Quantum electrodynamics in the presence of dielectrics and conductors. V. The extinction and moment theorems for correlation functions and relativistic aspects of blackbody fluctuations

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Dynamical laws describing the space-time development of the response functions and second-order correlation functions of the electromagnetic fields in a linear dielectric medium are presented in the form of a number of differential and integral equations. The boundary conditions on the response functions are expressed as extinction theorems, which are particularly useful for systems involving rough surfaces and metallic gratings. Higher-order correlation functions are shown to be related to the linear-response functions in a manner analogous to the moment theorem for Gaussian random processes. A modified form of the fluctuation-dissipation theorem is obtained and is used to calculate free-space blackbody fluctuations in a moving frame. Fluctuations in a moving dielectric are obtained from the transformation of fields under Lorentz transformation. Contact with the earlier works of Mehta and Wolf, Eberly and Kujawski, and the recent work of Baltes and co-workers on blackbody radiation is made wherever possible. Finally, the correlation functions in a moving frame are used to discuss the relaxation of a moving atom. The discussion in the last section is for the case of free space.

I. INTRODUCTION

In part I of this series of papers,[1] we showed how the linear-response theory can be used with ease to compute various types of electromagnetic-field correlation functions in the presence of dielectric bodies. In subsequent papers,[1] we used such correlation functions to discuss a number of surface-dependent effects. However, some fundamental aspects of the present theory, such as equations of motion, boundary conditions etc., were not elaborated upon in detail. In this paper, we consider such questions, discuss the relation of the higher-order correlation functions (which are needed, e.g., in the treatment of the decay of metastable states[2]) to response functions, and present a generalized fluctuation-dissipation theorem appropriate to relativistic ensembles.[3,4]

In Sec. II, we obtain a number of differential and integral equations satisfied by the response functions $\chi_{ijEE}, \chi_{ijEH}, \chi_{ijHE}, \chi_{ijHH}$. Both first-order and second-order differential equations are given. These equations in turn lead to a set of equations for the correlation functions $\mathcal{E}_{ij}, \mathcal{G}_{ij}, \tilde{\mathcal{G}}_{ij}, \mathcal{H}_{ij}$. The response functions satisfy much simpler equations than correlation functions. However, for the free-space case, the equations for correlation functions are simple.

In Sec. III, the boundary conditions are presented in the form of an extinction theorem.[5-9] A simple example is given to illustrate the use of this theorem. In Sec. IV we prove that the thermal fluctuations in a linear dielectric (occupying arbitrary domain) are Gaussian. This, in general, is not true for a nonlinear medium. In Sec. V, we obtain a modified form of the fluctuation-dissipation theorem that is applicable to relativistic ensembles. A number of the symmetry properties of the correlation functions in a moving frame are discussed. The correlation functions in a moving frame are used to discuss the relaxation of a moving atom in Sec. VI.

In part VI of this series we will present the theory of Lippmann fringes.[10] In part VII, we will deal with the general problem of scattering from rough surfaces as well as scattering from the dielectric inhomogeneities.[11] Use will be made of the integral equations of Sec. II as well as the extinction theorems of Sec. III. In particular, we will dwell upon Smith-Purcell radiation, Wood anomalies, and surface polariton emission.[12] In part VIII, we use the moment theorem of Sec. IV and the response functions of paper I to examine the decay of metastable states in presence of dielectric interface.

II. EQUATIONS OF MOTION FOR THE RESPONSE FUNCTIONS AND THE CORRELATION FUNCTIONS

In the usual optical coherence theory, great significance has been attached to the dynamical equations[13] satisfied by the correlation functions $\mathcal{E}_{ij}, \mathcal{H}_{ij}, \mathcal{G}_{ij}, \tilde{\mathcal{G}}_{ij}$. Such equations have been used to discuss the propagation of optical coherence. We will be using the notation of this series of papers. The correlation functions $\mathcal{E}_{ij}$, etc., are defined by Eqs. (I 2.16)–(I 2.19). We have discussed in paper I the relation of the correlation functions to different response functions χ_{ijEE},

χ_{ijEH}, etc. [cf. relations (I 2.20)–(I 2.23)]. In this section, we obtain the general equations satisfied by the response functions and correlation functions. We will present both differential and integral equations.

We first consider the differential equations satisfied by the response functions. The response functions are to be obtained from the solution of Maxwell's equations

$$\begin{aligned}&\nabla\times\vec{E}=ik_0(\vec{B}+4\pi\vec{\mathfrak{M}}),\\&\nabla\cdot(\vec{B}+4\pi\vec{\mathfrak{M}})=0,\\&\nabla\times\vec{H}=-ik_0(\vec{D}+4\pi\vec{\mathcal{P}}),\\&\nabla\cdot(\vec{D}+4\pi\vec{\mathcal{P}})=0,\quad k_0=\omega/c.\end{aligned}\tag{2.1}$$

Here $\mathfrak{M}$ and $\vec{\mathcal{P}}$ denote, respectively, the external magnetization and polarization, $\vec{D}$ and $\vec{B}$ are the usual electric and magnetic induction. From Eqs. (2.1), one can easily obtain the equations satisfied by response functions. Upon taking the functional derivative of (2.1) with respect to $\vec{\mathcal{P}}$, $\vec{\mathfrak{M}}$, and restricting our attention to the case of a linear dielectric characterized by local dielectric and permeability functions

$$\begin{aligned}&D_i(\vec{r},\omega)=\epsilon_{ij}(\omega)E_j(\vec{r},\omega),\\&B_i(\vec{r},\omega)=\mu_{ij}(\omega)H_j(\vec{r},\omega),\end{aligned}\tag{2.2}$$

we obtain the equations

$$\epsilon_{ijk}\partial_j^{(1)}\tilde{\chi}_{kp}=C_{il}^{(1)}ik_0\tilde{\chi}_{lp}+4\pi ik_0\delta_{ip}\delta(\vec{r}_1-\vec{r}_2)I^{(1)},\tag{2.3}$$

$$\partial_i^{(1)}C_{ij}^{(2)}\tilde{\chi}_{jp}=-4\pi\partial_p^{(1)}\delta(\vec{r}_1-\vec{r}_2)I^{(2)},\tag{2.4}$$

where $\partial_j^{(1)}$ denotes the differentiation $\partial/\partial r_{1j}$, and r_{1j} is the jth component of the vector $\vec{r}_1$. In the above equations, we have also not displayed the argument $(\vec{r}_1,\vec{r}_2,\omega)$ of $\tilde{\chi}$. In Eqs. (2.3) and (2.4), $\tilde{\chi}_{kp}$, $C_{il}^{(1)}$, $C_{il}^{(2)}$, $I^{(1)}$, $I^{(2)}$ are matrices defined by

$$\tilde{\chi}_{kp}(\vec{r}_1,\vec{r}_2,\omega)=\begin{pmatrix}\chi_{kpEE}(\vec{r}_1,\vec{r}_2,\omega)\\\chi_{kpEH}(\vec{r}_1,\vec{r}_2,\omega)\\\chi_{kpHE}(\vec{r}_1,\vec{r}_2,\omega)\\\chi_{kpHH}(\vec{r}_1,\vec{r}_2,\omega)\end{pmatrix},\tag{2.5}$$

$$I^{(1)}=\begin{pmatrix}0\\1\\-1\\0\end{pmatrix},\quad I^{(2)}=\begin{pmatrix}1\\0\\0\\1\end{pmatrix},$$

$$C_{il}^{(1)}=\begin{pmatrix}0&0&\mu_{il}&0\\0&0&0&\mu_{il}\\-\epsilon_{il}&0&0&0\\0&-\epsilon_{il}&0&0\end{pmatrix},\quad C_{il}^{(2)}=\begin{pmatrix}\epsilon_{il}&0&0&0\\0&\epsilon_{il}&0&0\\0&0&\mu_{il}&0\\0&0&0&\mu_{il}\end{pmatrix},\tag{2.6}$$

and ϵ_{ijk} is the completely antisymmetric tensor of Levicivita. $\tilde{\chi}_{kp}$ has the symmetry property

$$\tilde{\chi}_{pk}(\vec{r}_2,\vec{r}_1,\omega)=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\tilde{\chi}_{kp}(\vec{r}_1,\vec{r}_2,\omega),\tag{2.7}$$

which follows from Eqs. (I 2.5), (I 2.6), and the time-reversal invariance. On using Eq. (2.7) in Eqs. (2.3) and (2.4), we obtain equations involving the derivatives with respect to $\vec{r}_2$:

$$\epsilon_{ijk}\partial_j^{(2)}\tilde{\chi}_{pk}=-ik_0C_{il}^{(1)}\tilde{\chi}_{pl}-4\pi ik_0\delta_{ip}\delta(\vec{r}_1-\vec{r}_2)I^{(1)},\tag{2.8}$$

$$\partial_i^{(2)}C_{ij}^{(2)}\tilde{\chi}_{pj}=-4\pi\partial_p^{(2)}\delta(\vec{r}_1-\vec{r}_2)I^{(2)},\tag{2.9}$$

where the argument of $\tilde{\chi}$ is again $\vec{r}_1,\vec{r}_2,\omega$. The response function $\tilde{\chi}_{ij}$ can be expressed in terms of the equilibrium correlation function by using Eqs. (I 2.5), (I 2.6), and the fluctuation-dissipation theorem (I 2.10) as

$$\tilde{\chi}_{ij}(\vec{r}_1,\vec{r}_2,\omega)=\frac{1}{\pi}\int_{-\infty}^{+\infty}d\omega'[\omega'-(\omega+i\nu)]^{-1}\tanh(\tfrac{1}{2}\beta\omega')\times\tilde{S}_{ij}(\vec{r}_1,\vec{r}_2,\omega'),\tag{2.10}$$

where ν is an infinitesimal number, and where $\tilde{S}_{ij}$ is the column matrix

$$\tilde{S}_{ij}=\begin{pmatrix}\mathcal{E}_{ij}^{(S)}\\\mathcal{G}_{ij}^{(S)}\\\tilde{\mathcal{G}}_{ij}^{(S)}\\\mathcal{H}_{ij}^{(S)}\end{pmatrix}.\tag{2.11}$$

Hence, Eqs. (2.3), (2.4), (2.8), (2.9), and (2.10) lead to the following equations for the correlation functions (with $\hat{O}$ denoting the operation $\hat{O}f(\omega)=\int_{-\infty}^{+\infty}(d\omega'/\pi)[\omega'-\omega-i\nu]^{-1}\tanh(\frac{1}{2}\beta\omega')f(\omega')$):

$$\hat{O}(\epsilon_{ijk}\partial_j^{(1)}\tilde{S}_{kp} - ik_0 C_{il}^{(1)}\tilde{S}_{lp}) = 4\pi i k_0 \delta_{ip}\delta(\vec{r}_1 - \vec{r}_2)I^{(1)}, \tag{2.12}$$

$$\hat{O}\partial_i^{(1)} C_{ij}^{(2)}\tilde{S}_{jp} = -4\pi\partial_p^{(1)}\delta(\vec{r}_1 - \vec{r}_2)I^{(2)}, \tag{2.13}$$

and the equations obtained from (2.8) and (2.9) involving the derivatives with respect to $\vec{r}_2$. It should be noted that the operator $\hat{O}$ acts only on S and not on $C_{ij}^{(1)}$, $C_{ij}^{(2)}$. It is clear from Eqs. (2.12) and (2.13) that the correlation functions satisfy quite complex equations. One of the main reasons for the complexity of the above equations is the *dispersion* of the dielectric and permeability functions.

For the free space ($\epsilon_{ij} = \mu_{ij} = \delta_{ij}$) the above equations simplify considerably. The results are

$$\begin{aligned} &\partial_i^{(1)}\tilde{S}_{ip} = 0, \quad \epsilon_{ijk}\partial_j^{(1)}\tilde{S}_{kp} = ik_0 C_{il}^{(1)}\tilde{S}_{lp}, \\ &\partial_i^{(2)}\tilde{S}_{pi} = 0, \quad \epsilon_{ijk}\partial_j^{(2)}\tilde{S}_{pk} = -ik_0 C_{il}^{(1)}\tilde{S}_{pl}. \end{aligned} \tag{2.14}$$

Needless to say, Eqs. (2.14) also follow easily from Eqs. (2.3), (2.4), (2.8), and (2.9), by taking their real and imaginary parts, and using the fluctuation-dissipation relations (I 2.20)–(I 2.23) and

$$\begin{aligned} &\chi''_{ijEE}(\chi''_{ijHH}) = \mathrm{Im}\chi_{ijEE}(\mathrm{Im}\chi_{ijHH}), \\ &\chi''_{ijEH}(\chi''_{ijHE}) = -i\,\mathrm{Re}\chi_{ijEH}(-i\,\mathrm{Re}\chi_{ijHE}). \end{aligned} \tag{2.15}$$

These are the equations satisfied by symmetrized correlation functions. Normally ordered and antinormally ordered correlation functions satisfy similar equations, since different types of correlation functions are related by [cf. Eqs. (I 3.13)–(I 3.16) and (I 3.22)–(I 3.25)]

$$\begin{aligned} &\mathcal{E}_{ij}^{(N)}(\vec{r}_1, \vec{r}_2, \omega) \propto \eta(-\omega)\mathcal{E}_{ij}^{(S)}(\vec{r}_1, \vec{r}_2, \omega), \\ &\mathcal{E}_{ij}^{(A)}(\vec{r}_1, \vec{r}_2, \omega) \propto \eta(\omega)\mathcal{E}_{ij}^{(S)}(\vec{r}_1, \vec{r}_2, \omega). \end{aligned} \tag{2.16}$$

Here, $\eta(\omega)$ is the step function $\eta(\omega) = 1$ if $\omega > 0$; zero otherwise. Equations for normally ordered correlation functions obtained by combining Eqs. (2.14) and (2.16) coincide with those of Mehta and Wolf.[13] It should be noted that for the dielectric case, simple equations like (2.14) are not found because of the complex nature of $\epsilon(\omega)$. The complex nature of $\epsilon(\omega)$ leads to coupled equations involving both real and imaginary parts of χ, whereas for free space, such equations decouple and lead to simpler Eqs. (2.14). It should also be noted that Eqs. (2.3), (2.4), (2.8), and (2.9) are valid for arbitrary stationary radiation fields not necessarily in thermal equilibrium.

We next note that on combining Eqs. (2.3), (2.4), (2.8), and (2.9), we can obtain a set of second-order equations involving derivatives both with respect to $\vec{r}_1$ and $\vec{r}_2$. Further, it is also possible to obtain a set of decoupled equations for χ_{ijEE}. The order of the decoupled equations is, in general, four or less, depending on the nature of the medium under consideration. For an isotropic medium, such equations are of second order. One can, for example, obtain on taking the curl of (2.3), and using (2.4), the second-order equation

$$\begin{aligned} [\nabla^2 - k_0^2(C^{(1)})^2]\tilde{\chi}_{kp} &= -4\pi\partial_p^{(1)}\partial_q^{(1)}\delta(\vec{r}_1 - \vec{r}_2)I^{(2)}(C^{(2)})^{-1} - 4\pi i k_0\epsilon_{qsp}\partial_s^{(1)}\delta(\vec{r}_1 - \vec{r}_2)I^{(1)} + 4\pi k_0^2 C^{(1)} I^{(1)}\delta_{pq}\delta(\vec{r}_1 - \vec{r}_2) \\ &= (\nabla^2 + k_0^2\epsilon\mu)\hat{O}\tilde{S}_{qp}. \end{aligned} \tag{2.17}$$

For free space, Eq. (2.17) reduces to

$$(\nabla^2 + k_0^2)\tilde{\chi}_{qp} = -4\pi\partial_p^{(1)}\partial_q^{(1)}\delta(\vec{r}_1 - \vec{r}_2)I^{(2)} - 4\pi k_0^2\delta_{pq}\delta(\vec{r}_1 - \vec{r}_2)I^{(2)} - 4\pi i k_0 I^{(1)}\epsilon_{qsp}\partial_s^{(1)}\delta(\vec{r}_1 - \vec{r}_2). \tag{2.18}$$

From Eq. (2.18), we easily find that χ'' satisfies the equation

$$(\nabla^2 + k_0^2)\tilde{\chi}''_{ij} = 0, \tag{2.19}$$

and hence the second-order equation for the correlation function will be

$$(\nabla^2 + k_0^2)\tilde{S}_{ij} = 0. \tag{2.20}$$

Thus each of the correlation functions $\tilde{S}_{ij}$ satisfies the Helmholtz equation, which is also the case if ϵ and μ are assumed to be real, isotropic, positive, and their dispersion is ignored. We wish to emphasize that the equations for the response functions are more general than those for the correlations, in the sense that response functions have much more information than correlation functions. Moreover, the boundary conditions on the response functions for the case of domains involving dielectrics are relatively simpler than those for the correlation functions. However, to study how the coherence function propagates from one plane to another, it is better to use equations like (2.20) (cf. with the vanCitter-Zernike theorem, Ref. 5, p. 508). The boundary conditions to be used in the solution of Eqs. (2.3), (2.4), (2.8), and (2.9) are the usual Maxwell boundary conditions. In Sec. III, we will discuss another formulation of boundary conditions. In this connection, we note that it is also possible to obtain a set of integral equations for the coherence functions and the response functions. These equations follow from

Maxwell's equations in the integral form. We consider, for simplicity, the following geometrical situation: Domain V is occupied by a dielectric characterized by local ϵ_{ij} and μ_{ij}, and the rest of the space denoted by $\tilde{V}$ is filled by vacuum. We assume that the external polarization and magnetization charges are located in vacuum. The integral form of Maxwell's equations for such a medium is[8]

$$\vec{D}(\vec{r}_<) = \vec{E}^{(0)}(\vec{r}_<) + \frac{1}{4\pi}\nabla\times\nabla\times\int_V d^3r'[\vec{D}(\vec{r}') - \vec{E}(\vec{r}')]\mathcal{G}_0(\vec{r}_< - \vec{r}') + \frac{ik_0}{4\pi}\nabla\times\int_V d^3r'[\vec{B}(\vec{r}') - \vec{H}(\vec{r}')]\mathcal{G}_0(\vec{r}_< - \vec{r}'), \tag{2.21}$$

$$\vec{E}(\vec{r}_>) = \vec{E}^{(0)}(\vec{r}_>) + \frac{1}{4\pi}\nabla\times\nabla\times\int_V d^3r'[\vec{D}(\vec{r}') - \vec{E}(\vec{r}')]\mathcal{G}_0(\vec{r}_> - \vec{r}') + \frac{ik_0}{4\pi}\nabla\times\int_V d^3r'[\vec{B}(\vec{r}') - \vec{H}(\vec{r}')]\mathcal{G}_0(\vec{r}_> - \vec{r}'),$$

$$\vec{r}_< \in V, \quad \vec{r}_> \in \tilde{V}, \tag{2.22}$$

with similar equations for the magnetic field. Here, $\mathcal{G}_0$ is given by

$$\mathcal{G}_0(\vec{r} - \vec{r}') = \exp(ik_0|\vec{r} - \vec{r}'|)/|\vec{r} - \vec{r}'|. \tag{2.23}$$

It should be noted that $\vec{\nabla}\cdot\vec{E}^{(0)} = 0$ for the points $\vec{r}_<$, as the charges are located in $\tilde{V}$. From Eqs. (2.21) and (2.22) one obtains, on taking the functional derivatives, the equations

$$C_{ij}^{(2)}\tilde{\chi}_{jp}(\vec{r}_<, \vec{r}_>, \omega) = \tilde{\chi}_{ip}^{(0)}(\vec{r}_<, \vec{r}_>, \omega) + \frac{1}{4\pi}\epsilon_{ijk}\epsilon_{klm}\partial_j^<\partial_l^<(C_{mt}^{(2)} - \delta_{mt})\int_V d^3r'\,\tilde{\chi}_{tp}(\vec{r}', \vec{r}_>, \omega)\mathcal{G}_0(\vec{r}_< - \vec{r}')$$

$$+ \frac{ik_0}{4\pi}\epsilon_{ijk}\partial_j^< C_{kt}\int_V d^3r'\,\tilde{\chi}_{tp}(\vec{r}', \vec{r}_>, \omega)\mathcal{G}_0(\vec{r}_< - \vec{r}'),$$

$$C_{kt} = C_{kt}^{(1)}, \quad \text{with } \epsilon_{kt} \to \epsilon_{kt} - \delta_{kt}, \quad \mu_{kt} \to \mu_{kt} - \delta_{kt}. \tag{2.24}$$

Note that $\tilde{\chi}_{ij}(\vec{r}_>, \vec{r}_>', \omega)$ for $\vec{r}_> \in \tilde{V}$ satisfies Eq. (2.24) with $C_{ij}^{(2)}$ on the left-hand side of Eq. (2.24) replaced by δ_{ij}; i.e.,

$$\tilde{\chi}_{ip}(\vec{r}_>, \vec{r}_>'', \omega) = \tilde{\chi}_{ip}^{(0)}(\vec{r}_>, \vec{r}_>'', \omega) + \frac{1}{4\pi}\epsilon_{ijk}\epsilon_{klm}\partial_j^>\partial_l^>(C_{mt}^{(2)} - \delta_{mt})\int_V d^3r'\,\tilde{\chi}_{tp}(\vec{r}', \vec{r}_>'', \omega)\mathcal{G}_0(\vec{r}_> - \vec{r}')$$

$$+ \frac{ik_0}{4\pi}\epsilon_{ijk}\partial_j^> C_{kt}\int_V d^3r'\,\tilde{\chi}_{tp}(\vec{r}', \vec{r}_>'', \omega)\mathcal{G}_0(\vec{r}_> - \vec{r}'). \tag{2.25}$$

The philosophy here is to solve Eq. (2.24) for the interior response functions (i.e., for points $\vec{r}_<$), and use this solution in Eq. (2.25) to obtain the exterior response function for points $\vec{r}_>$. On combining Eqs. (2.10), (2.24), and (2.25), we will obtain the equations for the correlation functions.

Finally, we note that Baltes and co-workers, in a series of recent papers,[14] have studied the blackbody fluctuations in finite cavities which are assumed to be perfectly conducting. They expressed the blackbody correlation tensors in terms of the eigenfunctions of the cavity. Their expressions for correlation tensors follow easily from our formulation if we express χ_{ijEE} as a series of the vector eigenfunctions (obtained by using the vector boundary conditions $\vec{n}\times\vec{E} = 0$, $\vec{n}\cdot\vec{\nabla}\times\vec{E} = 0$, $\vec{n}$ being the unit outward normal to the surface bounding the cavity), and use the relations (k_λ being the eigenvalue)

$$\mathrm{Im}(k_0^2 - k_\lambda^2)^{-1} = (\pi/2k_0)[\delta(k_0 + k_\lambda) - \delta(k_0 - k_\lambda)],$$

$$\mathcal{E}_{ij}^{(N)}(\vec{r}_1, \vec{r}_2, \omega) = \hbar\eta(-\omega)(1 + \coth\tfrac{1}{2}\beta\omega\hbar)\chi_{ijEE}''(\vec{r}_1, \vec{r}_2, \omega). \tag{2.26}$$

Once the spectral tensor $\mathcal{E}_{ij}^{(N)}$ is known, the remaining spectral tensors $\mathcal{H}_{ij}^{(N)}$, $\mathcal{G}_{ij}^{(N)}$, and $\tilde{\mathcal{G}}_{ij}^{(N)}$ can be obtained by using Eq. (2.14).

III. THE EXTINCTION THEOREM FOR THE CORRELATION FUNCTIONS

As emphasized by Wolf and others, the usual Maxwell boundary conditions on the electromagnetic fields at the surface of two adjoining media are really the "saltus" type of conditions.[8] For example, even for an isotropic dielectric medium in the absence of any surface currents and charges, the normal component of the electric field is discontinuous. The value of the discontinuity is really known only after one has obtained the solution. To use the usual Maxwell boundary condition, one needs the general solution of Maxwell equations both inside and outside the medium. Several years ago, Lalor and Wolf[6] showed that the usual reflectivity formulas can be obtained by solving Maxwell's equations subject to the Ewald-Oseen extinction theorem of molecular optics.[5] This method of solving the problem requires only the solution of Maxwell's equations inside the medium. Pattanayak and Wolf[7] obtained the extinction theorem in the most general form, valid for an arbitrary type of medium. The present author used the generalized extinction theorem to obtain the dispersion relations for surface polaritons.[15] We will now formulate a generalized extinction theorem for the response functions and the correlation functions. Since knowledge of the response functions leads to the correlation functions, it is sufficient to solve the extinction theorem for response functions. The extinction theorem for the correlation functions is much more involved than the one for the response functions. It should be borne in mind that no simple extinction theorem for the correlation function is expected, as correlation functions *do not in general* satisfy outgoing boundary conditions at infinity.

We assume that the dielectric medium occupies volume V bounded by the surface S, and the rest of the domain, denoted by $\tilde{V}$, is free space. We also assume that external probes are located in the region $\tilde{V}$. Let $\vec{E}^0$ be the field produced by such probes. $\vec{E}^{(0)}$, in general, is not a transverse field. The generalized extinction theorem for our model medium reads[9] (recall that $\nabla\cdot\vec{E}^{(0)}=0$, for $\vec{r}_<\in V$)

$$\vec{E}^{(0)}(\vec{r}_<)+\frac{1}{4\pi k_0^2}\nabla\times\nabla\times\int dS\{\vec{E}(\vec{r}\,')\partial_n'\mathcal{G}_0(\vec{r}_<-\vec{r}\,')-\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\partial_n'\vec{E}(\vec{r}\,')+\vec{n}\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\vec{\nabla}'\cdot\vec{E}(\vec{r}\,') -ik_0\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\vec{n}\times[\vec{B}(\vec{r}\,')-\vec{H}(\vec{r}\,')]\}=0, \tag{3.1}$$

$$\vec{H}^{(0)}(\vec{r}_<)+\frac{1}{4\pi k_0^2}\nabla\times\nabla\times\int dS\{\vec{H}(\vec{r}\,')\partial_n'\mathcal{G}_0(\vec{r}_<-\vec{r}\,')-\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\partial_n'\vec{H}(\vec{r}\,')+\vec{n}\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\vec{\nabla}'\cdot\vec{H}(\vec{r}\,') +ik_0\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\vec{n}\times[\vec{D}(\vec{r}\,')-\vec{E}(\vec{r}\,')]\}=0,$$

$$\mathcal{G}_0(\vec{r}-\vec{r}\,')=(\exp ik_0|\vec{r}-\vec{r}\,'|)/|\vec{r}-\vec{r}\,'|,\quad \vec{r}_<\in V,\quad \vec{r}_>\in\tilde{V}, \tag{3.2}$$

where n is the unit outward normal to the surface S, and ∂_n' denotes the outward normal derivative. On taking functional derivatives with respect to $\vec{\mathcal{P}}$ and $\vec{\mathfrak{M}}$, we obtain the following extinction theorems for the response functions:

$$\tilde{\chi}_{ij}^{(0)}(\vec{r}_<,\vec{r}_>,\omega)+\frac{1}{4\pi k_0^2}\epsilon_{ikl}\epsilon_{lmq}\partial_k^<\partial_m^<\int dS[\tilde{\chi}_{qj}(\vec{r}\,',\vec{r}_>,\omega)\partial_n'\mathcal{G}_0(\vec{r}_<-\vec{r}\,')-\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\partial_n'\tilde{\chi}_{qj}(\vec{r}\,',\vec{r}_>,\omega) +n_q\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\partial_p'\tilde{\chi}_{pj}(\vec{r}\,',\vec{r}_>,\omega)-ik_0\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\epsilon_{qsp}n_sC_{pv}\tilde{\chi}_{vj}(\vec{r}\,',\vec{r}_>,\omega)]=0. \tag{3.3}$$

Equation (3.3) should now be used as a boundary condition to solve the equations for $\tilde{\chi}_{ij}$ obtained in Sec. II. $\tilde{\chi}_{ij}$, of course, satisfies homogeneous equations, as the probes are assumed to be in the region V. On combining Eqs. (2.10) and (3.3), we obtain a type of extinction theorem for correlation functions. The extinction theorem (3.3) enables one to solve for the interior response functions. The exterior response functions will be given by

$$\tilde{\chi}_{ij}(\vec{r}_>,\vec{r}_>'',\omega)=\tilde{\chi}_{ij}^{(0)}(\vec{r}_>,\vec{r}_>'',\omega) +\frac{1}{4\pi k_0^2}\epsilon_{ikl}\epsilon_{lmq}\partial_k^>\partial_m^>\int dS[\tilde{\chi}_{qj}(\vec{r}\,',\vec{r}_>'',\omega)\partial_n'\mathcal{G}_0(\vec{r}_>-\vec{r}\,')-\mathcal{G}_0(\vec{r}_>-\vec{r}\,')\partial_n'\tilde{\chi}_{qj}(\vec{r}\,',\vec{r}_>'',\omega) +n_q\mathcal{G}_0(\vec{r}_>-\vec{r}\,')\partial_p'\tilde{\chi}_{pj}(\vec{r}\,',\vec{r}_>'',\omega)-ik_0\epsilon_{qsp}n_s\mathcal{G}_0(\vec{r}_>-\vec{r}\,')C_{pv}\tilde{\chi}_{vj}(\vec{r}\,',\vec{r}_>'',\omega)]. \tag{3.4}$$

The extinction theorem (3.3) in a sense relates the free-space correlation functions to the correlation functions inside the medium.[16] Relations (3.3) and (3.4) are quite useful in calculations, particularly in cases of rough surfaces and metallic gratings, as will be demonstrated in part VII. Here we consider a simple situation. For an isotropic nonmagnetic medium, Eq. (3.3) becomes for the response function χ_{ijEE}

$$\chi_{ijEE}^{(0)}(\vec{r}_<,\vec{r}_>,\omega)+\frac{1}{4\pi k_0^2}\epsilon_{ikl}\epsilon_{lmq}\partial_k^<\partial_m^<\int dS[\chi_{qjEE}(\vec{r}\,',\vec{r}_>,\omega)\partial_n'\mathcal{G}_0(\vec{r}_<-\vec{r}\,')-\mathcal{G}_0(\vec{r}_<-\vec{r}\,')\partial_n'\chi_{qjEE}(\vec{r}\,',\vec{r}_>,\omega)]=0. \tag{3.5}$$

We also have from Eq. (2.17) the differential equations for χ_{ijEE}:

$$(\nabla^2 + k_0^2\epsilon)\chi_{ijEE} = 0\,,$$
$$\partial_i^< \chi_{inEE}(\vec{r}_<, \vec{r}_>, \omega) = 0\,. \qquad (3.6)$$

For the case of a plane interface it is easily shown, using the angular spectrum for χ,

$$\chi_{ijEE}(\vec{r}, \vec{r}', \omega) = \iint du\, dv\, \chi_{ijEE}(u, v, \omega; \vec{r}') \times \exp(iux + ivy + iwz)\,,$$
$$w^2 = k_0^2\epsilon - u^2 - v^2\,, \qquad (3.7)$$

that the solution of Eq. (3.6) subject to (3.5) is the same as that given by Eq. (IV 6.12). Finally, we note that if our probes are located in region V, then Eq. (3.3) holds with the term $\chi^{(0)} = 0$. However, now $\tilde{\chi}$ satisfies a more complicated equation (2.17). The value of the extinction theorem in the treatment of scattering due to surface perturbations will be shown in part VII of this series.[11]

IV. MOMENT THEOREM FOR THE FLUCTUATIONS IN A LINEAR DIELECTRIC

In part I of this series of papers, we obtained a number of response functions for space-time domains involving linear dielectrics. Such response functions yielded, via the fluctuation-dissipation theorem, the second-order coherence functions such as $\mathcal{E}_{ij}$, $\mathcal{H}_{ij}$, $\mathcal{G}_{ij}$, and $\tilde{\mathcal{G}}_{ij}$. The question now arises—what are the higher-order correlation functions? We will now show that the higher-order correlation functions for thermal fluctuations can be expressed in terms of second-order ones, by relations analogous to those which hold for Gaussian fluctuations. This result is true only for linear dielectrics. For the case of an electromagnetic field in free space, the Gaussian character is usually proved by finding the diagonal coherent-state representation of the density operator and then showing that the weight function in this representation is Gaussian in nature.[17]

It is clear, from Maxwell's equations in a linear dielectric, that the response of the field variables $\vec{E}$ and $\vec{H}$ to applied polarization and magnetization is always linear, i.e., the response of field variables to orders greater than one in external forces is zero. From the usual perturbation theory, one finds that the higher-order response functions are connected to the *repeated* commutators[18] of the field variables at different space-time points. Hence, for fluctuations in a linear dielectric, all the repeated commutators should vanish. From this, we conclude that for a linear dielectric the commutator of two field operators at different space-time points is a c number. We can indeed make the identification

$$\begin{aligned}
\chi''_{ijEE}(\vec{r}, \vec{r}', t-t') &= \tfrac{1}{2}[E_i(\vec{r}, t), E_j(\vec{r}', t')]\,,\\
\chi''_{ijEH}(\vec{r}, \vec{r}', t-t') &= \tfrac{1}{2}[E_i(\vec{r}, t), H_j(\vec{r}', t')]\,,\\
\chi''_{ijHE}(\vec{r}, \vec{r}', t-t') &= \tfrac{1}{2}[H_i(\vec{r}, t), E_j(\vec{r}', t')]\,,\\
\chi''_{ijHH}(\vec{r}, \vec{r}', t-t') &= \tfrac{1}{2}[H_i(\vec{r}, t), H_j(\vec{r}', t')]\,.
\end{aligned} \qquad (4.1)$$

Such a result was anticipated in part III, and indeed has been used in III and IV to examine one-photon transition probabilities as well as the radiation reaction fields. This result, in conjunction with the fact that equilibrium density operator is $e^{-\beta H}$, enables us to prove the moment theorem.[19]

We first consider the even-order correlation functions of the field operators. Consider two fourth-order correlation functions and their Fourier transforms, defined by

$$\Gamma_{ijkl}(t_1, t_2, t_3) = \langle A_i(t_1)A_j(t_2)A_k(t_3)A_l(0)\rangle\,, \qquad (4.2)$$

$$\Gamma_{jkli}(t_1, t_2, t_3) = \langle A_j(t_2)A_k(t_3)A_l(0)A_i(t_1)\rangle\,, \qquad (4.3)$$

$$= \frac{1}{(2\pi)^3}\int d\omega_1\, d\omega_2\, d\omega_3\, \Gamma_{jkli}(\{\omega_i\}) \times \exp\Big(-i\sum_i \omega_i t_i\Big)\,. \qquad (4.4)$$

The operators A_i stand for any of the field operators $\vec{E}$ and $\vec{H}$. From the definition of the correlation functions it is easy to show, using the eigenstates of the Hamiltonian H, that (putting $\hbar = 1$)

$$\Gamma_{ijkl}(\{\omega\}) = (2\pi)^3 \sum e^{-\beta E_\mu}(A_i)_{\mu\nu}(A_j)_{\nu\lambda}(A_k)_{\lambda\sigma}(A_l)_{\sigma\mu}\,\delta(\omega_1 + \omega_{\mu\nu})\delta(\omega_2 + \omega_{\nu\lambda})\delta(\omega_3 + \omega_{\lambda\sigma})/\mathrm{Tr}(e^{-\beta H})\,,$$
$$\omega_{\mu\nu} = E_\mu - E_\nu\,, \qquad (4.5)$$

$$\Gamma_{jkli}(\{\omega\}) = (2\pi)^3 \sum e^{-\beta E_\nu}(A_j)_{\nu\lambda}(A_k)_{\lambda\sigma}(A_l)_{\sigma\mu}(A_i)_{\mu\nu}\,\delta(\omega_1 + \omega_{\mu\nu})\delta(\omega_2 + \omega_{\nu\lambda})\delta(\omega_3 + \omega_{\lambda\sigma})/\mathrm{Tr}(e^{-\beta H})\,, \qquad (4.6)$$

and hence the Fourier transforms of the correlation functions Γ_{ijkl} and Γ_{jkli} are related by

$$\Gamma_{jkli}(\{\omega\}) = e^{-\beta\omega_1}\Gamma_{ijkl}(\{\omega\})\,. \qquad (4.7)$$

We now use the fact that the two-time commutator of any two operators is a c number [relation (4.1)] to obtain

$$\Gamma_{jkli}(\{t\}) = \Gamma_{ijkl}(\{t\}) + \langle A_k(t_3)A_l\rangle [A_j(t_2), A_i(t_1)] + \langle A_j(t_2)A_k(t_3)\rangle [A_l, A_i(t_1)] + \langle A_j(t_2)A_l\rangle [A_k(t_3), A_i(t_1)]. \quad (4.8)$$

We next use the fluctuation-dissipation theorem (I2.10) to obtain the relation between $\Gamma_{kl}(\omega)$, defined by

$$\Gamma_{kl}(\omega) = \int_{-\infty}^{+\infty} d\tau\, \Gamma_{kl}(\tau) e^{i\omega\tau},$$

$$\Gamma_{kl}(\tau) = \langle A_k(\tau)A_l(0)\rangle,$$

and the response function $\chi_{kl}(\omega)$

$$\Gamma_{kl}(\omega) = \frac{1}{2}\int_{-\infty}^{+\infty} d\tau\, e^{i\omega\tau} \langle \{A_k(\tau), A_l(0)\} + [A_k(\tau), A_l(0)]\rangle$$
$$= 2(1 - e^{-\beta\omega})^{-1}\chi''_{kl}(\omega). \quad (4.9)$$

We now take the Fourier transform of Eq. (4.8) and use Eqs. (4.1), (4.7), and (4.9) to eliminate the correlation function Γ_{jkli}; we then find that

$$(1/2\pi)\Gamma_{ijkl}(\{\omega\}) = \delta(\omega_1 + \omega_2)\,\Gamma_{ij}(\omega_1)\Gamma_{kl}(\omega_3) + \delta(\omega_1 + \omega_3)\Gamma_{ik}(\omega_1)\Gamma_{jl}(\omega_2) + \delta(\omega_2 + \omega_3)\Gamma_{jk}(\omega_2)\Gamma_{il}(\omega_1), \quad (4.10)$$

i.e., the Fourier transform of the fourth-order correlation function can be expressed in terms of the second-order correlation functions. In the time domain, Eq. (4.10) leads to the result

$$\Gamma_{ijkl}(t_1, t_2, t_3) = \Gamma_{ij}(t_1 - t_2)\Gamma_{kl}(t_3) + \Gamma_{ik}(t_1 - t_3)\Gamma_{jl}(t_2) + \Gamma_{il}(t_1)\Gamma_{jk}(t_2 - t_3). \quad (4.11)$$

Relation (4.11) is analogous to the moment theorem for Gaussian random processes. A similar moment relation holds for the higher-order correlation functions. In place of Eqs. (4.7) and (4.8), we now have

$$\Gamma_{i_2, i_3 \ldots i_{2n}, i_1}(\{\omega\}) = e^{-\beta\omega_1}\Gamma_{i_1, i_2 \ldots i_{2n}}(\{\omega\}), \quad (4.12)$$

$$\Gamma_{i_2, i_3 \ldots i_{2n}, i_1}(t_{i_1}\cdots t_{i_{2n}}) = \Gamma_{i_1, i_2 \ldots i_{2n}}(t_{i_1}\cdots t_{i_{2n}}) + \sum_{\alpha=2}^{2n} [A_{i_\alpha}(t_{i_\alpha}), A_{i_1}(t_1)]\,\Gamma_{i_2 \ldots \not{i}_\alpha \ldots i_{2n}}(t_{i_2}\cdots \not{t}_{i_\alpha}\cdots t_{i_{2n}}),$$
$$t_{i_{2n}} \equiv 0, \quad (4.13)$$

where $\Gamma_{i_2 \ldots \not{i}_\alpha \ldots i_{2n}}$ denotes a $(2n-2)$th-order correlation; e.g.,

$$\Gamma_{i_2, \not{i}_3 \ldots i_{2n}} \equiv \langle A_{i_2}(t_{i_2})A_{i_4}(t_{i_4})\cdots A_{i_{2n}}\rangle.$$

On combining Eqs. (4.12) and (4.13), we see that the $2n$th-order correlation function has been expressed in terms of $(2n-2)$th-order and second-order correlation functions. On repeating the above process, we will be able to express the $2n$th-order correlation function in terms of second-order functions. It should be noted that $\Gamma_{i_1, i_2 \ldots i_{2n}}$ will in general have $2n!/2^n n!$ terms. Using a similar procedure for the odd-order correlation functions, we find that the $(2n-1)$th-order correlation function can be expressed in terms of second- and first-order function. Since the mean values $\langle A_i(t)\rangle$ have been chosen to be zero, it follows that all the odd-order correlation functions vanish. We have proved these results for arbitrary operators A_i which are such that the commutator of any two operators at different times is a c number. Hence, the moment theorem will also hold if some of the operators A_i stand for the positive- or negative-frequency parts of the field operators. Note that if some of the Γ_{ij} vanish, then the number of terms in the moment theorem is appreciably reduced; e.g., if $A_i = E_i^{(+)}, A_j = E_j^{(+)}$, then the corresponding $\Gamma_{ij} = 0$. In this case, the fourth-order correlation function will have only two terms:

$$\langle E_i^{(+)}(\vec r_1, t_1)E_j^{(+)}(\vec r_2, t_2)E_k^{(-)}(\vec r_3, t_3)E_l^{(-)}(\vec r_4, t_4)\rangle = \langle E_i^{(+)}(\vec r_1, t_1)E_k^{(-)}(\vec r_3, t_3)\rangle\langle E_j^{(+)}(\vec r_2, t_2)E_l^{(-)}(\vec r_4, t_4)\rangle + \langle E_i^{(+)}(\vec r_1, t_1)E_l^{(-)}(\vec r_4, t_4)\rangle\langle E_j^{(+)}(\vec r_2, t_2)E_k^{(-)}(\vec r_3, t_3)\rangle, \quad (4.14)$$

where, according to Eq. (I3.22), one has for the antinormally ordered correlation function in terms of the response function

$$\langle E_i^{(+)}(\vec r_1, t_1)E_k^{(-)}(\vec r_3, t_3)\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{-i\omega(t_1 - t_3)}\eta(\omega)(1 + \coth\tfrac{1}{2}\beta\omega)\chi''_{ikEE}(\vec r_1, \vec r_3, \omega). \quad (4.15)$$

We have thus shown that the thermal fluctuations in a linear dielectric are completely determined in terms of χ_{ij}'s. Free thermal fields are, of course, included in our treatment. The above method of proof is easily generalized to the case of fermions. We have elsewhere discussed the bunching and antibunching effects in thermal beams using the above results and their generalizations for fermions.

V. BLACKBODY FLUCTUATIONS IN A MOVING FRAME AND A MODIFIED FLUCTUATION-DISSIPATION THEOREM

In this section, we will discuss how the linear-response theory can be used in the treatment of blackbody fluctuations in a moving frame. For the study of the correlation functions of blackbody radiation in a moving frame,[3,4,20] the equilibrium state of the radiation field is characterized by

$$\rho = e^{-\beta K}/\mathrm{Tr}\, e^{-\beta K}, \tag{5.1}$$

$$\beta = \beta_0\gamma, \quad \beta_0 = 1/K_B T_0, \quad \gamma = (1 - v^2/c^2)^{-1/2}, \quad K = H - \vec{\Pi}\cdot\vec{v}, \tag{5.2}$$

where T_0 is the rest-frame temperature, $\vec{v}$ denotes the velocity of the moving frame and Π_i is the ith component of the momentum operator for the radiation field. In the rest frame, the density matrix was simply

$$\rho = e^{-\beta_0 H}/\mathrm{Tr}\, e^{-\beta_0 H}, \tag{5.3}$$

which was used in the derivation of the fluctuation-dissipation theorem, namely (I2.10)

$$S_{ij}(\vec{r}, \vec{r}', \omega) = \hbar \coth(\tfrac{1}{2}\beta_0 \omega\hbar)\, \chi''_{ij}(\vec{r}, \vec{r}', \omega). \tag{5.4}$$

It is now clear that with Eq. (5.1) as the ensemble describing the radiation field, the fluctuation-dissipation theorem (5.4) will be different. We recall that the fluctuation-dissipation theorem is very basic in our treatment of correlation functions, and hence we need its modified form appropriate to ensemble (5.1). To get the modified form of the fluctuation-dissipation theorem, we need to know the form of the momentum operator $\vec{\Pi}$ and its properties. Needless to say, the momentum operator inside a dielectric medium does not appear to have a unique form.[21,22] For the momentum operator, we will adopt the expression given by Abraham

$$\vec{\Pi} = \frac{1}{4\pi c}\int : \vec{E}\times\vec{H} : d^3r, \tag{5.5}$$

as the energy and momentum form a four-vector.[21] In Eq. (5.5) the colons denote the normal ordering. It should be noted that in a moving dielectric, the relation between electric induction $\vec{D}$, magnetic induction $\vec{B}$, and $\vec{E}$, $\vec{H}$ is quite involved[21]:

$$\vec{D} = \epsilon(\vec{E} + \vec{v}\times\vec{B}/c) - \vec{v}\times\vec{H}/c, \quad \vec{B} = \vec{H} + (\vec{D} - \vec{E})\times\vec{v}/c, \tag{5.6}$$

where we have assumed that in the rest frame our medium is nonmagnetic. Equations (5.6) can be solved for $\vec{D}$ and $\vec{B}$ as functions of $\vec{E}$ and $\vec{H}$. It is clear that a moving dielectric behaves like an anisotropic magnetoelectric medium,[23] and hence the response functions in a moving frame would be obtained from the solution of much more complicated algebraic equations [cf. the solution of Maxwell's equations in a magnetoelectric medium[23]]. Therefore, most of the analysis of the present section will be restricted to the case of fluctuations in free space, and only at the end will we make some remarks concerning fluctuations in a moving dielectric.

Since $\vec{\Pi}$ is the momentum operator, it is expected to be the generator of translations; i.e.,

$$e^{i\vec{\Pi}\cdot\vec{x}}\begin{Bmatrix}\vec{E}(\vec{r})\\ \vec{H}(\vec{r})\end{Bmatrix} e^{-i\vec{\Pi}\cdot\vec{x}} = \begin{Bmatrix}\vec{E}(\vec{r}-\vec{x})\\ \vec{H}(\vec{r}-\vec{x})\end{Bmatrix}. \tag{5.7}$$

We verify in the Appendix that this is indeed the case. The calculation in the Appendix is done in r space, i.e., without ever using the mode decomposition of the field operators.

Having proved property (5.7), we can now obtain the modified form of the fluctuation-dissipation theorem. We express the symmetrized correlation functions in the form

$$2S_{ij}(\vec{r}, \vec{r}', t-t') = Z^{-1}\,\mathrm{Tr}\{e^{-\beta K}\{e^{iHt}A_i(\vec{r})e^{-iHt}, e^{iHt'}A_j(\vec{r}')e^{-iHt'}\}\}$$
$$= Z^{-1}\,\mathrm{Tr}\{e^{-\beta K}\{e^{iK(t-t')}e^{i\vec{\Pi}\cdot\vec{v}(t-t')}A_i(\vec{r})e^{-iK(t-t')}e^{-i\vec{\Pi}\cdot\vec{v}(t-t')}, A_j(\vec{r}')\}\}, \quad Z = \mathrm{Tr}\, e^{-\beta K},$$

which on using Eq. (5.7) becomes

$$2S_{ij}(\vec{r}, \vec{r}', t-t') = Z^{-1}\,\mathrm{Tr}\{e^{-\beta K}\{e^{iK(t-t')}A_i[\vec{r} - \vec{v}(t-t')]\, e^{-iK(t-t')}, A_j(\vec{r}')\}\},$$

which on using the relation $f(\vec{r}+\vec{x}) = e^{\vec{x}\cdot\vec{\nabla}}f(\vec{r})$, can be written as

$$2S_{ij}(\vec{r}, \vec{r}', t-t') = Z^{-1}e^{-(t-t')\vec{v}\cdot\vec{\nabla}}\,\mathrm{Tr}\{e^{-\beta K}\{e^{iK(t-t')}A_i(\vec{r})e^{-iK(t-t')}, A_j(\vec{r}')\}\}. \tag{5.8}$$

A similar procedure enables us to write the response function as

$$2\chi''_{ij}(\vec{r},\vec{r}',\tau)=Z^{-1}e^{-\tau\vec{v}\cdot\vec{\nabla}}\mathrm{Tr}\{e^{-\beta K}[e^{iK\tau}A_i(\vec{r})e^{-iK\tau},A_j(\vec{r}')]\}. \tag{5.9}$$

We have thus expressed the time evolution of the operators in Eqs. (5.8) and (5.9) by the unitary transformation e^{iKt}. The operator K is the same as the one appearing in the definition of the ensemble. Hence, we will obtain in the usual manner the fluctuation-dissipation relation

$$\int_{-\infty}^{+\infty}d\tau\, e^{i\omega\tau}\mathrm{Tr}\{e^{-\beta K}\{e^{iK\tau}A_i(\vec{r})e^{-iK\tau},A_j(\vec{r}')\}\}=\coth\frac{\beta\omega}{2}\int_{-\infty}^{+\infty}d\tau\, e^{i\omega\tau}\,\mathrm{Tr}\{e^{-\beta K}[e^{iK\tau}A_i(\vec{r})e^{-iK\tau},A_j(\vec{r}')]\}. \tag{5.10}$$

On combining Eqs. (5.8)–(5.10), we obtain the important relation

$$\int_{-\infty}^{+\infty}d\tau\, e^{i\omega\tau}e^{\tau\vec{v}\cdot\vec{\nabla}}S_{ij}(\vec{r},\vec{r}',\tau)=\coth\frac{\beta\omega}{2}\int_{-\infty}^{+\infty}d\tau\, e^{i\omega\tau}e^{\tau\vec{v}\cdot\vec{\nabla}}\chi''_{ij}(\vec{r},\vec{r}',\tau). \tag{5.11}$$

This equation can be simplified, as we can express the response functions and correlation functions in terms of their Fourier transforms in the form

$$\chi''_{ij}(\vec{r},\vec{r}',\tau)=\frac{1}{(2\pi)^4}\int d^3K\,d\omega\,\chi''_{ij}(\vec{K},\omega)e^{i\vec{K}\cdot(\vec{r}-\vec{r}')-i\omega\tau}. \tag{5.12}$$

Recall that we are dealing with the translationally invariant case. On combining Eqs. (5.11) and (5.12) we obtain

$$S_{ij}(\vec{K},\omega+\vec{K}\cdot\vec{v})=\coth(\tfrac{1}{2}\beta\omega)\chi''_{ij}(\vec{K},\omega+\vec{K}\cdot\vec{v}), \tag{5.13}$$

or, equivalently, the equation

$$S_{ij}(\vec{K},\omega)=\coth[\tfrac{1}{2}\beta_0\gamma(\omega-\vec{K}\cdot\vec{v})]\chi''_{ij}(\vec{K},\omega). \tag{5.14}$$

In coordinate space, Eq. (5.14) leads to the relation

$$S_{ij}(\vec{r}-\vec{r}',\omega)=\coth[\tfrac{1}{2}\beta_0\gamma(\omega+i\vec{v}\cdot\vec{\nabla}_{\vec{r}-\vec{r}'})]\chi''_{ij}(\vec{r}-\vec{r}',\omega). \tag{5.15}$$

Equation (5.14) [(5.15)] is our modified form of the fluctuation-dissipation theorem. It is interesting to note that in the modified form, the Doppler-shifted frequency $\omega\to\gamma(\omega-\vec{K}\cdot\vec{v})$ appears. For the free electromagnetic fields, the response functions χ'' are nonvanishing only for $|K|=\omega/c$; hence, $|\vec{K}\cdot\vec{v}|=Kv|\cos\theta|=(\omega/c)v|\cos\theta|\leq\omega v/c\leq\omega$. Therefore, at zero temperature, Eqs. (5.14) and (A3) imply that

$$S_{ij}(\vec{K},\omega)=\chi''_{ij}(\vec{K},|\omega|)\epsilon_i\epsilon_j, \tag{5.16}$$

which is equivalent to the statement that the vacuum is Lorentz invariant. From Eq. (5.15), or equivalently from time-reversal invariance, one obtains the following properties for the blackbody spectral tensors:

$$\begin{aligned}\mathcal{E}_{ij}(\vec{r},\vec{r}',\omega,\vec{v})&=\mathcal{E}^*_{ij}(\vec{r},\vec{r}',\omega,-\vec{v}),\\ \mathcal{H}_{ij}(\vec{r},\vec{r}',\omega,\vec{v})&=\mathcal{H}^*_{ij}(\vec{r},\vec{r}',\omega,-\vec{v}),\\ \mathcal{G}_{ij}(\vec{r},\vec{r}',\omega,\vec{v})&=-\mathcal{G}^*_{ij}(\vec{r},\vec{r}',\omega,-\vec{v}),\\ \tilde{\mathcal{G}}_{ij}(\vec{r},\vec{r}',\omega,\vec{v})&=-\tilde{\mathcal{G}}^*_{ij}(\vec{r},\vec{r}',\omega,-\vec{v}).\end{aligned} \tag{5.17}$$

Thus in a moving frame, $\mathcal{E}_{ij}$ is no longer real. At zero temperature, $\mathcal{E}_{ij}$ is real, as the zero-temperature correlations are independent of velocity [Eq. (5.16)]. One also has, from Eq. (5.15),

$$\begin{aligned}S_{ji}(\vec{r}'-\vec{r},\omega,\vec{v})&=\coth[\tfrac{1}{2}\beta_0\gamma(\omega+i\vec{v}\cdot\vec{\nabla}_{\vec{r}'-\vec{r}})]\\&\quad\times\chi''_{ji}(\vec{r}'-\vec{r},\omega)\\&=\coth[\tfrac{1}{2}\beta_0\gamma(\omega-i\vec{v}\cdot\vec{\nabla}_{\vec{r}-\vec{r}'})]\\&\quad\times\chi''_{ij}(\vec{r}-\vec{r}',\omega)\epsilon_i\epsilon_j\\&=S_{ij}(\vec{r}-\vec{r}',\omega,-\vec{v})\epsilon_i\epsilon_j,\end{aligned} \tag{5.18}$$

where ϵ_i denotes the parity of the operator A_i under time reversal. Using Eq. (5.18) and the fact that χ''_{ijEE} and χ''_{ijHH} (χ''_{ijEH} and χ''_{ijHE}) are even (odd) functions of $r-r'$ [Eqs. (I4.7) and (I4.8)], we obtain the relations

$$\begin{aligned}\mathcal{E}^{(S)}_{ji}(\vec{r},\vec{r}',\omega,\vec{v})&=\mathcal{E}^{(S)}_{ij}(\vec{r},\vec{r}',\omega,\vec{v})=\mathcal{H}^{(S)}_{ij}(\vec{r},\vec{r}',\omega,\vec{v}),\\ \mathcal{G}^{(S)}_{ji}(\vec{r},\vec{r}',\omega,\vec{v})&=-\mathcal{G}^{(S)}_{ij}(\vec{r},\vec{r}',\omega,\vec{v})=-\tilde{\mathcal{G}}^{(S)}_{ji}(\vec{r},\vec{r}',\omega,\vec{v}).\end{aligned} \tag{5.19}$$

Having studied the symmetry properties of the blackbody spectral tensors in a moving frame, we now obtain the normally ordered correlation functions. These can be obtained using Eqs. (I3.13)–(I3.16) and (5.15). We have, for example,

$$\mathcal{E}^{(N)}_{ij}(\vec{r},\omega)=2\eta(-\omega)\{\exp[\beta_0\gamma(|\omega|-i\vec{v}\cdot\vec{\nabla})]-1\}^{-1}\chi''_{ijEE}(\vec{r},|\omega|) \tag{5.20}$$

$$=\frac{2\eta(-\omega)}{4\pi k_0}\left(k_0^2\delta_{ij}+\frac{\partial^2}{\partial x_i\partial x_j}\right)\int d^3K\,\delta(K-k_0)e^{i\vec{K}\cdot\vec{r}}\{\exp[\beta_0\gamma(|\omega|+\vec{K}\cdot\vec{v})]-1\}^{-1},\quad k_0=|\omega|/c, \tag{5.21}$$

where in obtaining Eq. (5.21) we have used relations (I4.7), (A7), and $\chi''_{ijEE}=\mathrm{Im}\chi_{ijEE}$. The result (5.21), and similar ones for the other cross-spectral tensors, are equivalent to the Eberly and Kujawski results.[3,4] The higher-order correlations can be obtained from the Gaussian property of Sec. IV.

We close this section with few remarks concerning the fluctuations in a moving dielectric. In this case it is better to use directly the transformation laws of the field, which can be obtained from the transformation laws of the tensors[21]

$$F_{ik}=\begin{pmatrix} 0 & B_3 & -B_2 & -iE_1 \\ -B_3 & 0 & B_1 & -iE_2 \\ B_2 & -B_1 & 0 & -iE_3 \\ +iE_1 & iE_2 & iE_3 & 0 \end{pmatrix},$$
$$J_{ik}=\begin{pmatrix} 0 & H_3 & -H_2 & -iD_1 \\ -H_3 & 0 & H_1 & -iD_2 \\ H_2 & -H_1 & 0 & -iD_3 \\ iD_1 & iD_2 & iD_3 & 0 \end{pmatrix}. \quad (5.22)$$

Note that for fluctuations in free space, Eberly and Kujawski[3] also used this procedure and showed that the results were identical to those obtained with ensemble (5.1). If we consider a frame moving in the x direction, then the transformation laws for $\vec{E}$ and $\vec{H}$ are (assuming a nonmagnetic medium)

$$E_1=\overline{E}_1,\quad E_2=\gamma[\overline{E}_2-(v/c)\overline{H}_3],\quad E_3=\gamma[\overline{E}_3+(v/c)\overline{H}_2],$$
$$H_1=\overline{H}_1,\quad H_2=\gamma[\overline{H}_2+(v/c)\overline{D}_3],\quad H_3=\gamma[\overline{H}_3-(v/c)\overline{D}_2], \quad (5.23)$$

where the fields carrying a bar are fields in the rest frame. Hence, it is clear from (5.23) that the correlation functions in the moving frame will be given, for example, by

$$\mathcal{E}_{11}^{(S)}=\overline{\mathcal{E}}_{11}^{(S)},\quad \mathcal{E}_{22}^{(S)}=\gamma^2\left(\overline{\mathcal{E}}_{22}^{(S)}+\frac{v^2}{c^2}\overline{\mathcal{H}}_{33}^{(S)}-\frac{v}{c}(\overline{\mathcal{G}}_{23}^{(S)}+\overline{\overline{\mathcal{G}}}_{32}^{(S)})\right),$$
$$\mathcal{H}_{11}^{(S)}=\overline{\mathcal{H}}_{11}^{(S)},\quad \mathcal{H}_{22}^{(S)}=\gamma^2\left(\overline{\mathcal{H}}_{22}^{(S)}+\frac{v^2}{c^2}\epsilon^2\overline{\mathcal{E}}_{33}^{(S)}+\frac{v}{c}\epsilon(\overline{\mathcal{G}}_{23}^{(S)}+\overline{\overline{\mathcal{G}}}_{32}^{(S)})\right), \quad (5.24)$$

where we still have to transform the argument of the correlation functions, as $\overline{\mathcal{E}}_{11}$ depends on $\overline{\vec{r}},\overline{t}$. The two sets of coordinates are related by

$$\overline{x}=\gamma(x+vt),\quad \overline{y}=y,\quad \overline{z}=z,$$
$$\overline{t}=\gamma(t+vx/c^2), \quad (5.25)$$

and hence we have the relation

$$\mathcal{E}_{11}^{(S)}(\vec{r},t)=\overline{\mathcal{E}}_{11}^{(S)}[\gamma(x+vt),y,z,\gamma(t+vx/c^2)], \quad (5.26)$$

which in the Fourier domain reads as

$$\mathcal{E}_{11}^{(S)}(\vec{K},\omega)=\overline{\mathcal{E}}_{11}^{(S)}[\gamma(K_x+\omega v/c^2),K_y,K_z,\gamma(\omega+vK_x)]. \quad (5.27)$$

The rest-frame response functions can be found by using the procedure of Sec. IV of paper I. Here, we quote the results:

$$\chi_{ijEE}(\vec{r},\omega)=\epsilon^{-1}\chi_{ijHH}(\vec{r},\omega)=\left(k_0^2\delta_{ij}+\frac{\partial^2}{\epsilon\partial x_i\partial x_j}\right)\frac{e^{ikr}}{r},$$
$$\chi_{ijEH}(\vec{r},\omega)=-\chi_{ijHE}(\vec{r},\omega)=-ik_0\epsilon_{ijl}\frac{\partial}{\partial x_l}\frac{e^{ikr}}{r},$$
$$k=k_0\epsilon^{1/2},\quad k_0=|\omega|/c. \quad (5.28)$$

Assuming ϵ real and positive, ignoring its dispersion [otherwise the expressions for $\chi''_{ij}(\vec{K},\omega)$ are quite involved], and using Eqs. (5.28) and (I2.20)–(I2.23), we find the following for the rest-frame spectral tensors:

$$\overline{\mathcal{E}}_{ij}^{(S)}(\vec{K},\omega)=\overline{\mathcal{H}}_{ij}^{(S)}/\epsilon$$
$$=(4\pi^2/\epsilon)\delta(K^2-k^2)\coth(\tfrac{1}{2}\beta_0\omega)\times(K^2\delta_{ij}-K_iK_j),$$
$$\overline{\mathcal{G}}_{ij}^{(S)}(\vec{K},\omega)=-\overline{\overline{\mathcal{G}}}_{ij}^{(S)}(\vec{K},\omega)$$
$$=\coth(\tfrac{1}{2}\beta_0\omega)\,\delta(K-k)(2\pi^2/\epsilon^{1/2})\epsilon_{ijl}K_l. \quad (5.29)$$

Hence, on combining Eqs. (5.27) and (5.29), we find that the cross-spectral tensor $\mathcal{E}_{11}^{(S)}(\vec{K},\omega)$ in a moving frame is given by

$$\mathcal{E}_{11}^{(S)}(\vec{K},\omega)=(4\pi^2/\epsilon)(K_y^2+K_z^2)\coth[\tfrac{1}{2}\beta_0\gamma(\omega+vK_x)]\delta\left[K^2-\frac{\omega^2}{c^2}\epsilon+(1-\epsilon)\frac{\gamma^2v^2}{c^2}\left(K_x^2+\frac{\omega^2}{c^2}+\frac{2\omega K_x}{v}\right)\right]. \quad (5.30)$$

We thus see that even at zero temperature, the correlation function becomes velocity dependent in contrast to the free-space case. This is expected since, unlike the free-space case, the basic equations involve the velocity, due to relation (5.6).

In the special case one has at zero temperature,

$$\mathcal{E}_{11}^{(S)}(\vec{r}=0,\omega)=\frac{\tfrac{2}{3}(\omega^3/c^3)\epsilon^{1/2}(1-v^2/c^2)^2}{(1-v^2\epsilon/c^2)^2}, \quad (5.31)$$

which is relevant when one considers the lifetimes.

VI. THERMAL RELAXATION OF A MOVING ATOM

As an application of the correlation functions of blackbody radiation in a moving frame, we consider the thermal relaxation of a moving atom. It is best to work in a frame moving with the velocity of the atom. In this frame the atom is at rest; however, the atom sees transformed fields. The interaction Hamiltonian in the moving frame, in the dipole approximation, has the form

$$H_1 = -\int \vec{p}\cdot\vec{E}\,d^3r = -\vec{p}\cdot\vec{E}(0), \tag{6.1}$$

where $\vec{p}$ is the atomic polarization and $\vec{E}$ the electric field operator in the moving frame, and the atom is assumed to be at the origin of the moving frame. The relaxation can be studied in the usual manner by obtaining the master equation for the reduced density operator ρ corresponding to the atomic system [cf. paper IV, Sec. II]. It is found that ρ, in interaction picture, satisfies the relation

$$\frac{\partial\rho}{\partial t} = -\sum_{\alpha\beta}\int_0^\infty d\tau\{\mathcal{E}^{(S)}_{\alpha\beta}(0,0,\tau)[p_\alpha(t),[p_\beta(t-\tau),\rho(t)]] + \chi''_{\alpha\beta EE}(0,0,\tau)[p_\alpha(t),\{p_\beta(t-\tau),\rho(t)\}]\}. \tag{6.2}$$

For the two-level-atom case, Eq. (6.2) leads to, on making the rotating-wave approximation [cf. our derivation of (17.1) in Ref. 24], the following equations for the mean values:

$$\frac{\partial}{\partial t}\langle S^\pm\rangle = [\pm i(\omega+\Omega)-\Gamma]\langle S^\pm\rangle,$$
$$\frac{\partial}{\partial t}\langle S^z\rangle = -2\Gamma(\langle S^z\rangle - \langle S^z\rangle_{eq}). \tag{6.3}$$

Here ω is the energy separation between two levels of the atom, Γ and Ω are given by [cf. Eq. (IV 2.17)]

$$\Gamma = \sum_{\alpha\beta} d_\alpha d_\beta \mathcal{E}^{(S)}_{\alpha\beta}(0,0,\omega), \quad \Gamma_0 = \tfrac{2}{3}|d|^2 k_0^3, \quad k_0 = \omega/c, \tag{6.4}$$

$$\Omega = 2\sum_{\alpha\beta} d_\alpha d_\beta \,\mathrm{Im}\int_0^\infty d\tau\, \mathcal{E}^{(S)}_{\alpha\beta}(0,0,\tau)e^{i\omega\tau}, \tag{6.5}$$

d is the dipole matrix element,

$$\langle S^z\rangle_{eq} = -\tfrac{1}{2}\Gamma_0/\Gamma, \tag{6.6}$$

and Γ_0^{-1} is the usual lifetime of the excited atom in the moving frame. The lifetime will be different when viewed from the rest frame. We can write $\Gamma = \Gamma_0 + \Gamma_{vT}$, where Γ_{vT} is the velocity- and temperature-dependent contribution, which is found, on using Eqs. (6.4), (5.12), and (5.14), to be

$$\Gamma_{vT} = \frac{1}{4\pi^3}\int d^3K \sum_{\alpha\beta} d_\alpha d_\beta \chi''_{\alpha\beta EE}(\vec{K},\omega) \times\{\exp[\beta_0\gamma(\omega - \vec{K}\cdot\vec{v})]-1\}^{-1}. \tag{6.7}$$

Using equations (5.29) with $\epsilon = 1$, and doing the delta-function integration, one can reduce Eq. (6.7) to

$$\Gamma_{vT} = \frac{k_0^3}{2\pi}\int d\Omega(|d|^2 - |\vec{d}\cdot\vec{K}|^2/k_0^2) \times\{\exp[\beta_0\gamma(\omega-\vec{K}\cdot\vec{v})]-1\}^{-1}, \tag{6.8}$$

where $d\Omega$ denotes integration over the solid angle. In the special case when the dipole moment is along the z axis and the atom is also moving in the z direction, Eq. (6.8) reduces to

$$\Gamma_{vT} = 4k_0^3|d|^2\sum_1^\infty e^{-nx}\left[\frac{c^2}{n^2x^2v^2}\cosh\left(\frac{nxv}{c}\right) - \frac{c^3}{n^3x^3v^3}\sinh\left(\frac{nxv}{c}\right)\right],$$
$$x = \beta_0\gamma\omega. \tag{6.9}$$

The corresponding velocity- and temperature-dependent contribution to the shift Ω is also easily computed. The numerical values of Γ_{vT} can be read from curves (3a) and (3b) of Eberly and Kujawski.[4] One finds that at a given temperature, increasing v leads to a decrease in the temperature-dependent contribution to the damping. Similarly, a decrease in ω leads to a decrease in Γ_{vT}.

We now treat spontaneous emission from a moving atom in some detail. The mean number of photons in some mode ks in the moving frame will be

$$N_{ks}(\infty) = \langle a^\dagger_{ks}a_{ks}\rangle = |g_{ks}|^2[(\omega_{ks}-\omega)^2+\Gamma_0^2]^{-1}, \tag{6.10}$$

$$g_{ks} = -i(2\pi ck/L^3)^{1/2}(\vec{d}\cdot\vec{\epsilon}_{ks}), \tag{6.11}$$

where $\vec{\epsilon}_{ks}$ is the polarization vector of the mode ks. Note that here the treatment used in Ref. 24 applies, since the atom in the moving frame is at rest [for a derivation of (6.10) we refer the reader to Sec. 10 of Ref. 24]. One should now transform the spectrum (6.10) to the fixed frame. This can be achieved by using relations like (5.24) with $\epsilon = 1$. In the nonrelativistic case, the transformation is simple—one simply has to replace ω_{ks} by the Doppler-shifted frequency, and therefore

$$N_{ks} = |g_{ks}|^2[(\omega_{ks}-\omega-\vec{K}\cdot\vec{v})^2+\Gamma_0^2]^{-1}. \tag{6.12}$$

It should be noted that Lamb,[25] in his treatment of gas lasers, took the following as the interaction Hamiltonian between the moving atom and the radiation field:

$$H_1 \approx -\vec{p}(t)\cdot\vec{E}(\vec{v}t, t), \tag{6.13}$$

where $\vec{E}$ is now the rest-frame field. This Hamiltonian ignores the relativistic effects; for example, a moving electric dipole will appear as magnetoelectric dipole in the fixed frame. If now a mode decomposition of the field is used, then Eq. (6.13), for the case of a two-level atom, becomes

$$H_1 = \sum_{ks} g_{ks}(t) a_{ks}(S^+ e^{i\omega t} + \text{H.c.})e^{-i\omega_{ks}t} + \text{H.c.}, \tag{6.14}$$

$$g_{ks}(t) = g_{ks} e^{i\vec{k}\cdot\vec{v}t}. \tag{6.15}$$

Therefore, the coupling constant becomes time dependent. This is the viewpoint adopted by Haken[26] in the treatment of gas lasers. Thus, many of the results on spontaneous emission obtained in Ref. 24 can be generalized to the case of a moving atom by using Eq. (6.14). For example, Eqs. (7.23) and (7.24) of Ref. 24 lead to the following results for the mean number of photons and the rate of change in the mode ks:

$$N_{ks}(t) = |g_{ks}|^2 \iint_0^t dt_1\, dt_2 \langle S^+(t_1)S^-(t_2)\rangle \times \exp[-i(\omega_{ks} - \vec{k}\cdot\vec{v})(t_1 - t_2)], \tag{6.16}$$

$$\sigma_{ks}(t) \equiv \dot{N}_{ks}(t) = |g_{ks}|^2 \int_0^t d\tau \langle S^+(t)S^-(\tau)\rangle \times \exp[-i(\omega_{ks} - \vec{k}\cdot\vec{v})(t-\tau)] + \text{H.c.}, \tag{6.17}$$

where $\langle S^+(t)S^-(\tau)\rangle$ is the correlation function calculated in a frame moving with the velocity of the atom. On using Eq. (6.16) and Eq. (10.5) of Ref. 24,

$$\langle S^+(t)S^-(\tau)\rangle = \exp[i\omega(t-\tau) - \Gamma_0(t+\tau)], \tag{6.18}$$

we regain the result (6.12) for the spectrum. Note that Eq. (6.12) is still to be averaged over the distribution of the velocity, which would lead to the standard result of the Doppler broadening. A similar type of analysis can be carried out to discuss the saturation absorption spectroscopy.[27] For this purpose the result (6.17) and that of Sec. 18 of Ref. 24 are needed. Finally note that the present treatment still ignores recoil effects, which can be treated by a proper quantization of the center-of-mass motion. We will solve this problem elsewhere.

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APPENDIX: PROOF OF RELATION (5.7)

In this appendix we verify by a calculation in r space that $\vec{\Pi}$ is the generator of translations. To verify it we first calculate the commutator

$$\begin{aligned} C_{li} &= [E_l(\vec{r}), \Pi_i] \\ &= \frac{1}{4\pi c}\epsilon_{ijk}\int\{[E_l(\vec{r}), E_j(\vec{r}')H_k(\vec{r}')]d^3r' \\ &\quad + [E_l(\vec{r}), [H_k^{(-)}(\vec{r}'), E_j^{(+)}(\vec{r}')]]\} \\ &= \frac{1}{4\pi c}\epsilon_{ijk}\int [E_l(\vec{r}), E_j(\vec{r}')H_k(\vec{r}')]d^3r', \end{aligned} \tag{A1}$$

where we have used the fact that for the radiation field, $[H_k^{(-)}, E_j^{(+)}]$ is a c number. On rewriting (A1) in terms of response functions [Eq. (4.1)], we get for the function C_{li}

$$C_{li} = \frac{1}{4\pi^2 c}\epsilon_{ijk}\int_{-\infty}^{+\infty} d\omega \int d^3r' [\chi''_{ljEE}(\vec{r}, \vec{r}', \omega)H_k(\vec{r}') + \chi''_{lkEH}(\vec{r}, \vec{r}', \omega)E_j(\vec{r}')], \tag{A2}$$

which, on using the symmetry property obtained from the time-reversal invariance [cf. our discussion following Eq. (I2.19)]

$$\begin{aligned} \chi''_{ljEE}(\vec{r}, \vec{r}', \omega) &= -\chi''_{ljEE}(\vec{r}, \vec{r}', -\omega), \\ \chi''_{ljEH}(\vec{r}, \vec{r}', \omega) &= +\chi''_{ljEH}(\vec{r}, \vec{r}', \omega), \end{aligned} \tag{A3}$$

becomes

$$C_{li} = \frac{1}{2\pi^2 c}\int_0^{\infty} d\omega \int d^3r' \epsilon_{ijk}\chi''_{lkEH}(\vec{r}, \vec{r}', \omega)E_j(\vec{r}'). \tag{A4}$$

We recall that in deriving relations of the form (A3), one uses the time-reversal invariance. Since our ensemble (5.1) involves velocities (which are odd variables), we should not only change the sign of ω but also of velocity in (A3). However, for electromagnetic fields in free space, the response functions are independent of velocity, and hence a relation of the form, for example,

$$\chi''_{ljEE}(\vec{r}, \vec{r}', \omega, \vec{v}) = -\chi''_{ljEE}(\vec{r}, \vec{r}', -\omega, -\vec{v}), \tag{A5a}$$

implies

$$\chi''_{ijEE}(\vec{r}, \vec{r}', \omega) = -\chi''_{ijEE}(\vec{r}, \vec{r}', -\omega). \tag{A5b}$$

The response function χ''_{ikEH} can be obtained from (I4.8) and the relation $\chi''_{ikEH} = -i\,\mathrm{Re}\chi_{ikEH}$, and hence we have

$$\chi''_{ikEH}(\vec{r}, \vec{r}', \omega) = -\frac{i\omega}{c}\epsilon_{ikm}\frac{\partial}{\partial x_m}\frac{\sin(\omega/c)|\vec{r}-\vec{r}'|}{|\vec{r}-\vec{r}'|}. \tag{A6}$$

Using (A6) and the relation

$$\frac{k_0 \sin k_0 R}{R} = \frac{1}{4\pi}\int d^3K\,\delta(K-k_0)e^{i\vec{K}\cdot\vec{R}}, \tag{A7}$$

we find that (A4) reduces to

$$C_{li} = -i\frac{\partial}{\partial x_i}E_l(\vec{r}), \tag{A8}$$

and hence, on rewriting (A8) in vectorial form, we get

$$[E_l(\vec{r}), \vec{\Pi}\cdot\vec{x}] = -i(\vec{x}\cdot\vec{\nabla})E_l(\vec{r}), \tag{A9}$$

which implies that

$$e^{i\vec{\Pi}\cdot\vec{x}}\vec{E}(\vec{r})e^{-i\vec{\Pi}\cdot\vec{x}} = \vec{E}(\vec{r}-\vec{x}). \tag{A10}$$

Thus, $\vec{\Pi}$ is the generator of translations. One can similarly prove Eq. (5.7) for the magnetic field operator. It is perhaps worth noting that on the right-hand side of (A10) we obtain $\vec{E}(\vec{r}-\vec{x})$ rather than $\vec{E}(\vec{r}+\vec{x})$. In the usual case, where q and p represent position and momentum operators, one gets $e^{ixp}qe^{-ixp} = q + x$. It also follows from (A9) that $\vec{\Pi}$ is a constant of motion.

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