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Reconstruction of an entangled state in cavity QED

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We suggest a scheme to reconstruct a two-mode entangled state in cavity QED by using the interaction of a *V*-configuration three-level atom and by driving the cavity field. After the atomic interaction with the cavity fields, the probability of the atom being in its initial ground state is found to be directly related to the two-mode Wigner characteristic function. The Wigner function and the four-dimensional density-matrix elements can be obtained for the two-mode entangled field by simple transformations. We consider both the cases where two entangled modes are prepared in one cavity or in two spatially separated cavities. [S1050-2947(99)03304-1]

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I. INTRODUCTION

In recent times, the subject of state reconstruction has become a major field of study in quantum optics [1,2]. The quasiprobability functions and density matrices have been measured experimentally for single-mode running fields using the homodyne scheme [3]. There are also novel proposals to reconstruct quantum states by photon statistics. Direct photodetection of a light field gives information only on the diagonal elements of its density matrix so an auxiliary device is needed before photodetection of the field; the device can be a beam splitter or a linear amplifier [4]. A quantum state has been experimentally reconstructed for a one-dimensional harmonic motion in a trap by considering the excitation probability for the ion's electronic state [5]. There have also been proposals for a magnetic field [6] and for an atomic state [7]. Quantum-state reconstruction schemes have been suggested for the fields in high-Q cavities [1,8,9]. However, most of the earlier reconstruction schemes are for single-mode fields apart from the work of Raymer *et al.* [10], where they study reconstruction of a two-mode running field. In this paper we are interested in reconstruction of a two-mode entangled cavity field.

Entangled states have been at the focus of discussions in quantum optics. Two-system entanglement [11] allows more diverse measurement schemes which can admit tests of local realism [12]. In the heart of quantum teleportation, computing, and cryptography, entanglement resides [13–15]. Proposals to entangle fields in two spatially separated cavities exist [16,17]. Furthermore, it has recently been suggested that an unknown atomic state can be teleported between two cavities which are entangled [18]. A two-level atom in its excited state passes sequentially through two resonant single-mode cavities and is found to be in its ground state after the second-cavity interaction. The atom could have deposited a photon either in the first cavity or in the second so that the final state $|\Psi_f\rangle$ of the two-cavity field is [16]

$$|\Psi_f\rangle = A_1|1,0\rangle + A_2|0,1\rangle, \tag{1}$$

where $|1,0\rangle$ denotes one photon in the first cavity and none in the second, and $|0,1\rangle$ vice versa. It is also possible to produce coherent state entanglement $|\Psi_c\rangle$ between two separate cavities [17]:

$$|\Psi_c\rangle = B_1|\alpha,0\rangle + B_2|0,\alpha\rangle, \tag{2}$$

where $|\alpha,0\rangle$ denotes the first cavity in the coherent state $|\alpha\rangle$ and the second in the vacuum. A two-level atom passes through two far-off resonant cavities where an external driving field is simultaneously coupled. The external driving field is switched on by the atom being in its excited state. By preparing the atom in a superposition of excited and ground states, the atomic switch is in a quantum superposition. The atomic quantum switch can entangle two cavities to be in the state (2).

Meystre [16] and Davidovich *et al.* [17] suggested that the probability of atomic inversion for the second atom would reflect the interference between two-component states. In this paper we propose a scheme to reconstruct two-mode entangled states in high-Q cavities. We assume two cases of cavity-field entanglement: (i) Entanglement of two-mode fields in a cavity, and (ii) entanglement of two-mode fields of which one mode is in a cavity and the other mode is in a spatially separate cavity. In this paper, we only examine the question of reconstruction of the two-mode entangled states in a cavity or in two separate cavities assuming that the entangled states have been prepared.

There have been studies on reconstructing a *single-mode* field in a cavity by probing it with two-level atoms [1]. In particular, Kim *et al.* found that the probability of atomic inversion after a two-level atom interacts with a cavity field is directly related to the Wigner characteristic function [9], which is the Fourier transform of the Wigner function. We consider the following scheme to reconstruct the state of a two-mode field. We first displace the original entangled state by coupling resonant classical fields to the cavities. We then prepare a *V*-configuration three-level atom in its ground state and send it to interact with cavity fields. Throughout the paper we assume high-Q cavities so that the temporal evolution of the combined atom-field system is almost reversible, described by a three-level Jaynes-Cummings-type interaction [19]. This condition fits well the current experimental situation where the cavity damping time is three orders of

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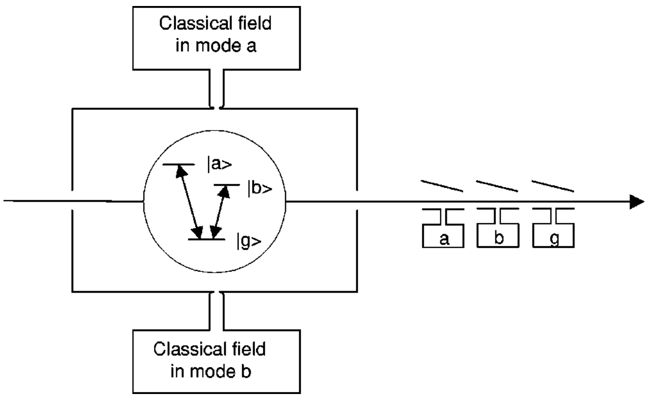

FIG. 1. Atomic interaction with an entangled two-mode field in a cavity.

magnitude larger than the atom-field interaction time [20]. After the interaction, the atoms are detected in one of the atomic energy eigenstates by state-selective field-ionization techniques. We show that the probability of the atom being in its initial ground state is directly related to the two-mode Wigner characteristic function [21].

We derive the atomic evolution in a driven two-mode cavity in Sec. II. We show the relation between the ground-state population density of the atom and the characteristic function for the two-mode Wigner function. We next extend this result to the case of quantum-state reconstruction for entanglement of two separate cavities in Sec. III.

II. ENTANGLEMENT IN A CAVITY

We prepare a high-Q cavity with a two-mode entangled state given by the density operator $\hat{\rho}_F$. Now we perform a displacement of the initial state in phase space by applying a unitary transformation

$$\hat{\rho}_F(\alpha,\beta)=\hat{D}_a(\alpha)\hat{D}_b(\beta)\hat{\rho}\hat{D}_b^\dagger(\beta)\hat{D}_b^\dagger(\alpha)$$

with the displacement operator

$$D_a(\alpha)=\exp(\alpha\hat{a}^\dagger-\alpha^*\hat{a});\quad D_b(\beta)=\exp(\beta\hat{b}^\dagger-\beta^*\hat{b}). \tag{3}$$

Here $\hat{a}(\hat{b})$ and $\hat{a}^\dagger(\hat{b}^\dagger)$ are the annihilation and creation operators of the field mode a (b) and $\alpha(\beta)$ is a complex number characterizing the amplitude and phase of the displacement. For a single-mode micromaser experiment, the displacement of the cavity field is carried out by coupling a resonant classical oscillator to the cavity field [22].

We inject a V-configuration three-level atom with two excited states $|a\rangle$ and $|b\rangle$ coupled to the common ground state $|g\rangle$. The atom interacts with a two-mode field in a perfect cavity. A field mode of the annihilation operator $\hat{a}$ is resonant with the $|a\rangle\leftrightarrow|g\rangle$ transition and the other field mode of the annihilation operator $\hat{b}$ is resonant with the $|b\rangle\leftrightarrow|g\rangle$ transition. The schematic representation of the atom-cavity interaction is sketched in Fig. 1. Under the rotating-wave approximation, the Hamiltonian in the interaction picture is

$$\hat{H}=\hbar\kappa_a(\hat{a}|a\rangle\langle g|+\hat{a}^\dagger|g\rangle\langle a|)+\hbar\kappa_b(\hat{b}|b\rangle\langle g|+\hat{b}^\dagger|g\rangle\langle b|), \tag{4}$$

where κ_a and κ_b are the coupling constants, respectively, for the $|a\rangle\leftrightarrow|g\rangle$ and $|b\rangle\leftrightarrow|g\rangle$ transitions.

The density operator for the combined atom-field system follows a unitary time evolution generated by the time evolution operator, $\hat{U}(t)=\exp(-i\hat{H}t/\hbar)$. The evolution operator has been studied for a Λ-configuration three-level atom interacting with a two-mode field [23,19]. Before the atom-field interaction, the atom is initially prepared in its ground state and the two-mode entangled state is displaced in the cavity. The initial atom-field density operator, thus, is

$$\hat{\rho}(0)=\hat{\rho}_F(\alpha,\beta)\otimes|g\rangle\langle g|. \tag{5}$$

The probability P_g of the atom being in its ground state after the interaction time t is then

$$P_g=\mathrm{Tr}_F\langle g|\hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t)|g\rangle, \tag{6}$$

where Tr_F is the trace over the field variables. Using the evolution operator $\hat{U}(t)$ for three-level systems [23,19] we find the result

$$P_g=\mathrm{Tr}[\rho_F(\alpha,\beta)\cos^2\sqrt{\kappa_a^2\hat{a}\hat{a}^\dagger+\kappa_b^2\hat{b}\hat{b}^\dagger}t]=\mathrm{Tr}_F[\hat{\rho}_F\cos^2\hat{\Theta}], \tag{7}$$

where the argument operator of the cosine function is

$$\begin{aligned}\hat{\Theta}&=\hat{D}_a^\dagger(\alpha)\hat{D}_b^\dagger(\beta)\sqrt{\kappa_a^2\hat{a}\hat{a}^\dagger+\kappa_b^2\hat{b}\hat{b}^\dagger}t\hat{D}_b(\beta)\hat{D}_a(\alpha)\\&=[\kappa_a^2(\hat{a}\hat{a}^\dagger+|\alpha|^2)+\kappa_b^2(\hat{b}\hat{b}^\dagger+|\beta|^2)+\kappa_a^2(\alpha^*\hat{a}+\alpha\hat{a}^\dagger)\\&\quad+\kappa_b^2(\beta^*\hat{b}+\beta\hat{b}^\dagger)]^{1/2}.\end{aligned} \tag{8}$$

When the displacements $\alpha=0$ and $\beta=0$, the probability P_g depends only on the energy distribution of the cavity field. However, with the displacement $\alpha\neq 0$ and $\beta\neq 0$, the atomic inversion may carry information on phase, i.e., off-diagonal elements of the density operator $\hat{\rho}$, as well.

One of the important ingredients of our reconstruction scheme is that the driving field has to be much stronger than the cavity field. It is normally true that photon statistics of the *strongly driven* cavity field is near Poissonian and its photon-number distribution has a dominant maximum at its mean photon number. Under this condition the Rabi oscillations in atomic inversion show collapses and revivals [24]. If we restrict ourselves to the atomic interaction before the first revival time, we can further approximate the argument operator $\hat{\Theta}$. In this regime, the argument of the cosine function in Eq. (8) is approximated by

$$\begin{aligned}\hat{\Theta}&\approx\sqrt{\kappa_a^2|\alpha|^2+\kappa_b^2|\beta|^2}t+\frac{\kappa_a^2}{2\sqrt{\kappa_a^2|\alpha|^2+\kappa_b^2|\beta|^2}}(\alpha^*\hat{a}+\alpha\hat{a}^\dagger)\\&\quad+\frac{\kappa_b^2}{2\sqrt{\kappa_a^2|\alpha|^2+\kappa_b^2|\beta|^2}}(\beta^*\hat{b}+\beta\hat{b}^\dagger).\end{aligned} \tag{9}$$

The first term on the right-hand-side of Eq. (9) is the Rabi frequency. Substituting the argument operator $\hat{\Theta}$ back to Eq. (7), we find the probability of the atom being in the ground state in the form

$$P_g(t) \approx \frac{1}{2} + \frac{1}{4}[e^{2i\Omega t}\,\mathrm{Tr}_F\,\hat{\rho}_F \hat{D}_a(\mu_a)\hat{D}_b(\mu_b) + \text{c.c.}], \tag{10}$$

where c.c. stands for the complex conjugate and the parameters are defined as

$$\Omega = \sqrt{\kappa_a^2|\alpha|^2 + \kappa_b^2|\beta|^2}, \tag{11}$$

$$\mu_a = \frac{i\kappa_a^2\alpha}{\sqrt{\kappa_a^2|\alpha|^2 + \kappa_b^2|\beta|^2}},$$

$$\mu_b = \frac{i\kappa_b^2\beta}{\sqrt{\kappa_a^2|\alpha|^2 + \kappa_b^2|\beta|^2}}.$$

Here, we can see that the probability P_g has a direct relation with the characteristic function

$$C_W(\eta,\xi) = \mathrm{Tr}[\hat{\rho}_F \hat{D}_a(\eta)\hat{D}_b(\xi)] \tag{12}$$

for the two-mode Wigner function [21] of the original cavity field.

Equations (10) and (12) show that experimentally measured data $1-2P_g$ are directly related to the two-mode Wigner characteristic function $C_W(\eta,\xi)$. A similar result was obtained for the quantum-state reconstruction of a single-mode cavity field [9]. However, it is not a mere extension of the single-mode case as we note that the measurement of the atom being in the ground state gives full information on *entanglement* of two modes. Measurement of phase information for a single mode is possible because of the initial displacement of the field but the measurement of the two-mode entanglement is not obvious. A measurement of atomic coherences is not needed but the ground-state population is enough to measure the entanglement. This is due to the fact that the ground state can be populated by either the $|a\rangle \to |g\rangle$ or $|b\rangle \to |g\rangle$ transition and the ground-state population reflects the interference between the $|a\rangle \to |g\rangle$ and $|b\rangle \to |g\rangle$ transitions. If the atom had a cascade configuration of the ground, intermediate, and excited energy states, the ground-state population would not give all the information on entanglement as the ground state is populated only by the transition from the intermediate state.

The phases of the driving fields determine which axes of the characteristic function the measured data refer to. The probability P_g is related to the real part of the characteristic function for $\Omega t = n\pi$ and the imaginary part for $\Omega t = (n + \frac{1}{4})\pi$, where $n = 0,1,2,\ldots$. By the Fourier transformation of the measured data we obtain the two-mode Wigner function. Measuring the characteristic function is important also because it gives all the statistical information of a quantum state and the density matrix of the state. Extending the single-mode operator identity [25], we obtain two-mode density-matrix elements in Fock basis:

$$\rho_{mnm'n'} = \frac{1}{\pi^2}\int d^2\eta \int d^2\xi\, C_W(\eta,\xi) \times {}_a\langle m|\hat{D}_a(-\eta)|n\rangle_{ab}\langle m'|\hat{D}_b(-\xi)|n'\rangle_b, \tag{13}$$

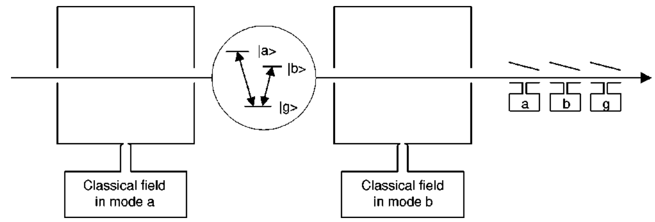

FIG. 2. A *V*-configuration three-level atom interacts with two cavities sequentially. Each cavity is a single-mode cavity. The first cavity field resonantly excites the $|g\rangle \leftrightarrow |a\rangle$ transition and the second excites the $|g\rangle \leftrightarrow |b\rangle$ transition.

where

$$ {}_a\langle m|\hat{D}_a(-\eta)|n\rangle_a = {}_a\langle n|\hat{D}_a(-\eta)|m\rangle_a^* = \sqrt{\frac{m!}{n!}} e^{-|\eta|^2/2}(\eta^*)^{n-m} L_m^{n-m}(|\eta|^2) \tag{14}$$

with the Laguerre polynomial $L_n^{m-n}(|\mu|^2)$.

Let us consider a Bell-type entangled state

$$|\Psi_B\rangle = \epsilon|0\rangle_a|0\rangle_b + \sigma|1\rangle_a|1\rangle_b \tag{15}$$

for which the nonvanishing matrix elements are

$$\rho_{0000} = |\epsilon|^2, \quad \rho_{0101} = \epsilon\sigma^*, \quad \rho_{1010} = \epsilon^*\sigma, \quad \rho_{1111} = |\sigma|^2. \tag{16}$$

Differences between the classical statistical mixture state and the entangled state appear due to the nonvanishing off-diagonal terms ρ_{0101} and ρ_{1010}, which can be obtained as the weighted integral of the characteristic function

$$\rho_{0101} = \frac{1}{\pi^2}\int d^2\eta \int d^2\xi\, C_W(\eta,\xi)\,\eta^*\xi^* \times \exp[-\tfrac{1}{2}(|\eta|^2 + |\xi|^2)] \tag{17}$$

which has been derived using Eqs. (13) and (14). Here the characteristic function is the measured data and the simple weighted integral gives the value of the density-matrix element.

Similar results can be derived for other types of states such as the coherent entangled state

$$|\Psi_A\rangle = \epsilon|\zeta\rangle_a|\upsilon\rangle_b + \sigma|\upsilon\rangle_a|\zeta\rangle_b, \tag{18}$$

where $|\zeta\rangle_a$ and $|\upsilon\rangle_b$ are coherent states.

III. ENTANGLEMENT OF TWO CAVITIES

We now consider the case of two spatially separate single-mode cavities that are entangled. After producing the entangled state of the density matrix $\hat{\rho}_F$, we couple the cavities with strong classical fields to displace the cavity fields. The displaced cavity fields in the two cavities will be represented by the density operator $\hat{\rho}_F(\alpha,\beta)$ as done earlier for the single cavity case. To reconstruct the quantum state, we send a *V*-configuration three-level atom to interact with the two driven cavities sequentially as shown in Fig. 2. We neglect the loss of cavity fields during the time when the atom flies through both the cavities. We also assume that the atom does not lose its coherence during the flight between two cavities. The $|g\rangle \leftrightarrow |a\rangle$ transition of the atom is resonant with the field mode a of the first cavity and the $|g\rangle \leftrightarrow |b\rangle$ transition is resonant with the field mode b of the second cavity.

We can easily derive the Hamiltonian for atomic interaction in the first and second cavities as we set, respectively, $\kappa_b=0$ and $\kappa_a=0$, for Eq. (4). We then calculate the evolution operators, $\hat{U}_a(t_a)$ and $\hat{U}_b(t_b)$, for atom-field interaction within the two cavities. Assuming the atom initially in its ground state, the density operator $\hat{\rho}(0)$ for the atom-field system is as shown in Eq. (5). After interaction with both the cavities, the probability P_g of the atom being in the ground state is

$$\begin{aligned} P_g &= \mathrm{Tr}_F[\langle g|\hat{U}_b(t_b)\hat{U}_a(t_a)\hat{\rho}(0)\hat{U}_a^\dagger(t_a)\hat{U}_b^\dagger(t_b)|g\rangle] \\ &= \mathrm{Tr}_F[\rho_F(\alpha,\beta)\cos^2(\sqrt{\hat{a}\hat{a}^\dagger}\kappa_a t_a)\cos^2(\sqrt{\hat{b}\hat{b}^\dagger}\kappa_b t_b)]. \end{aligned} \tag{19}$$

This probability can be rearranged as

$$P_g = \frac{1}{4} + \frac{1}{16}P_g^{ab} + \frac{1}{8}P_g^a + \frac{1}{8}P_g^b, \tag{20}$$

where

$$\begin{aligned} P_g^{ab} &= \mathrm{Tr}_F[\hat{\rho}_F(\alpha,\beta)(e^{2i\sqrt{\hat{a}\hat{a}^\dagger}\kappa_a t_a} + e^{-2i\sqrt{\hat{a}\hat{a}^\dagger}\kappa_a t_a}) \\ &\quad \times (e^{2i\sqrt{\hat{b}\hat{b}^\dagger}\kappa_b t_b} + e^{-2i\sqrt{\hat{b}\hat{b}^\dagger}\kappa_b t_b})], \\ P_g^a &= \mathrm{Tr}_F[\hat{\rho}_F(\alpha,\beta)(e^{2i\sqrt{\hat{a}\hat{a}^\dagger}\kappa_a t_a} + e^{-2i\sqrt{\hat{a}\hat{a}^\dagger}\kappa_a t_a})], \\ P_g^b &= \mathrm{Tr}_F[\hat{\rho}_F(\alpha,\beta)(e^{2i\sqrt{\hat{b}\hat{b}^\dagger}\kappa_b t_b} + e^{-2i\sqrt{\hat{b}\hat{b}^\dagger}\kappa_b t_b})]. \end{aligned} \tag{21}$$

In our quantum-state reconstruction scheme, the driving fields are assumed to be strong and the atomic interaction time is shorter than the first revival time. With a similar analysis to the single-cavity case, we can easily see that P_g^{ab} in Eq. (21) is related to the two-mode Wigner characteristic function, P_g^a to the single-mode Wigner characteristic function for the mode a in the first cavity, and P_g^b to the single-mode Wigner characteristic function for the mode b in the second cavity. In fact, P_g^a and P_g^b are the probabilities of the atom being in the ground state as the atom interacts only with the first or the second cavity, respectively. These marginal probabilities P_g^a and P_g^b can be measured in the supplementary experiments and we can get the two-mode Wigner characteristic function as we subtract the contributions of P_g^a and P_g^b from the probability P_g in Eq. (20).

However, for the Bell-type state, Eq. (15), the off-diagonal density-matrix element ρ_{0101} does not require the supplementary experiments of measuring the marginal probabilities because contributions of P_g^a and P_g^b vanish in the weighted integral, Eq. (17), as the weighting is an odd function.

IV. REMARKS

There are some causes of experimental errors. Atoms are sent to the cavity from an opening in a thermal oven and therefore have random Poisson distributed interaction time. Even though the width of the Poisson distribution can be as small as $\Delta t/t = 1\%$ [22], this can cause some errors. The probability of the atom being in the ground state is approximately given by Eq. (10) when the intensity of the driving field is strong. To get the characteristic function (12) from measurements of $P_g(t)$, the interaction times have to be precise. When the driving field is intense, the Rabi frequency is large and even a small fluctuation in the interaction time can be fatal. Thus in the experimental realization, the intensity of the driving field has to be carefully chosen. When the characteristic function is real, the envelope of the Rabi oscillations in $P_g(t)$ is the characteristic function as shown in Eq. (10). For the single-mode quantum-state reconstruction the error caused by the fluctuation of the interaction time has been considered by Kim *et al.* [9]. Similar conclusions apply to the two-mode problem.

In this paper we have been interested in reconstruction of small-amplitude quantum states. When an atom interacts with the strongly driven quantum state, there appear collapses and revivals in atomic inversion. The approximated probability $P_g(t)$ is correct only before the first revival so that we study the ground-state probabilities $P_g(t)$ before the revival and Fourier-transform them to get the Wigner function. The revival time depends on the intensity of the driving field. In the strong driving-field limit, as the collapse time is long we can collect enough experimental data to get the Wigner function or density-matrix elements. Atoms may pass the channeltron detectors without having been detected, in which case we have to restart the experiment. Once the atom is measured, the chance for the measurement to be wrong is negligible so the detection efficiency should not be an important obstacle.

Quantum entanglement is at the heart of current developments of quantum information theory. In this paper we suggested schemes for reconstructing the entangled states. We have considered the reconstruction schemes for the entanglement of two modes in a cavity and in two spatially separate cavities. We have shown that the probability of the V-configuration three-level atom being in its ground state is directly related to the two-mode Wigner characteristic function. The two-mode Wigner function and the density-matrix elements can be obtained from the characteristic function. We add that our emphasis has been on the entangled states though the formula (10) holds for all states of a two-mode field, for example, it would apply to the important case of two-mode squeezed vacuum in which the mode-mode correlation is also important.

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