

Tunneling and Traversal of Ultra-cold Atoms through Vacuum Induced Potentials

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We examine the passage of ultra-cold two-level atoms through the potential produced by the vacuum of the cavity field. The peak of the transmitted wave packet generally occurs at the instant given by the expression for phase time even if that instant is earlier than the instant at which incident wave packet's peak enters the cavity and thus the phase time can be considered as the appropriate measure of the time required for the atom to traverse the cavity. We show that the phase tunneling time for ultra-cold atoms could be both super - and sub - classical time and we show how this behaviour can be understood in terms of the momentum dependence of the phase of transmission amplitude. The passage of the atom through the cavity is unique as it involves a coherent addition of the transition amplitudes corresponding to both barrier and well. New features such as the splitting of the wave packet arise from the entanglement between the center of mass motion and the electronic as well as field states in the cavity.

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I. INTRODUCTION

An important question of great interest in several disciplines of physics has been - what is the tunneling time or traversal time of a quantum mechanical particle through a potential. Various definitions have been proposed and the subject has been reviewed extensively [1–5]. Although there exists no unique definition for the tunneling time, an important experiment by Steinberg et al [6] supports the belief that tunneling time is consistent with the Wigner's phase time [7] defined as the frequency derivative of the transmission amplitude phase. Steinberg et al observed that tunneling velocity of a single photon wave packet passing through the forbidden midgap region of a photonic band-gap material is superluminal.

In this paper, we examine the passage of a cold atom through a high quality cavity. In particular, we enquire what is the passage time of the atom through the cavity. The question is a complicated one as we have here a coupling with three different types of the degrees of freedom - (a) atom's center of mass motion, (b) atom's electronic states and (c) photons. Besides in a high quality cavity, the resonant transitions are especially important. Thus in the passage through the cavity the atom can change its electronic state. We have found that the passage time can be defined through the complex transmission amplitude. The connection to potential problems is provided by the existing results in the context of micromazors [8,9]. An analysis in the the dressed state basis reveals that the interaction of a moving atom with a single mode vacuum field in a high quality cavity is equivalent to a combination of a potential barrier and a well. These potentials belong to the category of vacuum induced potentials and should be distinguished from the optical potentials produced by a far off resonant field interacting with an atom [10]. Having realized that the cavity field can act like a potential for an ultra-cold atom, one could calculate the time the atom takes to traverse the cavity using methods similar to those used say in the context of tunneling electrons through potential barriers.

The organization of the paper is as follows. In Sec. II, we formulate the model describing the propagation of ultra-cold atoms through a high quality cavity. In Sec. III, we calculate the phase time for ultra-cold atoms. We find that the phase time could also be negative, which is reminiscent of superluminal propagation of electromagnetic fields [11]. We explain this characteristic of phase time in terms of the dispersion of the phase of transmission amplitude. This is again very much analogous to the propagation of light which can be understood in terms of the dispersion of the medium. In Sec. IV, we present the numerical results for the time dependence of the wave packet. We show the correlation of the peaks of wave packets to the phase time of Sec. III. In Sec. V, we discuss aspects of phase time where the effects of the entanglement with the internal degrees of freedom is important. Finally, we summarize our results in Sec. VI.

II. MODEL SYSTEM AND SUMMARY OF ATOM - FIELD INTERACTION

We consider an ultra-cold, two-level atom in its excited state to be incident on a single mode cavity of length L as shown in Fig. 1. The frequency of the cavity field has been tuned to the frequency of the atomic transition between the excited state $|e\rangle$ and the ground state $|g\rangle$. The Hamiltonian describing this resonant atom-field interaction including the quantized motion of center-of-mass (c.m.) of the atom, is given by

$$H = \frac{p_z^2}{2m} + \hbar\omega_e |e\rangle\langle e| + \hbar\omega_g |g\rangle\langle g| + \hbar\omega a^\dagger a + \hbar g u(z)(\sigma a^\dagger + a\sigma^\dagger), \quad (1)$$

where g is the atom-field coupling constant and $\omega = \omega_e - \omega_g$ is the resonance frequency of atom-field interaction. The operators a (a^\dagger) annihilate (create) a photon of frequency ω and σ (σ^\dagger) are the lowering (raising) operator for the atomic transition. For simplicity, we approximate the mode function $u(z)$ of the cavity by a mesa function.

In a reference frame rotating with frequency ω the Hamiltonian of the atom-field interaction reads as

$$H_I = \frac{p_z^2}{2m} + \hbar g u(z)(\sigma a^\dagger + a\sigma^\dagger). \quad (2)$$

The operator $(\sigma a^\dagger + a\sigma^\dagger)$ is easily diagonalizable. It has eigenstates $|\phi^0\rangle$, $|\phi_{n+1}^\pm\rangle$ with eigenvalues 0 , $\pm\sqrt{n+1}$, respectively. The dressed eigenstates can be expanded in terms of eigenstates of the free Hamiltonian as $|\phi^0\rangle = |g, 0\rangle$ and $|\phi_{n+1}^\pm\rangle = \frac{1}{\sqrt{2}}(|e, n\rangle \pm |g, n+1\rangle)$. If we expand the combined state of atom-cavity system as

$$|\Psi\rangle = \chi_0(z, t)|\phi^0\rangle + \chi_+(z, t)|\phi_{n+1}^+\rangle + \chi_-(z, t)|\phi_{n+1}^-\rangle, \quad (3)$$

then the time dependent Schrodinger equation becomes

$$i\hbar \frac{\partial \chi_\alpha}{\partial t} = h_\alpha \chi_\alpha, \quad \alpha = \pm, 0. \quad (4)$$

Here $h_o = p_z^2/2m$, $h_\pm = p_z^2/2m \pm \hbar g u(z) \sqrt{n+1}$ are operators acting in the space of the center of mass variables. Clearly the cavity with fixed number of photons creates a barrier and a well potential for the external motion of the atom corresponding to the dressed states $|\phi_{n+1}^\pm\rangle$ respectively as discussed in Ref. [8]. For the mesa mode function $u(z) = \theta(z)\theta(L-z)$, the cavity induced potentials are displayed in Fig. (2).

Since we need the transmission amplitude of the excited atom for further discussion we summarize the main results of Meyer et al [9]. Consider the initial atom-field state to be $|e, n\rangle$, i.e., the atom is in the excited state and the cavity field contains n photons. This initial atom-field state can be expanded in terms of dressed states as

$$|e, n\rangle = \frac{1}{\sqrt{2}} [|\phi_{n+1}^+\rangle + |\phi_{n+1}^-\rangle]. \quad (5)$$

From the above discussions, it is clear that the external motion of the excited atom experiences a coherent addition of a potential barrier and a well with potential energy $V = \hbar g \sqrt{n+1}$. We expand the initial wavepacket of the atom incident on the cavity as $\psi(z, 0) = \int dk A(k) e^{ikz} \theta(-z)$. The Heaviside's step function $\theta(-z)$ merely reflects the fact atomic wavepacket enters the cavity from the left side. The fourier amplitudes $A(k)$ are adjusted such that the center of the wave packet enters the cavity at time $t = 0$. The initial wave function of the atom-field system is therefore

$$|\Psi(z, 0)\rangle = \psi(z, 0) |e, n\rangle. \quad (6)$$

The wave function of the atom-field system after the atom has left the cavity is given by using Eq. (5)

$$\begin{aligned} |\Psi(z, t)\rangle &= \exp\left(\frac{-iH_I t}{\hbar}\right) \psi(z, 0) |e, n\rangle \\ &= \frac{1}{\sqrt{2}} \left[\exp\left(\frac{-iH_I t}{\hbar}\right) |\phi_{n+1}^+\rangle \psi(z, 0) + \exp\left(\frac{-iH_I t}{\hbar}\right) |\phi_{n+1}^-\rangle \psi(z, 0) \right]. \end{aligned} \quad (7)$$

It is to be noted that the first and second term in the above equation corresponds to the atom interacting with the potential barrier and well respectively. Denoting the reflection and transmission amplitudes as ρ_n^\pm , τ_n^\pm for the potential barrier (superscript +) and well (superscript -) respectively, we have

$$\rho_n^\pm = i\Delta_n^\pm \sin(k_n^\pm L) \exp(ikL) \tau_n^\pm, \quad (8)$$

$$\tau_n^\pm = \exp(-ikL) [\cos(k_n^\pm L) - i\Sigma_n^\pm \sin(k_n^\pm L)]^{-1}, \quad (9)$$

$$\begin{aligned} \Delta_n^\pm &= \frac{1}{2} \left(\frac{k_n^\pm}{k} - \frac{k}{k_n^\pm} \right), \\ \Sigma_n^\pm &= \frac{1}{2} \left(\frac{k_n^\pm}{k} + \frac{k}{k_n^\pm} \right), \end{aligned} \quad (10)$$

$$\begin{aligned} k_n^\pm &= \sqrt{\left(k^2 \mp \frac{2mg}{\hbar} \sqrt{n+1}\right)} \\ &= \sqrt{\left(k^2 \mp k_o^2 \sqrt{n+1}\right)}. \end{aligned} \quad (11)$$

Here $\hbar k$ is the atomic c.m. momentum and $\hbar^2 k_o^2/2m = \hbar g$ is the vacuum coupling energy. Carrying out the time evolution for the dressed states, we get the following wave function after the atom - field interaction :

$$\begin{aligned} |\Psi(z, t)\rangle &= \int dk A(k) e^{-i(\hbar k^2/2m)t} \{ [R_{e,n}(k) e^{-ikz} \theta(-z) + T_{e,n}(k) e^{ikz} \theta(z-L)] |e, n\rangle \\ &\quad + [R_{g,n+1}(k) e^{-ikz} \theta(-z) + T_{g,n+1}(k) e^{ikz} \theta(z-L)] |g, n+1\rangle \}, \end{aligned} \quad (12)$$

where

$$R_{e,n} = \frac{1}{2}(\rho_n^+ + \rho_n^-), \quad T_{e,n} = \frac{1}{2}(\tau_n^+ + \tau_n^-), \quad (13)$$

are the reflection and transmission amplitudes for the excited state of the atom and

$$R_{g,n+1} = \frac{1}{2}(\rho_n^+ - \rho_n^-), \quad T_{g,n+1} = \frac{1}{2}(\tau_n^+ - \tau_n^-). \quad (14)$$

are the reflection and transmission amplitudes for the ground state of the atom. Note that all the probability amplitudes for the excited or ground state of the atom depends on the coherent addition of amplitudes of the barrier and well. We have recently shown that transmission of an ultra-cold two-level atom in the excited state through two successive cavities depends strongly on this **coherent** addition of amplitudes. This leads to the splitting of the transmission resonances of the single cavity [12].

III. PHASE TIME FOR ULTRA-COLD ATOMS PASSING THROUGH A HIGH QUALITY CAVITY - ANALOG OF SUB - SUPERLUMINAL PROPAGATION

In the previous section, we have seen that dynamics of an ultra-cold atom passing through the cavity is reduced to the problem of reflection and transmission of an atom incident on the cavity induced potentials. In this section, we study in detail the transmission of the atom in the initial excited state through the cavity initially in vacuum state. The transmitted part of the atom - field system is then given by using Eq. (12)

$$|\Psi(z, t)\rangle = \int_{-\infty}^{\infty} dk A(k) e^{-i(\hbar k^2/2m)t} T_{e,0}(k) e^{ikz} \theta(z - L) |e, 0\rangle. \quad (15)$$

The transmission amplitude $T_{e,0} \equiv |T_{e,0}|e^{i\phi(k)}$ given by Eq. (13) depends on the vacuum coupling energy $\hbar g$. We consider a Gaussian wave packet $A(k) = \exp\left(-\frac{(k - \bar{k})^2}{\sigma^2}\right)$ of width σ and mean momentum \bar{k} for the incident atom. With this substitution for $A(k)$, the wave function including the normalization factor, is given by

$$|\Psi(z, t)\rangle = \frac{1}{(2\pi)^{3/4}} \sqrt{\frac{2}{\sigma}} \int_{-\infty}^{\infty} dk \exp\left(-\frac{(k - \bar{k})^2}{\sigma^2}\right) e^{-i(\hbar k^2/2m)t} |T_{e,0}| e^{i\phi(k)} e^{ikz} |e, 0\rangle, \quad z \geq L. \quad (16)$$

For small width σ the integrand in Eq. (16) has non vanishing value only in a small range of wave numbers k centered about the mean \bar{k} . Then, the envelope of the transmitted wave packet $|\langle e, 0|\Psi(z, t)\rangle|^2$ will be maximum when the total phase $\theta(k)$ of the integrand exhibits extremum at the wave number $k = \bar{k}$. Since we have assumed that the center of incident wave packet enters the cavity at time $t = 0$, this stationary phase condition gives the time at which the wave packet at the exit of the cavity $z = L$, is peaked as follows :

$$\left. \frac{\partial \theta(k)}{\partial k} \right|_{k=\bar{k}} = \left. \frac{\partial}{\partial k} [kL + \phi(k) - (\hbar k^2/2m)t] \right|_{k=\bar{k}} = 0, \quad (17)$$

which yields the phase tunneling time t_{ph}

$$t_{ph} = \left[\frac{m}{\hbar k} \left(\frac{\partial \phi}{\partial k} + L \right) \right]_{k=\bar{k}}. \quad (18)$$

The integral in Eq. (16) can be evaluated approximately by making Taylor expansion of the phase of transmission amplitude about the mean wave number $k = \bar{k}$. Keeping terms upto second order in the expansion and assuming $\sigma \ll \bar{k}$ to approximate $|T_{e,0}(k)| \approx |T_{e,0}(\bar{k})|$, the transmitted wave function is given at $z = L$ by

$$\begin{aligned} |\Psi(z, t)\rangle_{z=L} &\approx \frac{1}{(2\pi)^{3/4}} \sqrt{\frac{2}{\sigma}} \exp(i(\bar{k}L + \phi(\bar{k}) - \bar{E}t/\hbar)) |T_{e,0}(\bar{k})| \\ &\times \sqrt{\frac{2\pi}{\left(\frac{2}{\sigma^2} + i\alpha\right)}} \exp\left(\frac{-\bar{E}(t - t_{ph})^2}{m\left(\frac{2}{\sigma^2} + i\alpha\right)}\right) |e, 0\rangle, \end{aligned} \quad (19)$$

where $\bar{E} = \hbar^2 \bar{k}^2/2m$ is the average energy of the incident atom and the parameter $\alpha = \left. \frac{\hbar t}{m} - \frac{\partial^2 \phi}{\partial k^2} \right|_{k=\bar{k}}$ accounts for the spreading of the wave packet as it propagates. The maximum amplitude of the transmitted wave packet occurs

at time $t = t_{ph}$ given by the stationary phase assumption. It is very important to note that the phase time has no significance when either the Taylor expansion of the phase does not converge or additional terms more than the second order term are important in the expansion. In this general case, the transmitted wave packet will be deformed from the Gaussian shape and the concept of following the peak of the wave packet is meaningless. When there is no cavity $|T_{e,0}(k)| = 1$, $\phi(k) = 0$, then the phase time in Eq. (18) becomes $t_{ph} = mL/\hbar\bar{k} \equiv t_{cl}$ which is the classical time needed for the center of a free atomic wave packet to traverse a distance of length L . The phase tunneling time which a particle takes to traverse a **potential barrier**, has been studied extensively by Hartmann [2]. The tunneling time for a barrier is less than the time a free particle takes to traverse the same distance in free space. Here, we report such a superclassical traversal of the ultra-cold atom through the vacuum induced potentials. Note that the temperature of the atom will be in the range 10^{-7} to 10^{-8} K if the coupling constant g is in the range of 100-10 kHz and if the mean momentum $\bar{k}/k_o = 0.1$. It should be borne in mind that both barrier and well contribute to the traversal time of ultra-cold atoms. Using Eq. (18), we plot in Fig. 3 the phase time as a function of the mean wave number \bar{k} for the length of the cavity $k_oL = 10\pi$. The important result here is that the phase time exhibits the resonant behaviour of transmission probability and that the phase time is less than the classical time t_{cl} . In a different context viz. in the tunneling time of electrons passing through a finite superlattice, a similar resonant behaviour is found [4]. Another remarkable behaviour of phase time is that it can even be **negative**. Negative phase time implies that the peak of the transmitted wave packet emerges out even before the peak of the incident wave packet enters the interaction region. This can be understood from the interference between the incident wave and the wave that is reflected at the end of the cavity. From Eq. (18), we see that when the derivative of the phase of transmission amplitude is negative and its absolute value is greater than the length L of the cavity, the phase time becomes negative. Put in another way, when the phase function $\phi(k) + kL$ has negative slope, the phase time takes negative values. In Fig. 4, we show the phase time for the parameter $k_oL = \pi/2$. It is seen from the graph that for ultra-cold atoms ($\bar{k}/k_o \ll 1$) the phase time is negative. For fast atoms ($\bar{k}/k_o \gg 1$), the phase time approaches the classical time as the transmission probability becomes closer to unity. The phase time being negative is very similar to the concept of negative group velocity in the case of electro-magnetic pulse propagation. Here, the variation of the refractive index of the medium with respect to the frequency has steep negative slope leading to superluminal propagation [11]. To understand the negative phase time, we have also plotted in Fig. 5 the phase function $\phi(k) + kL$. The graph shows the expected negative slope for ultra-cold atoms.

IV. TIME DEPENDENCE OF THE WAVE PACKET FOR ULTRA-COLD ATOMS

To study the behaviour of actual envelope of the wave function, we evaluate numerically the integral Eq. (16) which describes the propagation of a Gaussian wave packet of an excited atom through the vacuum induced potentials. Garrett and McCumber [13] carried out a similar numerical integration for the electric field amplitude of a Gaussian light pulse passing through an anomalous dispersive medium. In Fig. 6(a), we show the numerical results for the normalized probability density $|\langle e, 0 | \Psi(z, t) \rangle|^2 / \sigma$ at the exit of the cavity $z = L$ as function of the time for the parameters $k_oL = \pi/2$, $\sigma/k_o = 0.01$, $\bar{k}/k_o = 0.1$. The peak of the transmitted wave packet occurs at the time $t/t_{cl} \approx -0.98$ which matches with the phase time in the Fig. 4 for the parameter $\bar{k}/k_o = 0.1$. Thus the wave packet appears to travel backwards in time in the sense of tracing the locus of maximum amplitude. The peak of the transmitted wave packet is formed even before the peak of the incident wave packet enters the cavity. For comparison, we have also plotted the envelope of the wave packet which travels through the same distance of length L in free space. The peak of the free wave packet occurs at the expected classical time. From the graph, we see that for ultra-cold atoms ($\bar{k}/k_o \ll 1$) the propagation of the atom through the cavity is faster than through the free space. In Fig. 6(b), we plot the envelope of the wave function for the parameters $k_oL = \pi/2$, $\sigma/k_o = 0.01$, $\bar{k}/k_o = 10$. For the case of fast atoms ($\bar{k}/k_o \gg 1$) the transmitted wave packet has maximum amplitude at the classical time ($t/t_{cl} \approx 1$) as expected from Fig. 4. Thus, the peak of the transmitted wave packet occurs at the instant given by the expression for phase time Eq. (18) even if that instant is earlier than the instant at which incident wave packet enters the cavity. While this is generally true for a narrow momentum distribution characterized by $\sigma \ll \bar{k}$ of the incident atom, strong deformation of the incident wave packet sometimes makes the phase time meaningless.

V. SPLITTING OF THE WAVE PACKET

We have so far considered only the propagation of the atomic wave packet in the initial excited state. But in a high quality the atom-field interaction leads to photon emission by the excited atom. We can also study the behaviour of the transmitted wave packet $|\langle g, 1 | \Psi(z, t) \rangle|^2$ for the ground state of the atom using Eq. (12). For the parameters of

Fig. 6(a), the phase time for the ground state $t_{ph}/t_{cl} \approx 0.45$ is positive but still a superclassical time. Numerical integration also gives the same time delay for the transmitted wave packet. In Fig. 7, we show the behaviour of the phase time for the wave packet corresponding to transmitted atom in the ground state. This behaviour is to be compared with that of the phase time for the transmission in the excited state (Fig. 4). The two phase times differ considerably for cold atoms. Generally, the difference in phase times for the ground and excited states of the atom results in **splitting** of the incident wave packet into two in the total transmission. But for the parameters of Fig. 6(a), the total transmission is dominated by the contribution from the ground state and hence the splitting is not seen. In Fig. 8, we plot the transmitted wave packet for the ground and excited states together for comparison. The graph shows the time delay between the atoms exiting the cavity in excited and ground states.

The **splitting** of the incident wave packet can also occur for a different reason as shown in the Fig. 9 for the parameters $k_o L = 10\pi$, $\sigma/k_o = 0.5$, $\bar{k}/k_o = 10$. It is seen that the probability density is zero at the classical time. This can be understood from the Rabi oscillations between the internal states of the fast atoms. For fast atoms ($k/k_o \gg 1$), the transmission amplitude can be approximated as $T_{e,o}(k) \approx \exp(-ikL)(\exp(ik_0^+L) + \exp(ik_0^-L))/2$ where k_0^\pm are given by Eq. (11). The transmission probability $|T_{e,o}|^2$ exhibits oscillatory behaviour as a function of momentum k of the incident atom. Moreover, at the mean wave number corresponding to the classical time, the transmission amplitude $T_{e,o}(\bar{k}) \approx \cos(gt_{cl}) = 0$. Thus the **correlation with the internal dynamics** (Rabi oscillations) of the atom leads to the splitting of the incident wave packet of the external motion. Obviously since the wave packet is deformed for these parameters, the phase time ($t_{ph}/t_{cl} \approx -0.62$) loses its physical significance and does not represent the peak to peak traversal time.

VI. SUMMARY

We have considered the propagation of a Gaussian wave packet of an excited two-level atom through a high quality cavity which is initially empty. The atom-field interaction is equivalent to the reflection and transmission of the wave packet through the potentials created by the dressed states. This is perhaps one of the rare examples in physics where the tunneling time would depend on the coherent addition of transmission amplitudes through a barrier and a well. The phase tunneling time can exhibit both super- and sub- classical traversal behaviour. For certain set of parameter, the phase tunneling time for cold atoms can even be negative. All this can be understood in terms of the dispersion characteristics of the phase of the transmission amplitude and is analogous to the dispersion of the refractive index which leads to super and subluminal propagation. Numerical integration is performed to calculate the time dependent wave packet at the exit of the cavity. In most cases the peak of the transmitted wave packet occurs at the instant given by the expression for the phase time even when it is negative. Thus the phase time can be considered as the approximate time required for the atom to traverse the cavity. Further, we demonstrate how the correlation between internal and external dynamics of the atom leads to the splitting of the wave packet, a feature which is unique to the traversal of atoms through a cavity. Finally, we note that the vacuum field for the initial state of the cavity does not limit the study of tunneling time of the atom. In a general Fock state, the potential energy of atom-field interaction with the cavity induced potentials will be different from that of vacuum field. Still, we can redefine the atom-field coupling constant of the interaction to include this change and the superclassical tunneling of ultra-cold atoms is a common feature for a general Fock state of the cavity field.

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- [1] R. Y. Chiao and A. M. Steinberg, in **Progress in Optics**, edited by E. Wolf (Elsevier, Amsterdam, 1997), Vol. 37, p. 345 ; E. H. Hauge and J. A. Stovneeng, *Rev. Mod. Phys.* **61**, 917 (1989); R. Landauer and Th. Martin, *Rev. Mod. Phys.* **66**, 217 (1994); V. S. Olkhovsky and E. Recami, *Phys. Reports.* **214**, 340 (1992).
- [2] T. E. Hartmann, *J. Appl. Phys.* **33**, 3427 (1962).
- [3] R. Y. Chiao, *Phys. Rev. A* **48**, R34 (1993); A. M. Steinberg and R. Y. Chiao, *ibid.* **49**, 2071 (1994); E. Bolda, J. C. Garrison and R. Y. Chiao, *ibid.* **49**, 2938 (1994).
- [4] Pedro Pereyra, *Phys. Rev. Lett.* **84**, 1772 (2000).
- [5] H. M. Nussenzveig, *Phys. Rev. A* **62**, 042107 (2000).
- [6] A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, *Phys. Rev. Lett.* **71**, 708 (1993).
- [7] E. P. Wigner, *Phys. Rev.* **98**, 145 (1955).
- [8] B. -G. Englert, J. Schwinger, A. O. Barut, and M. O. Scully, *Europhys. Lett.* **14**, 25 (1991); M. O. Scully, G. M. Meyer and H. Walther, *Phys. Rev. Lett.* **76**, 4144 (1996).
- [9] G. M. Meyer, M. O. Scully, and H. Walther, *Phys. Rev. A* **56**, 4142 (1997); M. Loffler, G. M. Meyer, M. Schroder, M. O. Scully, and H. Walther, *ibid.* **56**, 4153 (1997); M. Schroder, K. Vogel, W. P. Schleich, M. O. Scully, and H. Walther, *ibid.* **56**, 4164 (1997).
- [10] P. S. Jessen and I. H. Deutsch, *Adv. At. Mol. Phys.* **37**, 95 (1996).
- [11] L. J. Wang, A. Kuzmich and A. Dogariu, *Nature* **406**, 277 (2000).
- [12] G. S. Agarwal and R. Arun, *Phys. Rev. Lett.* **84**, 5098 (2000).
- [13] C. G. B. Garrett and D. E. McCumber, *Phys. Rev. A* **1**, 305 (1970).

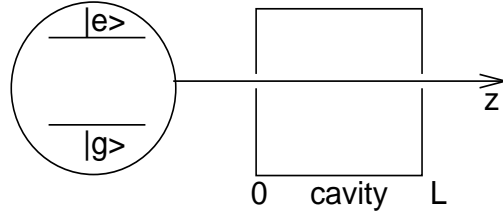


FIG. 1. The scheme of the high quality cavity with which the ultra-cold atom interacts.

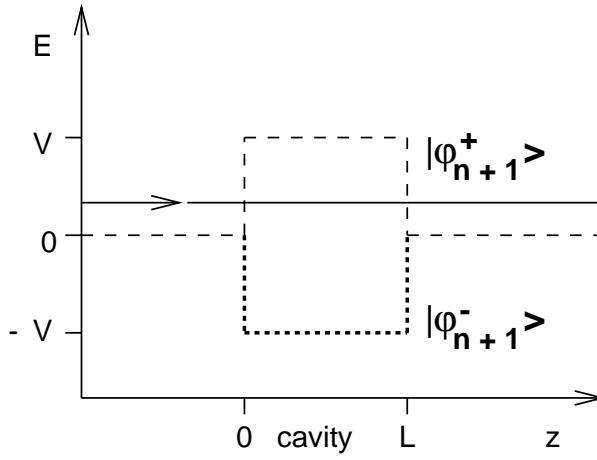


FIG. 2. Schematic representation of the energy E of the excited two-level atom incident upon a single mode cavity with n photons. The interaction is equivalent to reflection and transmission of the atom through a potential barrier (dashed) or potential well (dotted) with a potential energy $V = \hbar g \sqrt{n+1}$. The atom can be reflected or transmitted in either of the states $|e, n\rangle$ and $|g, n+1\rangle$.

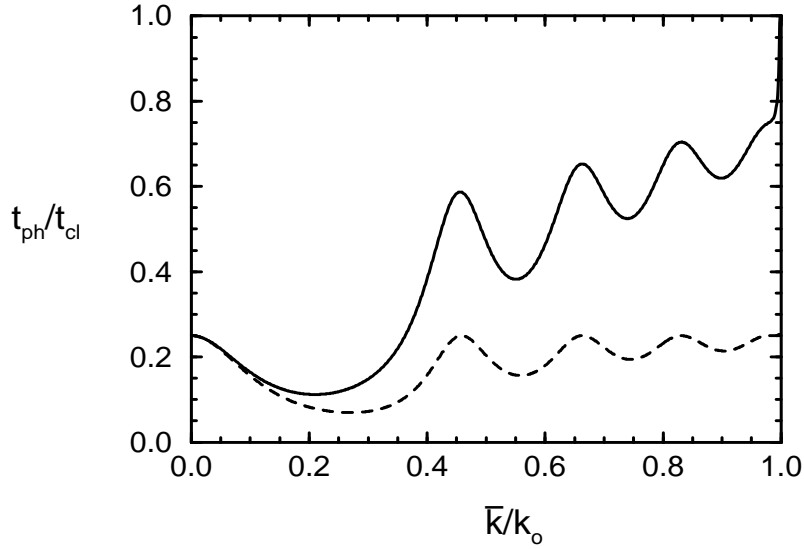


FIG. 3. The dependence of the dimensionless phase time (solid curve) for transmission in the excited state on the mean wave number \bar{k}/k_o of the incident atom for the parameter $k_o L = 10\pi$. The phase time follows the resonant behaviour of the transmission probability $|T_{e,0}|^2$ (dashed curve).

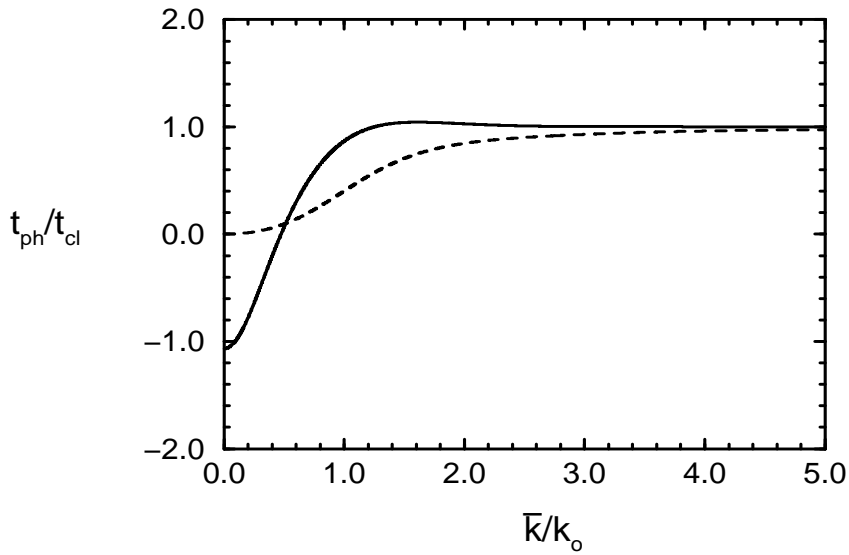


FIG. 4. The dimensionless phase time (solid curve) for transmission in the excited state as a function of the mean wave number \bar{k}/k_o of the incident atom for the parameter $k_o L = \pi/2$. The dashed curve represents the probability of transmission of the atom in the initial excited state ($|T_{e,0}|^2$) through the cavity.

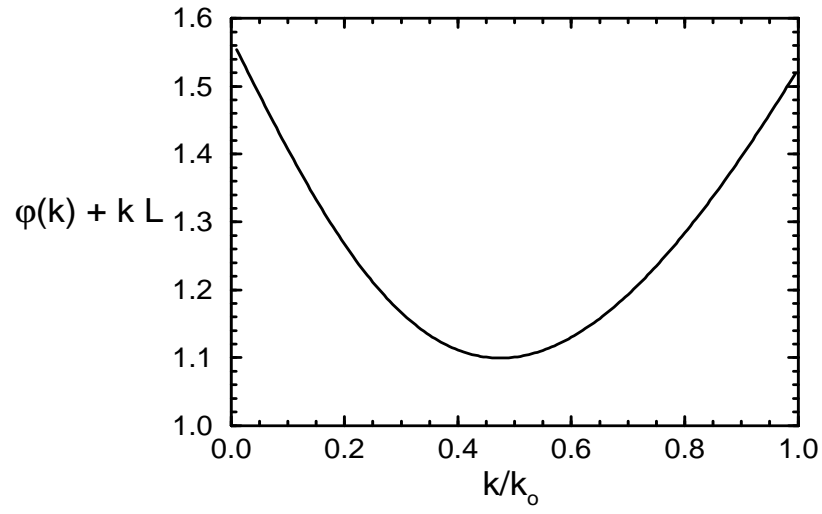


FIG. 5. The phase function $\phi + kL$ as a function of the wave number k/k_0 of the excited atom for the parameter $k_0L = \pi/2$.

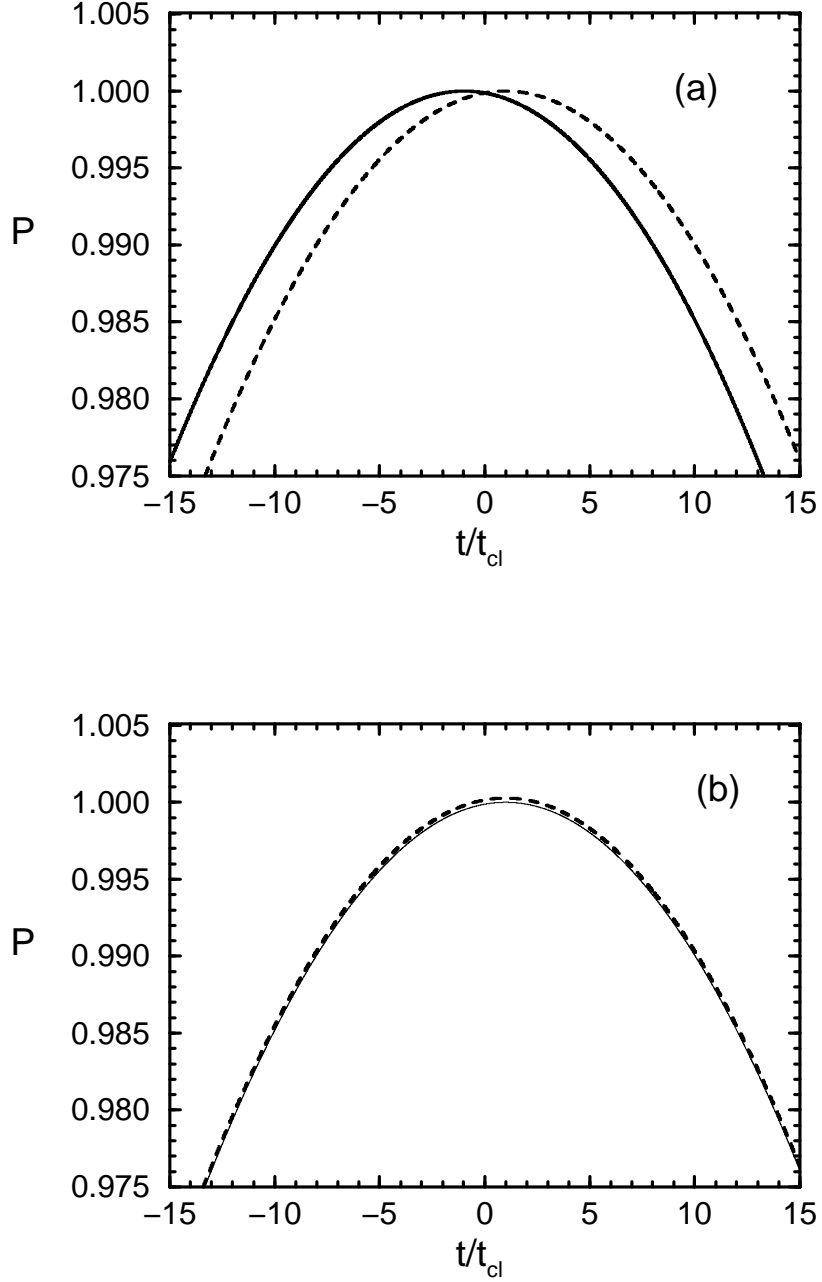


FIG. 6. The normalized probability density $P \equiv |\langle e, 0 | \Psi(z, t) \rangle|^2 / \sigma$ at $z = L$ as a function of dimensionless time t/t_{cl} . The solid (dashed) curve represents P after transmission through the cavity (free space). The parameters used for the calculation are $k_o L = \pi/2$, $\sigma/k_o = 0.01$ and (a) $\bar{k}/k_o = 0.1$, (b) $\bar{k}/k_o = 10$. Both the solid and dashed curves are normalized to unity.

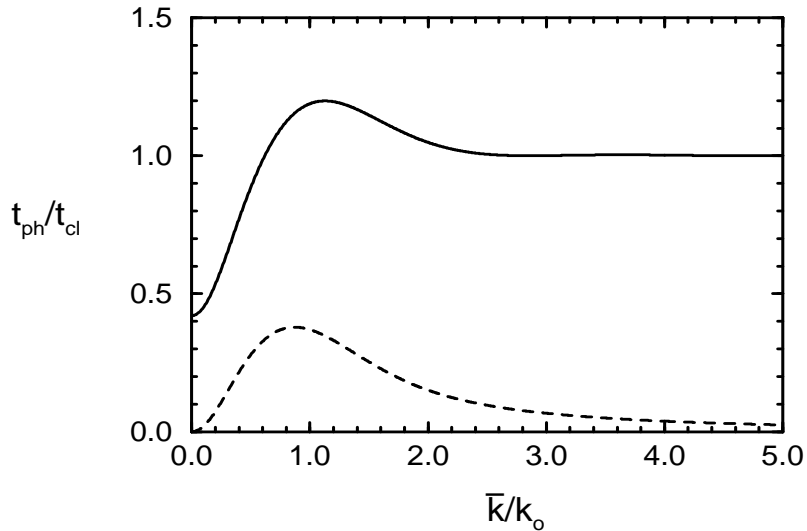


FIG. 7. The dimensionless phase time (solid curve) for transmission in the ground state as a function of the mean wave number \bar{k}/k_o of the incident atom for the parameter $k_o L = \pi/2$. The dashed curve represents the probability of transmission of the atom in the ground state ($|T_{g,1}|^2$) through the cavity.

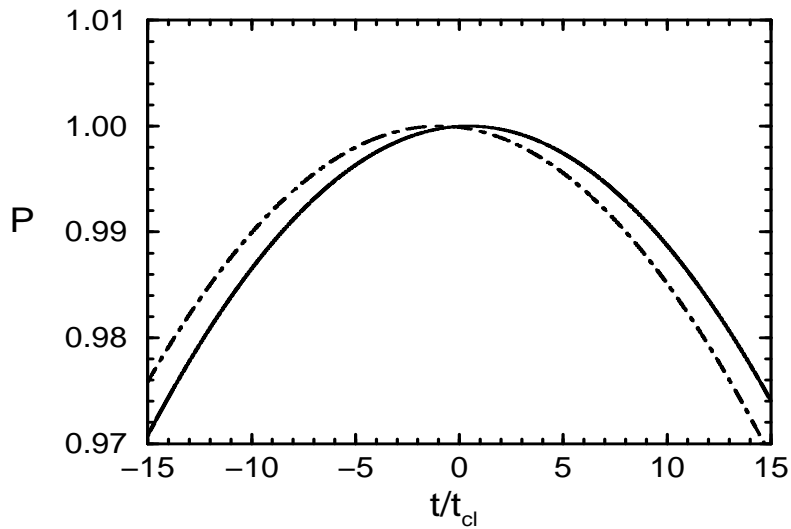


FIG. 8. The normalized probability density $P \equiv |\langle g, 1 | \Psi(z, t) \rangle|^2 / \sigma$ at $z = L$ as a function of dimensionless time t/t_{cl} . The parameters of Fig. 6(a) are used for the calculation and dot-dashed curve corresponds to $P \equiv |\langle e, 0 | \Psi(z, t) \rangle|^2 / \sigma$ at $z = L$. Both the solid and dot-dashed curves are normalized to unity.

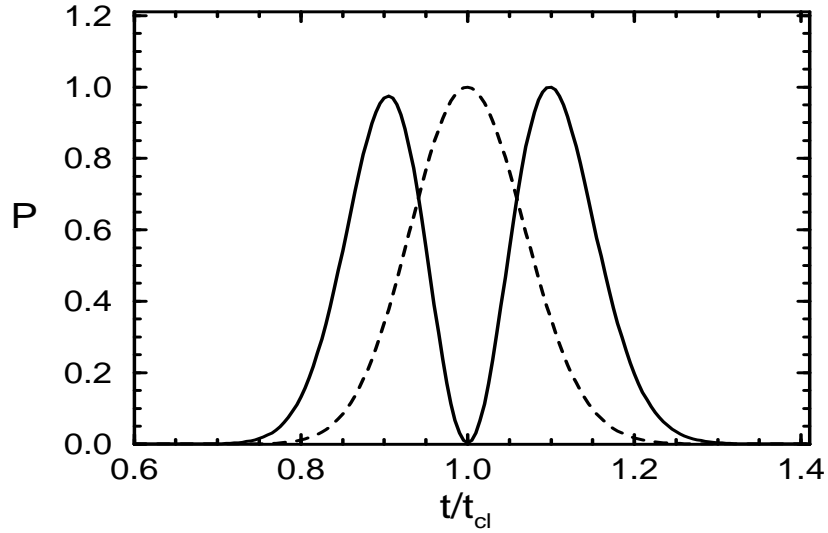


FIG. 9. The normalized probability density $P \equiv |\langle e, 0 | \Psi(z, t) \rangle|^2 / \sigma$ at $z = L$ as a function of dimensionless time t/t_{cl} . The solid (dashed) curve represents P after transmission through the cavity (free space). The parameters used for the calculation are $k_o L = 10\pi$, $\sigma/k_o = 0.5$, and $\bar{k}/k_o = 10$. Both the solid and dashed curves are normalized to unity.