\cos(\Delta E_{12} t)$.

If the calculations are done for waves of infinite spatial extent, the integrals occurring in the numerator and denominator of eq. (8) would be individually divergent, although the final result would turn out to be finite. However, it is better to do these calculations with wave packets of finite width a, as has been done above, and take the limit of $a \to 0$ in the end.

We have superposed the 3 mass eigenstates with the same momentum p but different E_i to form the neutrino state vector in eq. (2). Should one superpose different p_i but the same

E, or, different p_i and different E_i? This question has been studied in recent literature [4]. Such possibilities will be included in a future study of the group velocities of neutrino waves, which is under preparation.

For neutrinos whose energy is very large compared to their rest masses, the formula in eq. (11) may be written in the form

$$v_{\mu\mu} = \frac{v_1 \cos^4\theta + v_2 \sin^4\theta + (v_1 + v_2)\sin^2\theta\cos^2\theta \ \cos y \ e^{-(\delta p/p)^2 y^2/4}}{\cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta \ \cos y \ e^{-(\delta p/p)^2 y^2/4}} \ , \tag{14}$$

where $y = \Delta m^2 L/(2E)$, Δm^2 and L being the measured mass-squared difference and base-line distance and $\delta p = a/\sqrt{2}$ the width of the p-space wave packet. Thus the exponential damping factor multiplying the oscillatory factor $\cos y$ is simply $e^{-(\delta p/p)^2 y^2/4}$ where $\delta p/p$ is the fractional uncertainty in momentum.

We also note that the formula in eq. (11) and eq. (10) for two- and three- generations are valid, to the leading order, for propagation through matter at constant density when vacuum values of E_i and the mixing angles are replaced by their matter-dependent values.

From eq. (6), the normalised oscillation probability, that is, the probability for detecting flavour β at time t is

$$P_{\beta\alpha} = \int |A_{\beta\alpha}|^2 dx = \sum_i |C_i^{\beta\alpha}|^2 + \sum_{i>j} 2\mathrm{Re}\ (C_i^{\beta\alpha} C_j^{\beta\alpha *} e^{-i\Delta E_{ij} t}) e^{-(\delta p/p)^2 y^2/4} \ . \tag{15}$$

This differs from the usual oscillation formula by the factor $\exp(-(\delta p/p)^2 y^2/4)$ in the second term. In view of the successful neutrino oscillation phenomenology achived so far, we will assume that $a = \sqrt{2}\delta p$ is so small that this factor can be replaced by unity for all the terrestrial experiments as well as solar neutrino experiments.

Since the oscillation probability given in eq. (15) is the denominator in eq. (10), the group velocity can become very large if the oscillation probability is very small. In fact, it can become infinite if the oscillation probability at that distance is zero. This is the origin of superluminal propagation, as our analysis in the next section will clearly show.

We now come back to the interpretation of $\langle x\rangle_{\beta\alpha}$ defined in eq.(8) which is the basis of the above formulae. Note that in the denominator we have $\int dx|A_{\beta\alpha}|^2$ instead of $\sum_\beta \int dx|A_{\beta\alpha}|^2$ which is unity. Thus $\langle x\rangle_{\beta\alpha}$ must be interpreted as the *conditional measurement* of the position of the neutrino under the condition that only ν_α is detected. Here the probability amplitude for detecting it as ν_α itself is regarded as the wave function for normalising the expectation value of x. This is to be contrasted with the usual definition of the expectation value of x, independent of the flavour detected, namely

$$\langle x\rangle_\alpha = \frac{\sum_\beta \int \ x \ |A_{\beta\alpha}|^2 dx}{\sum_\beta \int |A_{\beta\alpha}|^2 dx} = \sum_\beta \int \ x \ |A_{\beta\alpha}|^2 dx \ . \tag{16}$$

We may distinguish this case by calling it the *unconditional measurement* of the expectation value of x.

Before we go to the numerical analysis, we make some general remarks. As already pointed out, the origin of the superluminal propagation is the vanishing of the oscillation probability $P_{\beta\alpha}$ in the denominator of eq. (10). In other words, superluminal propagation and the vanishing of the oscillation probability go together. Since the number of events also vanishes, one has the paradoxical situation of *unobservable superluminal propagation*. Our numerical analysis below is subject to this criticism.

In the particular case of the OPERA experiment, $P_{\beta\alpha}$ becomes the survival probability $P_{\mu\mu}$. There is no evidence for the survival probability $P_{\mu\mu}$ in OPERA becoming vanishingly small. Hence explanation of superluminality through enhancement of the group velocity in the standard oscillation frame-work as discussed above is untenable.

Our numerical analysis will be based on the group velocity derived from the conditional measurement of $\langle x \rangle_{\beta\alpha}$ defined in eq. (8), following Mecozzi and Bellini [2]. Alternatively, one could base the analysis on the group velocity derived from the unconditional measurement $\langle x \rangle_\alpha$ defined in eq. (16). Since this does not have the vanishing denominator, it will not lead to superluminal group velocity.

Actually, for the OPERA experiement, both $\langle x \rangle_{\beta\alpha}$ and $\langle x \rangle_\alpha$ will give essentially the same result for the group velocity since $P_{\mu\mu}$ at OPERA does not deviate very much from unity, as far as is known.

III. NUMERICAL ANALYSIS WITH THREE GENERATIONS

To obtain a realistic estimate for the group velocity of muon neutrinos, and its implications, we discuss the three generation scenario.

As shown in the previous section, the gaussian smearing will have no effect if $(\delta p/p)^2 \ll 1$. We will work under this assumption and make estimates for this quantity later, showing that it is indeed small. As a first approximation we neglect the matter effects.

Following eq. (10), we may write the "group velocity" of the superposition that starts as ν_μ and is detected as ν_μ (which we denote as v_μ for simplicity) as

$$\begin{aligned} v_\mu &= v_2 + S^\mu_{12}(v_1 - v_2) + S^\mu_{32}(v_3 - v_2), \\ &\equiv v_2 + \Delta v_\mu \ . \end{aligned} \tag{17}$$

The factors S_{ij} are given as

$$S^\mu_{12} = \frac{1}{P_{\mu\mu}} |U_{\mu 1}|^2[1 - 2|U_{\mu 2}|^2 \sin^2(\Delta E_{21}t/2) - 2|U_{\mu 3}|^2 \sin^2(\Delta E_{31}t/2)] \ ; \tag{18}$$

$$S^\mu_{32} = \frac{1}{P_{\mu\mu}} |U_{\mu 3}|^2[1 - 2|U_{\mu 1}|^2 \sin^2(\Delta E_{31}t/2) - 2|U_{\mu 2}|^2 \sin^2(\Delta E_{32}t/2)] \ , \tag{19}$$

where the denominator, which is simply the survival probability of ν_μ, is given by,

$$P_{\mu\mu} = 1 - 4 \sum_{j>i=1}^{3} |U_{\mu i}|^2 |U_{\mu j}|^2 \sin^2(\Delta E_{ij}t/2) \ . \tag{20}$$

Here

$$\frac{\Delta E_{ij}t}{2} = \frac{(E_i - E_j)t}{2} \approx 1.27 \frac{\Delta_{ij}(\mathrm{eV}^2)L(\mathrm{km})}{E(\mathrm{GeV})} \ , \tag{21}$$

where $\Delta_{ij} = m_i^2 - m_j^2$ and the mixing parameters are given in the basis where the charged lepton mass matrix is diagonal as

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}\ e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}\ e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}\ e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}\ e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}\ e^{i\delta} & c_{23}c_{13} \end{bmatrix} , \tag{22}$$

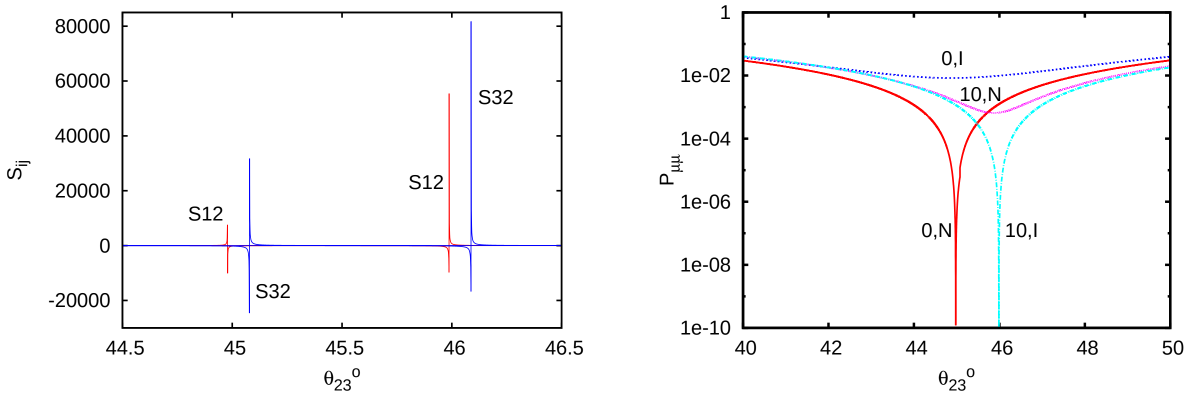

FIG. 1: (L) The terms S_{12} and S_{32} as a function of θ_{23}. The values on the left are for the normal hierarchy solution with $\theta_{13} = 0$ while those on the right are for the inverted hierarchy with $\theta_{13} = 10^\circ$. In both sets, the curves for S_{32} have been offset by $\theta_{23} = 0.1^\circ$ for clarity, else the two curves for S_{12} and S_{32} would overlap each other. In the former case, $\delta_{CP} = 0$ and $L/E = 611.165$ km/GeV while for the latter, $\delta_{CP} = 180^\circ$ and $L/E = 604.629$ km/GeV. (R) The denominator $P_{\mu\mu}$ plotted as a function of θ_{23} for the same parameters. The dips as $P_{\mu\mu} \to 0$ correspond to the sharp peaks in S_{ij} on the left.

where $c_{ij} = \cos\theta_{ij},\ s_{ij} = \cos\theta_{ij}$ and δ is the CP phase. The coefficients $U_{\mu i}$ correspond to the second row of the mixing matrix. Note that in the absence of any mixing v_μ in eq. (17) reduces to v_2, as it should.

We now present a numerical analysis of the possible values of v_μ with realistic parameter values for the mixing angles and mass squared differences. We use typical values/ranges for the mass-squared differences and neutrino mixing parameters: $\Delta_{21} = 7.6 \times 10^{-5}$ eV^2, $|\Delta_{32}| = 2 \times 10^{-3}$ eV^2, $\theta_{12} = 34^\circ$, $36^\circ \leq \theta_{23} \leq 54^\circ$, $\theta_{13} \leq 10^\circ$ while δ_{CP} is unknown [5]. The sign of Δ_{32} is not known and there are two possible hierarchies, $m_3^2 > m_2^2$ known as normal hierarchy (N) and $m_3^2 < m_2^2$ known as inverted hierarchy (I). We consider results for both these cases.

Note that the propagation can become superluminal when the S_{ij} are significantly enhanced over unity. Fig. 1 shows the values of θ_{23}, close to maximal mixing, where this enhancement is seen, for different mass hierarchies as well as values of θ_{13}. (Notice therefore that this measurement is sensitive to the mass ordering in the 2–3 sector which is as yet unknown).

This enhancement of the group velocity occurs for L/E (km/GeV) ~ 600, well within the range of the OPERA experiment, viz., $7 < L/E($ km/GeV$) < 730$.

However, it can be seen from the right hand panel of Fig. 1 that $P_{\mu\mu}$ is very close to zero precisely at these values, that is, the enhancement of S_{ij} is dominantly due to the vanishing of the denominator. (The maxima of S_{ij} are slightly offset from the minima of $P_{\mu\mu}$ because of the dependence of the numerator on the mixing parameters as well. In fact, when $\theta_{13} = 0$, $P_{\mu\mu}$ and S_{ij} both vanish near $\theta_{23} = 45^\circ$.) This in turn means that there will be hardly any events that survive at these values, for this conditional measurement!

Several remarks are in order:

- The first is that the mixing parameters as well as L/E need to be extremely fine-tuned in order to get this effect. However they are almost washed out by the suppression of the event rate due to small survival probability.

- The extreme enhancement of S_{ij} over unity is required in order to obtain superluminal velocities commensurate with the OPERA observation. This is because the coefficients of S_{ij} which are $(v_i - v_j)$ in eq. (17) behave as $\Delta_{ji}/(2p^2)$ and are small due to the highly suppressing factor of $2p^2$ in the denominator. It is unlikely that such small excesses will be measurable by any experiment. The larger the enhancement required, the more finely tuned are the parameters.

- Moreover, the fine-tuning in the value of L/E would imply that the enhancement of the velocity only occurs for a single E value when the base-line distance L is kept fixed, in contrast to the observed roughly constant enhancement over a range of $1 \lesssim E \lesssim 100$ GeV as observed in OPERA [1].

  One way of working around this limitation is to consider the energy dependence of the matter-dependent contributions. The usual electro-weak interactions in matter lead to the inclusion of a matter-dependent potential, V_{EW}, that alters the matter dependent mass-squared differences in a non-trivial way; however, the resulting change is not large enough to give the required enhancement of the factor of 14 in the term $\Delta^m_{32}L/(4E)$ at $\langle E \rangle = 17$ GeV.

  One possible solution is the inclusion of a new matter interaction V_{new} with the same energy behaviour, but about an order of magnitude larger than the usual electro-weak potential. Then, at energies around $\langle E \rangle = 17$ GeV relevant to the OPERA data, the expression for the matter-dependent mass-squared differences simplifies to

$$\Delta^m_{ij} \approx \Delta_{ij} + 2EV_{new} \,, \tag{23}$$

  so that the term occurring in Δv_μ becomes

$$\frac{\Delta^m_{ij} L}{4E} \approx \frac{\Delta_{ij} L}{4E} + \frac{V_{new} L}{2} \,. \tag{24}$$

  The energy dependence of this matter potential is exactly what is required so that the ratio Δ^m_{ij}/E is approximately independent of E for $E \sim$ 10s of GeV (but does not significantly alter the atmospheric neutrino analysis). Such an energy-independent contribution would remove the fine-tuning in L/E that currently occurs in the expressions for Δv_μ and would in principle uniformly allow for superluminal propagation of all velocities relevant to the OPERA analysis. However, while resolving the fine-tuning in L/E, the new matter potential enhances the relevant terms in a manner identical to the vacuum analysis, viz., through the vanishing of $P_{\mu\mu}$. Hence, it runs into the same difficulties with observing the effect as discussed earlier.

- In spite of what is said above, superluminal group velocity is a real effect and may be observable in future, as shown in the following example. Consider the 2-flavour case where we neglect m_2 and set $m_3 = \Delta_{32} \sim 2 \times 10^{-3}$ eV^2. For smaller energies of the order of KeVs, observable excesses of v_μ over unity can be obtained for a modest value of $P_{\mu\mu} \sim 0.03$. This corresponds to $\sin^2 2\theta_{23} = 0.97$, with the ratio L/E tuned so that $\sin^2(\Delta_{32}L/(4E)) = 1$ (achievable with energies in KeV and length in meters);

this leads to $v_\mu = 1 - \Delta_{32}/(2p^2) \times (0.5 - \cos 2\theta_{23}/P_{\mu\mu}) \sim 1 + 6\,\Delta_{32}/(2p^2)$. For energy E in KeV we get an enhancement $v_\mu - 1 = 6\,10^{-9}/E^2$; it may thus be possible to observe the effect since $P_{\mu\mu}$ is not zero. Of course, there are many experimental problems to be overcome before such an observation is achieved.

IV. OTHER IMPLICATIONS OF GROUP VELOCITY MEASUREMENT

We add a few remarks on the implication of the OPERA result independent of the fact that it is superluminal or subluminal. Consider a possible measurement of group velocities of electron and muon neutrinos in a future possible experiment. Following eq. (17), these velocities in vacuum may be written in the form

$$\begin{aligned} v_e &= v_1 + S^e_{21}(v_2 - v_1) + S^e_{31}(v_3 - v_1) \ ; \\ v_\mu &= v_2 + S^\mu_{12}(v_1 - v_2) + S^\mu_{32}(v_3 - v_2) \ , \end{aligned} \tag{25}$$

where S^μ_{ij} are given in eqs. (18) and (19) and S^e_{ij} are given by

$$S^e_{21} = \frac{|U_{e1}|^2[1 - 2|U_{e2}|^2 \sin^2(\Delta E_{12}t/2) - 2|U_{e3}|^2 \sin^2(\Delta E_{13}t/2)]}{1 - 4\sum_{j>i=1}^{3} |U_{ei}|^2|U_{ej}|^2 \sin^2(\Delta E_{ij}t/2)} \ , \tag{26}$$

and

$$S^e_{31} = \frac{|U_{e3}|^2[1 - 2|U_{e1}|^2 \sin^2(\Delta E_{13}t/2) - 2|U_{e2}|^2 \sin^2(\Delta E_{23}t/2)]}{1 - 4\sum_{j>i=1}^{3} |U_{ei}|^2|U_{ej}|^2 \sin^2(\Delta E_{ij}t/2)} \ . \tag{27}$$

Simultaneous measurement of these two velocities immediately gives information on the ordering of masses m_2 and m_3 since we already know that $m_2 > m_1$ from the solution to the solar neutrino problem. Unlike earlier proposed solutions [8], this does not require matter effects to resolve the issue. Even a short base-line experiment with neutrino factories with muon storage rings may help resolve the hierarchy issue. Note that the v_e and v_μ could refer to either neutrinos or antineutrinos. The caveat is that the effect is magnified only for a given set of parameter values including L/E.

V. CONCLUSIONS AND SOME REMARKS

To summarise, we have considered the superposition of 3 energy (mass) eigenstates with the same momentum p, but with different energies E_i to form a neutrino flavour state. We have included the effect of the finite width of wave packets of the same width and considered its effect on the group velocity of the neutrino flavour state.

When this is applied to the propagation of ν_μ for distances of the order of hundreds of kms and energies corresponding to the observation of muon neutrinos in the OPERA experiment we find the effect is very small. Effectively the finite width can be ignored as is done in Ref. [2].

However the effect of width increases with distance. For astronomical distances the effect of finite width is to reduce the group velocity to a weighted average of the velocities of the individual energy eigenstates. This explains why, even if the OPERA result is correct, there is no contradiction with the absence of superluminal propagation of neutrinos from supernova SN1987a.

Our analysis shows that superluminal propagation of neutrinos occurs whenever the oscillation probability corresponding to that measurement vanishes. For instance, muon neutrinos are dominantly observed at OPERA; these will exhibit superluminal behaviour precisely when the survival probability $P_{\mu\mu}$ vanishes. While $P_{\mu\mu}$ does not vanish for the parameter range of the OPERA measurement, in principle, this feature makes superluminal neutrinos effectively unobservable through conditional measurement as explained in section 2.

Finally we discuss the implication of the OPERA result irrespective of superluminality or otherwise. The neutrino mass ordering or hierarchy is not fully known. The present understanding is that this requires separate measurement of oscillations of neutrinos and anti-neutrinos in the presence of matter. The OPERA measurement of neutrino flavour "velocity" has added a new way of precision measurement of neutrino parameters and may have significance in the context of neutrino mass hierarchy (provided the parameters lie in an extremely narrow part of the known range of neutrino parameters). A detailed analysis including the effect of matter will be presented elsewhere.

After most of the work reported here was completed, we came across Ref.[9] which overlaps with parts of the present paper. We thank Tim R. Morris for bringing to our attention his paper Ref. [10] in which similar ideas have been discussed. In particular, the extreme fine-tuning of parameters required to produce velocities of the order observed in OPERA and the difficulty that the effect is seen only at fixed energy was pointed out in this paper. This paper points out the existence of multiple peaks, that would give rise to a strong energy dependence, in addition.

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