

# Nonclassical Interference Effects In The Radiation From Coherently Driven Uncorrelated Atoms

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We demonstrate the existence of new nonclassical correlations in the radiation of two atoms, which are coherently driven by a continuous laser source. The photon-photon-correlations of the fluorescence light show a spatial interference pattern not present in a classical treatment. A feature of the new phenomenon is, that bunched and antibunched light is emitted in different spatial directions. The calculations are performed analytically. It is pointed out, that the correlations are induced by state reduction due to the measurement process when the detection of the photons does not distinguish between the atoms. It is interesting to note, that the phenomena show up even without any interatomic interaction.

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Resonance fluorescence from a single atom driven by a coherent field was the first example allowing to observe nonclassical effects such as antibunching and subpoissonian statistics [1–3]. These experiments first done with atoms in a beam have later also been performed using single trapped ions [4,5]. Squeezing in resonance fluorescence has also been investigated [6]. Since the early work of Mollow [7], Carmichael and Walls [8] one has studied at length the quantum statistical characteristics of radiation produced by a cooperative system of two and more atoms [9,10]. Some evidence of cooperative effects have been demonstrated [11]. There are also some recent proposals to investigate cooperative effects including the interactions between the atoms [12,13].

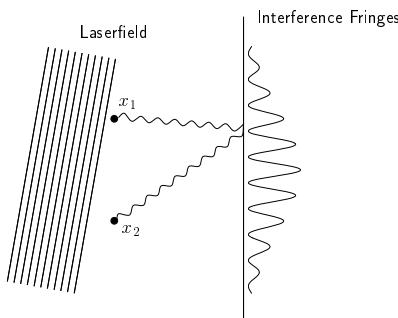


FIG. 1. Two atoms in a coherent laser field show interference fringes

In the present paper we study the two atom fluorescence leading to new nonclassical effects. We consider a situation, where the two atoms are driven coherently by a continuous laser source. These nonclassical effects are observed in photon-photon-correlations when state reduction occurs in the measurement process. We note, that in an earlier study by Mandel [14] correlation effects in a similar system have been discussed. However, in contrast to our paper there no continuously pumped atoms have been considered. We calculate nonclassical two photon correlations for all times including dissipation of the atoms. The dynamics resulting from the excitation by cw fields and by spontaneous emission is very important for the present results. We should mention that on the experimental side first order interference in the radiation produced by a prepared system of two atoms has been reported [15]. We also point out, that the creation of entangled states of distant atoms has been discussed in a different context [16].

It may be noted, that the trapped ion technology is fairly well advanced to envisage resonance fluorescence measurements with chains of trapped ions driven by coherent fields. One can also use single molecules in crystalline hosts, where recently nonclassical photon statistics have been measured [17].

We consider two identical atoms with the levels  $|e\rangle_i$  and  $|g\rangle_i$  ( $i = 1, 2$ ) at fixed positions  $\mathbf{x}_1$  and  $\mathbf{x}_2$  with dipole moment  $\mathbf{d}$  and transition frequency  $\omega$ . They are driven by a resonant external laser field with wavevector  $\mathbf{k}_L$ . We assume that the only dissipative terms are due to the spontaneous decays of the levels  $|e\rangle_{1,2}$ . In the rotating-wave, Markov and Born approximations, the time evolution of the system is given by the master equation [18]

$$\dot{\rho} = -i\Omega \sum_{\mu=1}^2 \left[ e^{[i(\mathbf{k}_L \cdot \mathbf{x}_\mu - \omega_L t)]} \sigma_\mu^+ + \text{h.c.}, \rho \right] - \sum_{\mu=1}^2 \gamma (\sigma_\mu^+ \sigma_\mu^- \rho + \rho \sigma_\mu^+ \sigma_\mu^- - 2\sigma_\mu^- \rho \sigma_\mu^+), \quad (1)$$

where  $2\Omega$  is the Rabi frequency of the atom laser system,  $\sigma_\mu^\pm$  are the atomic raising and lowering operators for atom number  $\mu$ , and  $2\gamma = \frac{4}{3}|\mathbf{d}|^2 \frac{\omega^3}{\hbar c^3}$  is the Einstein A coefficient for the single atom. As we assume the electromagnetic field outside the laser beam in the vacuum

state, the correlation functions can be written as

$$G^{(1)}(\mathbf{r}, t) = \sum_{\mu, \nu=1}^2 e^{[ik\mathbf{x}_{\mu, \nu} \cdot \hat{\mathbf{r}}]} \langle \sigma_{\mu}^{+}(t) \sigma_{\nu}^{-}(t) \rangle \quad (2)$$

$$G^{(2)}(\mathbf{r}_1, t; \mathbf{r}_2, t + \tau) = \sum_{\mu, \nu, \lambda, \rho=1}^2 e^{[ik(\mathbf{x}_{\lambda, \nu} \cdot \hat{\mathbf{r}}_1 + \mathbf{x}_{\rho, \mu} \cdot \hat{\mathbf{r}}_2)]} \times \langle \sigma_{\lambda}^{+}(t) \sigma_{\rho}^{+}(t + \tau) \sigma_{\mu}^{-}(t + \tau) \sigma_{\nu}^{-}(t) \rangle \quad (3)$$

with  $\mathbf{x}_{\mu, \nu} := \mathbf{x}_{\mu} - \mathbf{x}_{\nu}$  and  $\hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|}$ .

The two time correlations (3) can be calculated using the quantum regression theorem and the time evolution of the density operator. Therefore the normalized second order correlation function can be written as

$$g^{(2)}(\mathbf{r}_1, t; \mathbf{r}_2, t + \tau) = \frac{P(\mathbf{r}_2, t + \tau | \mathbf{r}_1, t)}{P(\mathbf{r}_2, t)} \quad (4)$$

where  $P(\mathbf{r}, t)$  is the probability of finding a photon at position  $\mathbf{r}$  at time  $t$ , and  $P(\mathbf{r}_2, t + \tau | \mathbf{r}_1, t)$  the conditional probability of finding a photon at position  $\mathbf{r}_2$  at time  $t + \tau$  premising the detection of a photon at point  $\mathbf{r}_1$  at time  $t$ . Using the inequality

$$\text{Tr}(\rho(\alpha I(\mathbf{r}_1) + \beta I(\mathbf{r}_2))^2) \geq 0 \quad \forall \alpha, \beta \quad (5)$$

leads us to

$$\prod_{i=1}^2 (g^{(2)}(\mathbf{r}_i; \mathbf{r}_i) - 1) \geq (g^{(2)}(\mathbf{r}_1; \mathbf{r}_2) - 1)^2 \quad (6)$$

for classical systems. This gives us a good possibility to estimate the nonclassical behavior of our system.

As there can always be found one state which does not interact with the laser field it is suggestive to use this state to build up the basis for the further calculations. We call this state  $|a\rangle$  and define

$$\begin{aligned} |e\rangle &:= |e, e\rangle \\ |s\rangle &:= \frac{1}{\sqrt{2}} (e^{-i\phi} |e, g\rangle + e^{i\phi} |g, e\rangle) \\ |a\rangle &:= \frac{1}{\sqrt{2}} (e^{-i\phi} |e, g\rangle - e^{i\phi} |g, e\rangle) \\ |g\rangle &:= |g, g\rangle \end{aligned} \quad (7)$$

with  $|i, j\rangle := |i\rangle_1 \otimes |j\rangle_2$ ,  $\phi := \frac{1}{2} \mathbf{k}_L \cdot \mathbf{x}_{12}$

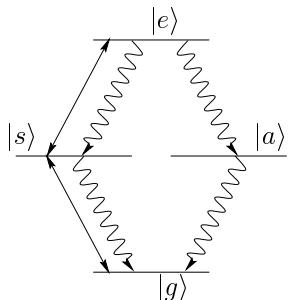


FIG. 2. Level scheme of the atomic system for the symmetrized set of states

In this representation the master equation reduces to the following sets of 6+3 equations

$$\begin{aligned} \dot{\rho}_{ee} &= 4(\alpha \rho_{es}^i - \rho_{ee}) \\ \dot{\rho}_{ss} &= 2(\rho_{ee} - \rho_{ss} + 2\alpha(\rho_{sg}^i - \rho_{es}^i)) \\ \dot{\rho}_{aa} &= 2(\rho_{ee} - \rho_{aa}) \\ \dot{\rho}_{es}^i &= -3\rho_{es}^i - 2\alpha(\rho_{ee} - \rho_{ss} + \rho_{eg}^r) \\ \dot{\rho}_{sg}^i &= 2\rho_{es}^i - \rho_{sg}^i + 2\alpha(1 - \rho_{ee} - \rho_{aa} - 2\rho_{ss} + \rho_{eg}^r) \\ \dot{\rho}_{eg}^r &= -2(\rho_{eg}^r + \alpha(\rho_{sg}^i - \rho_{es}^i)) \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{\rho}_{ea}^r &= -3\rho_{ea}^r - 2\alpha\rho_{sa}^i \\ \dot{\rho}_{sa}^i &= 2(\alpha(\rho_{ea}^r + \rho_{ag}^r) - \rho_{sa}^i) \\ \dot{\rho}_{ag}^r &= -2\rho_{ea}^r - 2\alpha\rho_{sa}^i - \rho_{ag}^r \end{aligned} \quad (9)$$

with  $\alpha := \frac{\Omega}{\sqrt{2}\gamma}$ ,  $\rho_{kl}^r := \Re(\rho_{kl})$  and  $\rho_{kl}^i := \Im(\rho_{kl})$ .

By solving the first set of equations (8) we find the diagonal elements in the steady-state to be:

$$\rho_{gg}^{SS} = \frac{(\gamma^2 + \Omega^2)^2}{(\gamma^2 + 2\Omega^2)^2} \quad (10)$$

$$\rho_{ss}^{SS} = \frac{\Omega^2 [2\gamma^2 + \Omega^2]}{(\gamma^2 + 2\Omega^2)^2} \quad (11)$$

$$\rho_{aa}^{SS} = \rho_{ee}^{SS} = \frac{\Omega^4}{(\gamma^2 + 2\Omega^2)^2} \quad (12)$$

This calculation does not depend on the direction of the driving laser, although it is included by the proper definition of our symmetric and antisymmetric state.

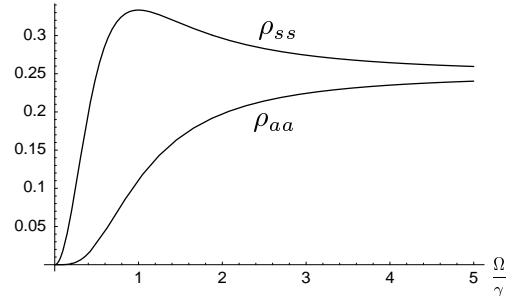


FIG. 3. Steady state populations of the symmetric and antisymmetric state as a function of the laser intensity  $\Omega$

If the laser direction is perpendicular to the ion separation ( $\phi = 0$ ) this solution corresponds to the earlier found solutions of Richter [10], where we neglect the dipole-dipole interaction. This is justified when the ion distance is in the range of several wavelengths of the atomic transition. In this case the solutions are completely independent of the separation of the atoms. For strong laser fields ( $\Omega \gg \gamma$ ) the populations are equal for each of the

atomic states ( $\rho_{gg}^{SS} = \rho_{ss}^{SS} = \rho_{ee}^{SS} = \rho_{aa}^{SS} = 1/4$ ).

We next discuss, how a detection event leads to state reduction and entanglement. After a detection at the point  $\mathbf{r}$  we have to find a new density operator in the following way

$$\rho(\mathbf{r}) = \frac{\sigma^-(\mathbf{r})\rho^{SS}\sigma^+(\mathbf{r})}{\langle\sigma^+(\mathbf{r})\sigma^-(\mathbf{r})\rangle}, \quad \text{where} \quad (13)$$

$$\sigma^\pm(\mathbf{r}) := \sigma_1^\pm + e^{i\mathbf{k}_L \mathbf{r} \cdot \mathbf{x}_{12}} \sigma_2^\pm. \quad (14)$$

If, for instance, the system is in state  $|e\rangle$  before a detection at  $\mathbf{r}$  with  $\hat{\mathbf{r}} \cdot \mathbf{x}_{12} = 0$ , we find it in  $|s\rangle$  after the detection. This explains, how entanglement could come into the system, as the system is transformed from a non-entangled state  $|e\rangle$  to an entangled state  $|s\rangle$  by a detection event.

Let us now check, if the atoms remain uncorrelated after a detection event i.e.  $\rho(\mathbf{r})$  factorizes into a product of density matrices  $\rho_i$  with  $\rho_i$  referring to the density matrix of the  $i$ -th atom. We find that this is not the case, for example

$$\Im(\rho(\mathbf{r})_{sa}) = \frac{\Omega^2 \sin \phi}{2(\Omega^2 + \gamma^2(1 + \cos \delta(\mathbf{r})))}, \quad \rho_{ee}(\mathbf{r}) = 0 \quad (15)$$

with  $\delta(\mathbf{r}) = (\mathbf{k}_L - k_L \hat{\mathbf{r}}) \cdot \mathbf{x}_{12}$ . On the other hand a factorized density matrix would imply,  $\Im(\rho(\mathbf{r})_{sa}) = 0$  after any detection event. Furthermore, the dependence of  $\Im(\rho(\mathbf{r})_{sa})$  on the coordinates of the two atoms through  $\delta(\mathbf{r})$  also points out the nonexistence of a factorized density matrix after detection. Thus there are values of  $\mathbf{r}$ , where the state reduction by the measurement leaves the system in an entangled state. With this mechanism the entanglement, which is necessary for nonclassical effects, is brought into the system simply by the detection of a single photon.

To calculate the intensity at position  $\mathbf{r}$  we use the relations

$$\langle \sigma_1^+ \sigma_1^- \rangle = \langle \sigma_2^+ \sigma_2^- \rangle = \rho_{ee} + \frac{1}{2} (\rho_{ss} + \rho_{aa}) \quad (16)$$

$$\langle \sigma_1^+ \sigma_2^- \rangle = \langle \sigma_2^+ \sigma_1^- \rangle^* = \frac{1}{2} e^{2i\phi} (\rho_{ss} - \rho_{aa}), \quad (17)$$

where we assumed the system to be in the steady state (i.e.  $\rho_{sa}^{SS} = 0$ ). We then find

$$G^{(1)}(\mathbf{r}, t) = (1 + \cos \delta(\mathbf{r})) [\rho_{ee}(t) + \rho_{ss}(t)] + (1 - \cos \delta(\mathbf{r})) [\rho_{ee}(t) + \rho_{aa}(t)]. \quad (18)$$

This leads to the well known interference fringes in the first order correlation function as studied in the experiment of Eichmann et al. [15].

In this case the system behaves as the well known double slit experiment where the atoms act as slits and the two optical paths interfere. Like in a system with prepared initial states the photons emitted by symmetric transitions  $|e\rangle \rightarrow |s\rangle$  and  $|s\rangle \rightarrow |g\rangle$  show inverse fringes compared to those emitted by antisymmetric transitions  $|e\rangle \rightarrow |a\rangle$  or  $|a\rangle \rightarrow |g\rangle$ . This follows from the  $1 \pm \cos \delta(\mathbf{r})$  terms in (18). One major difference is that the contrast depends on the intensity of the laser light so that the fringes disappear for higher laser intensities when  $\Omega \gg \gamma$ . The populations  $\rho_{ss}^{SS}$  and  $\rho_{aa}^{SS}$  equalize.

To get the second order correlation function we solve the master equation by calculating the Liouville Operator and use for the initial state the density matrix given by (13). This procedure is also equivalent to using the quantum regression theorem.

Remarkably enough we are able to give an analytical expression for the intensity-intensity correlation:

$$\begin{aligned} g^{(2)}(\mathbf{r}_1, 0; \mathbf{r}_2, \tau) = 1 + & \frac{e^{-3t}}{4\nu^2(s + \cos \delta_1)(s + \cos \delta_2)} \times \\ & \{ 4e^{2t} \nu^2 s \sin \delta_1 \sin \delta_2 + s [e^{t} \nu^2 s + (s - 1)^2] \cos \delta_1 \cos \delta_2 \\ & - e^{3t/2} \nu s^2 [2\nu \cos(\nu t) + 3 \sin(\nu t)] \\ & + 2e^{3t/2} \nu s (\cos \delta_1 + \cos \delta_2) [(2s - 3) \sin(\nu t) - 2\nu \cos(\nu t)] \\ & + e^{t/2} \nu (2e^t \cos \delta_1 \cos \delta_2 + s \sin \delta_1 \sin \delta_2) \\ & [2\nu(s - 2) \cos(\nu t) + (5s - 6) \sin(\nu t)] \\ & + \frac{1}{4} \cos \delta_1 \cos \delta_2 [(s(s(4s - 33) + 64) - 36) \cos(2\nu t) \\ & + 2\nu(s - 2)(5s - 6) \sin(2\nu t)] \} \end{aligned} \quad (19)$$

with  $s := 2 \left( \frac{\Omega}{\gamma} \right)^2 + 1$ ,  $\delta_i = \delta(\mathbf{r}_i)$  and  $\nu := \frac{1}{2} \sqrt{8s - 9}$  being parameters for the intensity of the driving field and  $t := \gamma \tau$  is the time scaled with the atoms spontaneous emission rate. For  $\tau = 0$  Eq. (19) reduces to

$$g^{(2)}(\mathbf{r}_1, 0; \mathbf{r}_2, 0) = \frac{s^2 \cos^2 \left( \frac{\delta_1 - \delta_2}{2} \right)}{(s + \cos \delta_1)(s + \cos \delta_2)} \quad (20)$$

Assuming measurements with only one detector at position  $\mathbf{r}$ ,  $g^{(2)}(\mathbf{r}, 0; \mathbf{r}, 0)$  shows interferences fringes with maxima larger and minima smaller than one (Fig 4). This means that the system is emitting super-poissonian light in one and sub-poissonian light in the other direction. This behavior has no analogy in the first order correlation i.e. in the two slit experiment.

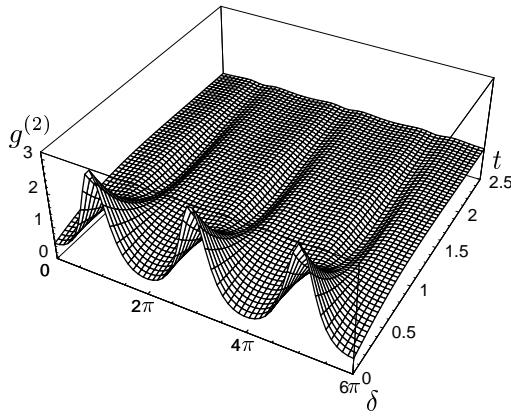


FIG. 4. Second order correlation function for one detector with  $\delta_1 = \delta_2 = \delta$ ,  $\Omega = 0.8\gamma$

For two detectors the situation is different. For any detector position with  $\delta_2 = (2n+1)\pi + \delta_1$ ,  $g^{(2)}(\mathbf{r}_1, 0; \mathbf{r}_2, 0)$  vanishes completely, while at other positions we find maximum detection probability (Fig 5). After a detection at position  $\mathbf{r}_1$  with  $\delta_1 = 2n\pi$  we find  $\rho_{ee} = \rho_{aa} = 0$ . Therefore there is no probability to detect any photon at a position with  $\delta_2 = (2n+1)\pi$  at the same time, as no emission into the antisymmetric channel could take place (18). So we again find fringes as a function of  $\delta_2$ . For  $\delta_1 = (2n+1)\pi$  we detect an emission on the antisymmetric channel first, so we find  $g^{(2)}(\mathbf{r}_1, 0; \mathbf{r}_2, 0)$  vanishing for positions  $\mathbf{r}_2$  with  $\delta_2 = 2n\pi$  for the same reasons as above (Fig 6).

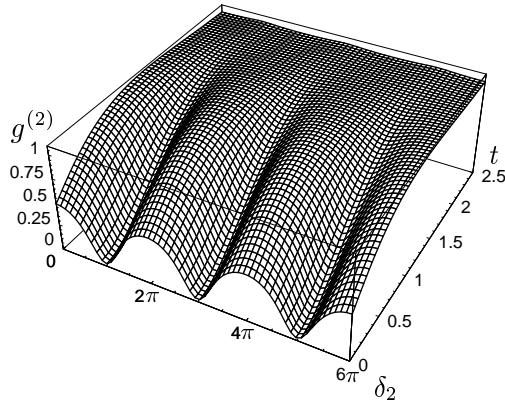


FIG. 5. Second order correlation function for two detectors with  $\delta_1 = 2n\pi$ ,  $\Omega = 0.8\gamma$

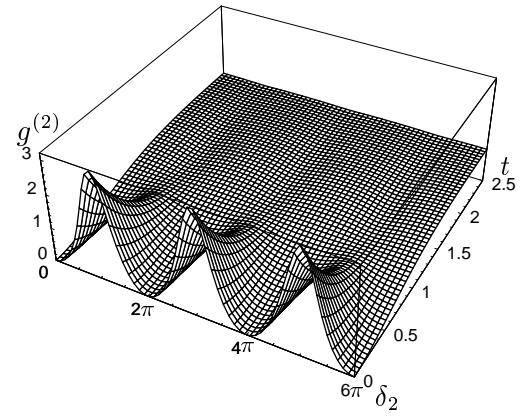


FIG. 6. Second order correlation function for two detectors with  $\delta_1 = (2n+1)\pi$ ,  $\Omega = 0.8\gamma$

After any detection on a specific channel the system needs time for re-excitation to emit to the orthogonal channel; on the other hand cascade emissions could only take place at the same channel. Note that the behavior of  $g^{(2)}(\mathbf{r}_1, 0; \mathbf{r}_2, \tau)$  is well understood in terms of our symmetrized basis (7). These results show also that the **inequality** (6) could be violated by choosing the two detector positions in a way that  $g^{(2)}(\mathbf{r}_1, 0; \mathbf{r}_1, 0) > 1$  and  $g^{(2)}(\mathbf{r}_2, 0; \mathbf{r}_2, 0) < 1$ . In this manner we find nonclassical behavior in our system as consequence of the detection induced entanglement of the two atoms. We note that dependence of the type  $\cos^2$  in the two photon correlations has also been derived by Mandel in his work on radiation from atoms prepared in atomic coherent states [14]. In contrast to that we are dealing here with the more general case of continuous excitation of the atoms including dissipation.

In conclusion it has been shown by analytic calculations, that a system of two atoms coherently excited by a cw laser source shows nonclassical features being dependent on the distance of the atoms, although there is no assumed interaction between the atoms. This can be understood by measurement induced entanglement, which comes into the system if the detection does not distinguish between the two atoms, i.e. the detected photon does not carry which-way-information. On the first glance the system studied seems to be similar to the two slit experiment, however the nonclassical effects discussed here do not show up in the first order correlation and can only be seen in the second order correlation as it should be the case for a quantum phenomenon.

To see the reported effects single ions stored in a linear rf-trap can be used; remaining micromotion could be overcome by a phase-sensitive detection [19]. In this paper the case was discussed, that the detector/or detectors do not discriminate the light emitted by the individual atoms. There are, however, further interesting

phenomena observable when the which-way-information is available. One example is that the excitation of one atom and the selective observation of the fluorescence from the other one opens, among other effects, the possibility to investigate the dipole-dipole interaction between the atoms. These results will be described elsewhere.

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