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FUZZY MODELS FOR IMAGE PROCESSING AND APPLICATIONS

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Various uncertainties involved in pattern recognition problems and the relevance of fuzzy set theory in handling them are, first of all, explained. This is followed by different image ambiguity measures based on fuzzy entropy and fuzzy geometry of image subsets, and a discussion on the flexibility in choosing membership functions. Some illustrations of commonly used fuzzy image processing operations are then provided, along with their significance, features and applications to some real life problems, e.g., motion frame analysis, character recognition, IRS image analysis. Most of the algorithms and tools described here were developed by the author with his colleagues. An extensive bibliography is also provided.

Key Words : Pattern Recognition; Vision; Fuzzy Systems; Soft Computing; Image Analysis

1 Introduction

Pattern recognition and machine learning form a major area of research and development that encompasses the processing of pictorial and other non-numerical information obtained from interaction between science, technology and society. A motivation for this spurt of activity in this field is the need for the people to communicate with computing machines in their natural mode of communication. Another important motivation is that scientists are also concerned with the idea of designing and making intelligent machines that can carry out certain tasks as we human beings do. The most salient outcome of these is the concept of future generation computing systems.

Machine recognition of patterns can be viewed as a two-fold task, consisting of learning the invariant and common properties of a set of samples characterizing a class, and of deciding that a new sample is a possible member of the class by noting that it has properties common to those of the set of samples. Therefore, the task of pattern recognition by a computer can be described as a transformation from the measurement space M to the feature space F and finally to the decision space D.

When the input pattern is a gray tone image, some processing tasks such as enhancement, filtering, noise reduction, segmentation, contour extraction and skeleton extraction are performed in the measurement space, in order to extract salient features from the image pattern. This is what is basically known as image processing. The ultimate aim is to make its understanding, recognition and interpretation from the processed information available from the image pattern. Such a complete image recognition/interpretation system is called a vision system which may be viewed as consisting of three levels namely, low level, mid level and high level corresponding to M, F and D with an extent of overlapping among them.

In a pattern recognition or vision system, uncertainty can arise at any phase of the aforesaid tasks resulting in from the incomplete or imprecise input information, the ambiguity/vagueness in input image, the ill-defined and/or overlapping boundaries among the classes or regions, and the indefiniteness in defining/extracting features and relations among them. Any decision taken at a particular level will have an impact on all higher level activities. It is therefore required for a recognition system to have sufficient provision for representing these uncertainties involved at every stage, so that the ultimate output (results) of the system can be associated with the least uncertainty (and not be affected or biased very much by the earlier or lower level decisions).

The present paper describes various fuzzy set theoretic tools and explores their effectiveness in representing/describing various uncertainties that might arise in an image recognition system and the ways these can be managed in making a decision. Some examples of uncertainties that arise often in the process of recognizing a pattern are given in Section 2. Section 3 provides a definition of image, and describes various fuzzy set theoretic tools for measuring information on grayness ambiguity and spatial ambiguity in an image. Concepts of bound functions and spectral fuzzy sets characterising the flexibility in membership functions are discussed in Section 4. Their applications to formulate some low level vision operations (e.g., enhancement, segmentation, skeleton extraction and edge detection), whose outputs are crucial and responsible for the overall performance of a vision system, are then presented in Section 5. Some real life applications (e.g., motion frame analysis, character recognition and remote sensing image analysis) of these methodologies and tools are described in Section 6. Sections 7 and 8 provide the discussion and conclusions respectively.

2 Uncertainties in a Recognition System and Relevance of Fuzzy Set Theory

Some of the uncertainties which one encounters often while designing a pattern recognition or vision[1,2] system will be explained in this section. Let us consider, first of all, the problem of processing and analyzing a gray tone image pattern. A gray tone image possesses some ambiguity within the pixels due to the possible multivalued levels of brightness. This pattern indeterminacy is due to inherent vagueness rather than randomness. The conventional approach to image analysis and recognition consists of segmenting (hard partitioning) the image space into meaningful regions, extracting its different features (e.g. edges, skeletons, centroid of an object), computing the various properties of and relationships among the regions, and interpreting and/or classifying the image. Since the regions in an image are not always crisply defined, uncertainty can arise at every phase of the aforesaid tasks. Any decision taken at a particular level will have an impact on all higher level activities. Therefore, a recognition system (or vision system) should have sufficient provision for representing the uncertainties involved at every stage i.e., in defining image regions, its features and relations among them, and in their matching, so that it retains as much as possible the information content of the original input image for making a decision at the highest level. The ultimate output (result) of the system will then be associated with least uncertainty (and unlike conventional systems it will not be biased or affected very much by the lower level decisions). For example, consider the problem of object extraction from a scene. Now, the question is "How can someone define exactly the target or object region in a scene when its boundary is ill-defined?" Any hard thresholding made for its extraction will propagate the associated uncertainty to the following stages, and this might affect its feature analysis and recognition. Similar is the case with the tasks of contour extraction and skeleton extraction of a region.

From the aforesaid discussion, it becomes therefore convenient, natural and appropriate to avoid committing ourselves to a specific (hard) decision (e.g., segmentation/thresholding, edge detection and skeletonization) by allowing the segments or skeletons or contours to be fuzzy subsets of the image; the subsets being characterized by the possibility (degree) of a pixel belonging to them. Prewitt[3] first suggested that the results of image segmentation should be fuzzy subsets, rather than ordinary subsets. Similarly, for describing and interpreting ill-defined structural information in a pattern, it is natural to define primitives (line, corner, curve etc.) and relations among them using labels of fuzzy sets. For example, primitives which do not lend themselves to precise definition may be defined in terms of arcs with varying grades of membership from 0 to 1 representing its belonging to more than one class. The production rules of a grammar may similarly be fuzzified to account for the fuzziness in physical relation among the primitives; thereby increasing the generative power of a grammar for syntactic recognition[4] of a pattern.

The incertitude in an image pattern may be explained in terms of grayness ambiguity or spatial (geometrical) ambiguity or both. Grayness ambiguity means "indefiniteness" in deciding a pixel as white or black. Spatial ambiguity refers to "indefiniteness" in shape and geometry (e.g., in defining centroid, sharp edge, perfect focusing etc.) of a region. There is another kind of uncertainty which may arise from the subjective judgement of an operator in defining the grades of membership of the object regions. This has been explained in Section 4 in terms of flexibility in membership function.

Let us now consider the problem of determining the boundary or shape of a class from its sampled points or prototypes. There are various approaches[5–7] described in the literature which attempt to provide an exact shape of the pattern class by determining the boundary such that it contains (passes through) some of the sample points. This need not be true. It is necessary to extend the boundaries to some extent to represent the possible uncovered portions by the sampled points. The extended portion should have lower possibility to be in the class than the portions explicitly highlighted by the sample points. The size of the extended regions should also decrease with the increase of the number of sample points. This leads one to define a multivalued or fuzzy (with continuum grade of belonging) boundary of a pattern class[8,9]. Similarly, the uncertainty in classification or clustering of image points or patterns may arise from the overlapping nature of the various classes or image properties. This overlapping may result from fuzziness or randomness. In the conventional classification technique, it is usually assumed that a pattern may belong to only one class, which is not necessarily true. A pattern may have degrees of membership in more than one class. It is, therefore, necessary to convey this information while classifying a pattern or clustering a data set.

In the following section we will be explaining various fuzzy set theoretic tools for image analysis (which were developed based on the realization that many of the basic concepts in pattern analysis, e.g., the concept of an edge or a corner, do not lend themselves to precise definition).

3 Image Ambiguity and Uncertainty Measures

An L level image X ($M\times N$) can be considered as an array of fuzzy singletons, each having a value of membership denoting its degree of possessing some property (e.g., brightness, darkness, edginess, blurredness, texture, etc.). In the notation of fuzzy sets one may therefore write that

$$X=\{\mu_X(x_{mn}):\ m=1,2,\ldots,M;\ n=1,2,\ldots,N\}; \quad \ldots (1)$$

where μ_X (x_{mn}) or, μ_{mn} denotes the grade of possessing such a property μ by the (m, n)th pixel. This property μ of an image may be defined using global information or local information or positional information or a combination of them depending on the problem. Again, the aforesaid information can be used in a number of ways (in their various functional forms), depending on individuals opinion and/or the problem to his hand, to define a requisite membership function for an image property. Basic principles and operations of image processing and pattern recognition in the light of fuzzy set theory are available in reference 10.

Let us now explain the various image information measures (arising from both fuzziness and randomness) and tools, and their relevance to different operations for image processing and analysis. These are classified mainly in two groups, namely grayness ambiguity and spatial ambiguity.

3.1 Grayness Ambiguity Measures

The definitions of some of the measures which were formulated to represent grayness ambiguity in an image X with dimension $M\times N$ and levels L (based on individual pixel as well as a collection of pixels) are listed below.

rth Order Fuzzy Entropy :

$$H^r(X)=(-1/k)\sum_i \{\mu(s_i^r)\log\{\mu(s_i^r)\} + \{1-\mu(s_i^r)\}\log\{1-\mu(s_i^r)\}\} \quad i=1,2,\ldots k \quad \ldots (2)$$

where s_i^r denotes the ith combination (sequence) of r pixels in X; k is the number of such sequences; and $\mu(s_i^r)$ denotes the degree to which the combination s_i^r, as a whole, possesses some image property μ.

Hybrid Entropy:

$$H_{hy}(X)=-P_w\log E_w - P_b\log E_b \quad \ldots (3)$$

with

$$E_w=(1/MN)\sum_m\sum_n \mu_{mn}\exp(1-\mu_{mn})$$

$$E_b=(1/MN)\sum_m\sum_n (1-\mu_{mn})\exp(\mu_{mn}) \quad \ldots (4)$$

$$m=1,2,\ldots M,\ n=1,2,\ldots,N.$$

Here μ_{mn} denotes the degree of "whiteness" of the (m,n)th pixel. P_w and P_b denote probability of occurrences of white (μ_{mn}=1) and black (μ_{mn}=0) pixels respectively; and E_w and E_b denote the average likeliness (possibility) of interpreting a pixel as white and black respectively.

Correlation:

$$C(\mu_1, \mu_2) = 1 - 4\left[\sum_m \sum_n \{\mu_{1mn} - \mu_{2mn}\}^2\right] / (X_1 + X_2)$$
$$= 1 \text{ if } X_1 + X_2 = 0 \qquad \ldots (5)$$

with

$$X_1 = \sum_m \sum_n \{2\mu_{1mn} - 1\}^2$$
$$X_2 = \sum_m \sum_n \{2\mu_{2mn} - 1\}^2 \qquad \ldots (6)$$
$$m=1, 2, \ldots, M : n=1, 2, \ldots N.$$

Here μ_{1mn} and μ_{2mn} denote the degree of possessing the properties μ_1 and μ_2 respectively by the (m,n)th pixel and C (μ_1, μ_2) denotes the correlation between two such properties μ_1 and μ_2 (defined over the same domain).

These expressions eqs. (2-6) are the versions extended to the two dimensional image plane from those defined[11,12] for a fuzzy set. $H^r(X)$ gives a measure of the average amount of difficulty in taking a decision whether any subset of pixels of size r possesses an image property or not. Note that, no probabilistic concept is needed to define it. If r=1, $H^r(X)$ reduces to (non-normalized) entropy as defined by De Luca and Termini[13]. $H_{hy}(X)$, on the other hand, represents an amount of difficulty in deciding whether a pixel possesses a certain property μ_{mn} or not by making a prevision on its probability of occurrence. (It is assumed here that the fuzziness occurs because of the transformation of the complete white (0) and black pixels (1) through a degradation process; thereby modifying their values to lie in the intervals [0, 0.5] and [0.5, 1] respectively). Therefore, if μ_{mn} denotes the fuzzy set 'object region' then the amount of ambiguity in deciding x_{mn} a member of object region is conveyed by the term hybrid entropy depending on its probability of occurrence. In the absence of fuzziness (i.e., with exact defuzzification of the gray pixels to their respective black or white version), H_{hy} reduces to the two state classical entropy of Shannon[14], the states being black and white. Since a fuzzy set is a generalized version of an ordinary set, the entropy of a fuzzy set deserves to be a generalized version of classical entropy by taking into account not only the fuzziness of the set but also the underlying probability structure. In that respect, H_{hy} can be regarded as a generalized entropy such that classical entropy becomes its special case when fuzziness is properly removed.

Note that eqs. (2) and (3) are defined using the concept of logarithmic gain function. Similar expressions using exponential gain function i.e., defining the entropy of an n-state system have been given by Pal and Pal[15–18].

$$H = \sum_i p_i e^{1-p_i}, \; i = 1, 2, \ldots, n. \qquad \ldots (7)$$

All these terms, which give an idea of '*indefiniteness*' or *fuzziness* of an image may be regarded as the measures of average intrinsic information which is received when one has to make a decision (as in pattern analysis) in order to classify the ensembles of patterns described by a fuzzy set.

$H^r(X)$ has the following properties:
Pr 1 : H^r attains a maximum if μ_i=*0.5* for all i.
Pr 2 : H^r attains a minimum if μ_i=0 or 1 for all i.
Pr 3 : $H^r \geq H^{*r}$, where H^{*r} is the rth order entropy of a sharpened version of the fuzzy set (or an image).
Pr 4 : H^r is, in general, not equal to $\overline{H}^r$, where $\overline{H}^r$ is the rth order entropy of the complement set.
Pr 5 : $H^r \leq H^{r+1}$ when all $\mu_i \in$ [0.5, 1].
$H^r \geq H^{r+1}$ when all $\mu_i \in$ [0, 0.5].

The '*sharpened*' or '*intensified*' version of X is such that

$$\mu_{x^*}(x_{mn}) \geq \mu_x(x_{mn}) \quad \text{if } \mu_x(x_{mn}) \geq 0.5$$
$$\text{and} \qquad \ldots (8)$$
$$\mu_{x^*}(x_{mn}) \leq \mu_x(x_{mn}) \quad \text{if } \mu_x(x_{mn}) \leq 0.5.$$

When r=1, the property *Pr* 4 is valid only with the 'equal' sign. The property *Pr* 5 (which does not arise for r=1) implies that H^r is a monotonically non-increasing function of r for $\mu_i \in$ [0, 0.5] and a monotonically nondecreasing function of r for $\mu_i \in$ [0.5, 1] (when the '*min*' operator has been used to get the group membership value). When all μ_i values are the same, $H^1(X)=H^2(X)=\ldots=H^r(X)$. This is because of the fact that the difficulty in taking a decision regarding possession of a property on an individual is the same as that of a group selected therefrom. The value of H^r would, of course, be dependent on the μ_i values.

Again, the higher the similarity among singletons (supports) the quicker is the convergence to the limiting value of H^r. Based on this observation, an index of similarity of supports of a fuzzy set may be defined as $S=H^1/H^2$ (when $H^2=0$, H^1 is also zero and S is taken as 1). Obviously, when $\mu_i \in [0.5, 1]$ and the min operator is used to assign the degree of possession of the property by a collection of supports, S will lie in $[0,1]$ as $H^r \le H^{r+1}$. Similarly, when $\mu_i \in [0, 0.5]$, S may be defined as H^2/H^1 so that S lies in $[0, 1]$. The higher the value of S the more alike (similar) are the supports of the fuzzy set with respect to the fuzzy property μ. This index of similarity can therefore be regarded as a measure of the degree to which the members of a fuzzy set are alike[19].

Therefore, the value of 1st order fuzzy entropy (H^1) can only indicate whether the fuzziness in a set is low or high. In addition to this, the value of H^r, $r > 1$ also enables one to infer whether the fuzzy set contains similar supports (or elements) or not. The similarity index thus defined can be successfully used for measuring inter class and intraclass ambiguity (i.e., class homogeneity and contrast) in pattern recognition and image processing problems.

$H^1(X)$ is regarded as a measure of the average amount of information (about the grey levels of pixels) which has been lost by transforming the classical pattern (two-tone) into a fuzzy (gray) pattern X. Further details on this measure with respect to image processing problems are available in references (10), (20-22). It is to be noted that $H^1(X)$ reduces to zero whenever μ_{mn} is made 0 or 1 for all (m, n), no matter whether the resulting defuzzification (or transforming process) is correct or not. In the following discussion it will be clear how H_{hy} takes care of this situation.

Let us now discuss some of the properties of $H_{hy}(X)$. In the absence of fuzziness when MNP_b pixels become completely black ($\mu_{mn}=0$) and MNP_w pixels become completely while ($\mu_{mn}=1$), then $E_w=P_w$, $E_b=P_b$ and H_{hy} boils down to the two state classical entropy

$$H_c = -P_w \log P_w - P_b \log P_b, \qquad \ldots (9)$$

the states being black and white. Thus H_{hy} reduces to H_c only when a proper defuzzification process is applied to detect (restore) the pixels. $|H_{hy} - H_c|$ can therefore be treated as an objective function for enhancement and noise reduction. The lower the difference, the lesser is the fuzziness associated with the individual symbol and the higher will be the accuracy in classifying them as their original value (white or black). (This property is lacking with the $H^1(X)$ measure and the measure of Xie and Bedrosian[23] which always reduces to zero or some constant value irrespective of the defuzzification process). In other words, $|H_{hy} - H_c|$ represents an amount of information which was lost by transforming a two tone image to a gray tone.

For a given P_w and P_b, ($P_w+P_b=1$, $0 \le P_w$, $P_b \le 1$), of all possible defuzzifications, the proper defuzzification of the image is the one for which H_{hy} is minimum.

If $\mu_{mn}=0.5$ for all (m,n) then $E_w=E_b$
and $H_{hy}=-\log(0.5 \exp 0.5)$ ... (10)
i.e., H_{hy} takes a constant value and becomes independent of P_w and P_b. This is logical in the sense that the machine is unable to make a decision on the pixels since all μ_{mn} values are 0.5.

3.2 Spatial Ambiguity Measures Based on Fuzzy Geometry of Image

Many of the basic geometric properties of and relationships among regions has been generalized to fuzzy subsets. Such an extension, called fuzzy geometry[24–28], includes the topological concept of connectedness, adjacency and surroundedness, convexity, area, perimeter, compactness, height, width, length, breadth, index of area coverage, major axis, minor axis, diameter, extent, elongatedness, adjacency and degree of adjacency. Some of these geometrical properties of a fuzzy digital image subset (characterized by piecewise constant membership function $\mu_X(x_{mn})$ or simply μ) are listed below with illustrations. These may be viewed as providing measures of ambiguity in the geometry (spatial domain) of an image.

Compactness:[24]

$$comp(\mu) = \frac{a(\mu)}{[p(\mu)]^2} \qquad \ldots (11)$$

where

$$a(\mu) = \sum \mu$$

and

$$p(\mu) = \sum_{i,j,k} |\mu(i) - \mu(j)||A(i,j,k)|. \quad \ldots (12)$$

Here, $a(\mu)$ denotes area of μ, and $p(\mu)$, the perimeter of μ, is just the weighted sum of the lengths of the arcs $A\ (i,\ j,\ k)$[24] along which the region $\mu(i)$ and $\mu(j)$ meet, weighted by the absolute difference of these values. Physically, compactness means the fraction of maximum area (that can be encircled by the perimeter) actually occupied by the object. In the non fuzzy case, the value of compactness is maximum for a circle and is equal to $1/4\pi$. In the case of the fuzzy disc, where the membership value is only dependent on its distance from the center, this compactness value is $\geq 1/4\pi$. Of all possible fuzzy discs compactness is therefore minimum for its crisp version.

Height and Width:[24]

$$h(\mu) = \sum_n \max_m \mu_{mn} \quad \ldots (13)$$

and

$$w(\mu) = \sum_m \max_n \mu_{mn} \quad \ldots (14)$$

So, height/width of a digital picture is the sum of the maximum membership values of each row/column.

Length and Breadth[26,27]*:*

$$l(\mu) = \max_m \left(\sum_n \mu_{mn} \right) \quad \ldots (15)$$

and

$$b(\mu) = \max_n \left(\sum_m \mu_{mn} \right). \quad \ldots (16)$$

The length/breadth of an image fuzzy subset gives its longest expansion in the column/row direction. If μ is crisp, μ_{mn}=0 or 1; then length/breadth is the maximum number of pixels in a column/row. Comparing eqs. (15) and (16) with (13) and (14) we notice that the length/breadth takes the summation of the entries in a column/row first and then maximizes over different columns/rows whereas, the height/width maximizes first the entries in a column/row and then sums over different columns/rows.

Index of Area Coverage:[26,27]

$$IOAC(\mu) = \frac{a(\mu)}{l(\mu)b(\mu)} \quad \ldots (17)$$

In the nonfuzzy case, the *IOAC* has value of 1 for a rectangle (placed along the axes of measurement). For a circle this value is $\pi r^2/(2r * 2r) = \pi/4$. *IOAC* of a fuzzy image represents the fraction (which may be improper also) of the maximum area (that can be covered by the length and breadth of the image) actually covered by the image.

Again, note the following relationships,

$$l(X)/h(X) \leq 1$$

and $\quad \ldots (18)$

$$b(X)/w(X) \leq 1.$$

When equality holds for eq. (18) the object is either vertically or horizontally oriented. Similarly, major axis, minor axis, center of gravity and density are also defined in reference 27.

Degree of Adjacency:[27]

The degree to which two crisp regions S and T of an image are adjacent is defined as

$$a(S,T) = \sum_{p \in BP(S)} \frac{1}{1 + |\mu(p) - r(q)|} \times \frac{1}{1 + d(p)} \quad \ldots (19)$$

Here $d(p)$ is the shortest distance between p and q, q is a border pixel (BP) of T and p is a border pixel of S. The other symbols have the same meaning as in the previous discussion.

The degree of adjacency of two regions is maximum (=1) only when they are physically adjacent i.e., $d(p)$=0 and their membership values are also equal i.e., $\mu(p)=r(q)$. If two regions are physically adjacent then their degree of adjacency is determined only by the difference of their membership values. Similarly, if the membership values of two regions are equal their degree of adjacency is determined by their physical distance only. The readers may note the difference between eq. (19) and the adjacency definition[24].

4 Flexibility in Membership Functions

Since the theory of fuzzy sets is a generalization of the classical set theory, it has greater flexibility to capture faithfully the various aspects of incompleteness or imperfection (i.e. deficiencies) in information of a situation. The flexibility of fuzzy set theory is associated with the elasticity property of the concept of its membership function. The grade of membership is a measure of the compatibility of an object with the concept represented by a fuzzy set. The higher the value of membership, the lesser will be the amount (or extent) to which the concept represented by a set needs to be stretched to fit an object.

Since the grade of membership is both subjective and dependent on context, some difficulty of adjudging the membership value still remains. In order words, the problem is how to assess the membership of an element to a set. This is an issue where opinions vary, giving rise to uncertainties. Two operators, namely "*Bound Functions*"[29] and "*Spectral Fuzzy Sets*"[30] have been defined to analyze the flexibility and uncertainty in membership function evaluation. These are explained below along with their significance in image analysis and pattern recognition problems.

Consider, for example, a "*bright image*" which may be considered as a fuzzy set. This is represented by an S-type function which is a nondecreasing function of gray value. Now, the question is, "*can any such nondecreasing function be taken to represent the above fuzzy set?*". Inituitively, the answer is 'no'. Bounds for such an S-type membership function μ have been reported[29] based on the properties of fuzzy correlation[11]. The correlation measure between two membership functions μ_1 and μ_2 relates the variation in their functional values.

The significance of the bound functions in selecting an S-type function μ for image segmentation problem has been reported[31]. It has been shown that for detecting a minimum in the valley region of a histogram, the window length w of the function $\mu : [0, w] \rightarrow [0,1]$ should be less than the distance between two peaks around that valley region. The ability to make the fuzzy set theoretic approach flexible and robust will be demonstrated further in Section 5.

The concept of spectral fuzzy sets is used where, instead of a single unique membership function, a set of functions reflecting various opinions on membership elements is available so that each membership grade is attached to one of these functions. By giving due respect to all the opinions available for further processing, it reduces the difficulty (*ambiguity*) in selecting a single function. A spectral fuzzy subset F having n supports is characterized by a set or a band (spectrum) of r membership functions (reflecting r opinions) and may be represented as

$$F = U_j\,[U_i \mu_F^i (x_j)/x_j], \quad x_j \in \Psi,$$
$$i = 1,2,\ldots,r; \quad j = 1,2,\ldots,n \qquad \ldots (20)$$

where r, the number of membership functions, may be called the cardinality of the opinion set. $\mu_F^i(x_j)$ denotes the degree of belonging of x_j to the set F according to the ith membership function. The various properties and operations related to it have been defined by Pal and Das Gupta[30]. The incertitude or ambiguity associated with this set is two-fold, namely ambiguity in assessing a membership value to an element (d_1) and ambiguity in deciding whether an element can be considered to be a member of the set or not (d_2).

The (dis) similarity between the concept of spectral fuzzy sets and those of the other tools such as probabilistic fuzzy set, interval-valued fuzzy set, fuzzy set of type 2 or ultra fuzzy set[32-36] (which have also considered the difficulty in settling a definite degree of fuzziness or ambiguity) has been explained in reference (30).

The concept has been found to be significantly useful[30] in segmentation of ill-defined regions where the selection of a particular threshold becomes questionable as far as its certainty is concerned. In other words, questions may arise like, "*where is the boundary*" or "*what is the certainty that a level 1, say is a boundary between object and background*". The opinions on these queries may vary from individual to individual because of the differences in opinion in assigning membership values to the various levels. In handling this uncertainty, the algorithm gives due respect to various opinions on membership of gray levels for object region, minimizes the image ambiguity $d(=d_1+d_2)$ over the resulting band of membership functions and then makes a soft decision by providing a set of thresholds (instead of a single one) along with their certainty values. A hard (crisp) decision obviously corresponds to one with maximum d value i.e., the level at which opinions differ most. The problems of edge detection and skeleton extraction (where incertitude arises from ill-defined regions and various opinions on membership values), and any expert system type application (where differences in experts' opinions leads to an uncertainty) may also be similarly handled within this framework.

5 Some Examples of Fuzzy Image Processing Operations

Let us now describe some algorithms to show how the aforesaid information measures and geometrical properties can be incorporated in handling uncertainties in various operations e.g., gray level thresholding, enhancement, contour detection and skeletonization by avoiding hard decisions, and providing output in both fuzzy and nonfuzzy (as a special case) versions. It is to be noted that these low level operations (particularly image segmentation and object extraction) play a major role in an image recognition system. As mentioned in Section 2, any error made in this process might propagate to feature extraction and classification.

5.1 Enhancement in Property Domain

The objective of enhancement techniques is to process a given image so that the result is more suitable than the original for a specific application. The term 'specific' is of course, problem oriented. The techniques used here are based on the modification of pixels in the fuzzy property domain of an image[10,20,21].

The contrast intensification operator on a fuzzy set **A** generates another fuzzy set **A**′=INT(**A**) in which the fuzziness is reduced by increasing the values of $\mu_A(x_{mn})$ which are above 0.5 and decreasing those which are below it. Define this INT operator by a transformation T_1 of the membership function μ_{mn} as

$$\begin{aligned} T_1(\mu_{mn}) &= T_1'(\mu_{mn}) = 2\mu_{mn}^2, \qquad 0 \le \mu_{mn} \le 0.5 \\ &= T_1''(\mu_{mn}) = 1 - 2(1-\mu_{mn})^2, \qquad \ldots (21) \\ & \qquad 0.5 \le \mu_{mn} \le 1 \\ & m=1,2,\ldots M,\ n=1,2,\ldots N \end{aligned}$$

In general, each μ_{mn} in X (eq. 1) may be modified to μ'_{mn} to enhance the image X in the property domain by a transformation function T_r where

$$\begin{aligned} \mu'_{mn} &= T_r(\mu_{mn}) = T_r'(\mu_{mn}), \quad 0 \le \mu_{mn} \le 0.5 \\ &= T_r''(\mu_{mn}), \quad 0.5 \le \mu_{mn} \le 1 \qquad \ldots (22) \\ & r = 1,2,\cdots \end{aligned}$$

The transformation function T_r is defined as successive applications of T_1 by the recursive relationship[20]

$$T_s(\mu_{mn}) = T_1\{T_{s-1}(\mu_{mn})\}, \quad s = 1,2,\cdots \qquad \ldots (23)$$

and $T_1(P_{mn})$ represents the operator INT defined in eq. (21).

As r increases, the enhancement function (curve) in μ_{mn}–μ'_{mn} plane tends to be stepper because of the successive application of INT. In the limiting case, as $r \to \infty, T_r$ produces a two-level (binary) image. It is to be noted here that, corresponding to a particular operation of T', one can use any of the multiple operations of T'', and vice versa, to attain a desired amount of enhancement. Similarly, some other enhancement functions can be used independently instead of these used in eq. (21).

The membership plane μ_{mn} for enhancing contrast around a cross-over point may be obtained from[11,20].

$$\mu_{mn} = G(x_{mn}) = [1 + (|\hat{x} - x_{mn}| / F_d)^{F_e}]^{-1} \qquad \ldots (24)$$

Where the position of cross-over points bandwidth and hence the symmetry of the curve are determined by the fuzzifiers F_e and F_d. When $\hat{x}=x_{\max}$ (maximum level in X), μ_{mn} represents an S type function. When $\hat{x}$=any arbitrary level l, μ_{mn} represents a π type function.

After enhancement in the fuzzy property domain, the enhanced spatial domain x'_{mn} may be obtained from

$$x'_{mn} = G^{-1}(\mu'_{mn}), \alpha \le \mu'_{mn} \le 1 \qquad \ldots (25)$$

where α is the value of μ_{mn} *when* x_{mn}=0.

Note that the aforesaid method provides a basic module of fuzzy enhancement. In practice one may use it with other smoothing, noise cleaning or enhancement operations for resulting in desired outputs. An extension of this concept to enhance the contrast among various ill-defined regions using multiple applications of π and $(1-\pi)$ functions has been described in references (21) and (37) for edge detection of X-ray images. The edge detection operators involve *max* and *min* operations. The article at reference (38) demonstrates, in this regard, an attempt to use a relaxation (iterative) algorithms for fast image enhancement utilizing various orders of S functions; convergence has also been analyzed.

Fuzzy image enhancement technique has also been applied recently by Krell *et al.*[39] for enhancing the quality of images taken by electronic postal imaging device needed by clinicians to verify the shape and the location of 'therapy beam' with respect to the patients anatomy. Other enhancement operators recently developed are available in references (40) and (41).

5.2 Optimum Enhancement Operator Selection

When an image is processed for visual interpretation, it is ultimately up to the viewers to judge its quality for a specific application and how well a particular method works. The process of evaluation of image quality therefore becomes subjective which makes the definition of a well processed image an elusive standard for comparison of algorithm performance. Again, it is customary to have an iterative process with human interaction in order to select an appropriate operator for obtaining the desired processed output. For example, consider the case of contrast enhancement using a nonlinear functional mapping. Not every kind of nonlinear function will produce a desired (meaningful) enhanced version. The questions that automatically arise are "Given an arbitrary image which type of nonlinear functional form will be best suited without prior knowledge on image statistics (e.g., in remote applications like space autonomous operations where frequent human interaction is not possible) for highlighting its object?" and "Knowing the enhancement function how can one quantify the enhancement quality for obtaining the optimal one?". Regarding the first question, even if the image statistics are given, it is possible only to estimate approximately the function required for enhancement and the selection of the exact functional form still needs human interaction in an iterative process. The second question, on the other hand, needs individual judgement which makes the optimum decision subjective.

The method of optimization of the fuzzy geometrical properties and entropy has been found[42] to be successful, when applied on a set of different images, in providing quantitative indices in order to avoid such human iterative interaction in selecting an appropriate nonlinear function and to make the task of subjective evaluation objective.

5.3 Threshold Selection (Fuzzy Segmentation)

Given an L level image X of dimension $M \times N$ with minimum and maximum gray values l_{min} and l_{max} respectively, the algorithm for its fuzzy segmentation into object and background may be described as follows:

Step 1: Construct the membership plane using the standard S function as

$$\mu_{mn} = \mu(l) = S(l;\ a,b,c) \cdot \qquad \ldots (26)$$

or,

$$\mu_{mn} = \mu(l) = 1 - S(l;\ a,b,c) \qquad \ldots (27)$$

(depending on whether the object regions possess higher or lower gray values) with cross-over point b and band width $\Delta b = b - a = c - b$.

Step 2: Compute the parameter $I(X)$, where $I(X)$ represents either grayness ambiguity or spatial ambiguity, as stated in Section 3, or both.

Step 3: Vary b between l_{min} and l_{max} and select those b for which $I(X)$ has local minima or maxima depending on $I(X)$. (Maxima correspond to the correlation measure only.) Among the local minima/maxima, let the global one have cross-over point at s.

The level s, therefore, denotes the cross over point of the fuzzy image plane μ_{mn}, which has minimum grayness and/or geometrical ambiguity. The μ_{mn} plane then can be viewed as a fuzzy segmented version of the image X. For the purpose of nonfuzzy segmentation, we can take s as the threshold (or boundary) for classifying or segmenting an image into object and background.

Faster methods of computation of the fuzzy parameters are explained in reference (27). Note that $w=2\Delta b$ is the length of the window (such that $[0,\ w] \rightarrow [0,1]$) which was shifted over the entire dynamic range. As w decreases, the possibility of detecting some undesirable thresholds (spurious minima) increases because of the smaller value of Δb. On the other hand, an increase in w results in a higher value of fuzziness and thus leads towards the possibility of losing some of the weak minima. The criteria regarding the selection of membership functions and the length of window (i.e., w) have been reported in references (29) and (31) assuming continuous functions for both histogram and membership function. It is shown that, μ should satisfy the bound criteria derived based on the correlation (Section 4). Another way of handling this uncertainty using spectral fuzzy sets for providing a soft decision is explained in reference (30).

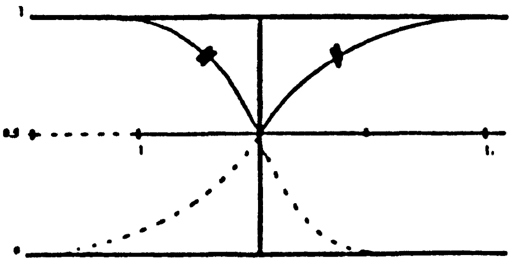

Fig. 1 Inverse π function (solid line) for computing object and background entropy

Let us now describe another way of extracting an object by minimizing higher order entropy (eq. 2) of both object and background regions using an inverse π function as shown by the solid line in Fig. 1. Unlike the previous algorithm, the membership function does not need any parameter selection to control the output.

Suppose s is the assumed threshold so that the gray level ranges $[1,s]$ and $[s+1, L]$ denote, respectively, the object and background of the image X. The inverse π-type function to obtain μ_{mn} values of X is generated by taking union of $S[x; \{s-(L-s)\}, s, L]$ and $1-S[x; 1, s, (s+s-1)]$, where S denotes the standard S function. The resulting function as shown by the solid line, makes μ lie in [0.5, 1]. Since the ambiguity (difficulty) in deciding a level as a member of the object or the background is maximum for the boundary level s, it has been assigned a membership value of 0.5. Ambiguity decreases as the gray value moves away from s on either side. The μ_{mn} thus obtained denotes the degree of belonging of a pixel x_{mn} to either object or background. Since s is not necessarily the mid point of the entire gray scale, the membership function may not be a symmetric one. Therefore, the task of object extraction is to:

Step 1: Compute the rth order fuzzy entropy of the object H_O^r and the background H_B^r considering only the spatially adjacent sequences of pixels present within the object and background respectively. Use the '*min*' operator to get the membership value of a sequence of pixels.

Step 2: Compute the total rth order fuzzy entropy of the partitioned image as $H_s^r = H_O^r + H_B^r$.

Step 3: Minimize H_s^r with respect to s to get the threshold for object background classification.

Referring back to the Section 3.1, it is seen that H^2 reflects the homogeneity among the supports in a set, in a better way than H^1 does. The higher the value of r, the stronger is the validity of this fact. Thus, considering the problem of object-background classification, the improper selection of the threshold is more strongly reflected by H^r than H^{r-1}.

The methods of object extraction (or segmentation) described above are all based on gray level thresholding. Another way of doing this task is by pixel classification. The details on this technique using *fuzzy c-means, fuzzy isodata, fuzzy dynamic clustering and fuzzy relaxation* are available in references (2), (10), (43-50). The fuzzy c-means (FCM) algorithm is a well known clustering algorithm used for pixel classification. Here we describe it in brief.

Fuzzy segmentation results in fuzzy partitions of $X=\{x_1, x_2, \ldots\ldots x_n\}$, where X denotes a set of n unlabeled column vectors in R^p (i.e., each element of X is a p dimensional feature vector). A fuzzy c-partition (c is an integer, $1 \le c \le n$) is the matrix $U=[\mu_{ik}]=1,2\ldots\ldots, c,\ k=1, \ldots\ldots, n$ which satisfies the following constraints:

$$\mu_{ik}\varepsilon[0,1], \sum_{i=1}^{c}\mu_{ik} = 1, \text{ and } 0<\sum_{k=1}^{c}\mu_{ik} <n, \text{ for all } i \,\&\, k.$$

Here the kth column of U represents membership values of x_k to the c fuzzy subsets and $\mu_{ik}=\mu_i(x_k)$ denotes the grade of membership of x_k in the ith fuzzy subset.

The FCM algorithm searches the local minimum of the following objective function:

$$J_m(U,V)=\sum_{k=1}^{n}\sum_{i=1}^{c}(\mu_{ik})^m\|x_k-v_i\|_A^2,\quad 1\le m\le\infty,$$

where U is a fuzzy c-partition of X, $\|.\|_A$ is any inner product norm, $V=\{v_1, v_2, \ldots\ldots v_c\}$ is a set of cluster centers, $v_i \in R^p$, and $m \in \{1, \infty\}$ is the weighting exponent on each fuzzy membership. For $m>1$ and $x_k \ne v_l$ for all I, k, it has been shown that $J_m(U, V)$ may be minimized only if

$$\mu_{ik} = 1/\sum_{j=1}^{c}\left(\frac{\|x_k-v_i\|_A^2}{\|x_k-v_j\|_A^2}\right)^{\frac{2}{m-1}}$$

and $v_i = \dfrac{\sum_{k=1}^{n}(\mu_{ik})^m x_k}{\sum_{k=1}^{n}(\mu_{ik})^m}, 1 \le i \le c.$

The FCM algorithm, when Euclidian distance norm is considered, can only be used for hyperspherical clusters with approximately equal dimensions. To cope with clusters having large variability in cluster shapes, densities, and the number of data points in each cluster, Gustafson and Kessel[51] used the scaled Mahalanobis distance in the FCM algorithms. By the use of a distance measure derived from maximum likelihood estimation methods, Gath and Geva[52] obtained an algorithm which is effective even when the clusters are ellipsoidal in shape and unequal in dimension. As the value of c i.e., the number of clusters is not always known, several cluster validity criteria have been suggested in the literature to find the optimum number of clusters. These include partition coefficient, classification entropy, properties coefficient, total within class distance of clusters, total fuzzy hyper volume of clusters, and partition density of clusters[50-53].

Generalizing FCM algorithm further, Dave[54] proposed the fuzzy c shells (FCS) algorithm to search for clusters that are hyper ellipsoidal shells. One of its advanced versions is seen to be better than Hough transformation (in terms of memory and speed of computation) when used for ellipse detection. It is also shown[54] that the use of fuzzy memberships improves the ability to attain global optima compared to the use of hard membership. For the same purpose, Krishnapuram et al.[55] proposed another algorithm which is claimed to be less time consuming than that of Dave.

For further information readers may consult references (56) to (58). Cannon *et al.*[57] describe a modified version of the FCM, which incorporate supervised training data. The article of Cannon et al.[57] describes an approach that reduces the computation required for the FCM, by utilising look up tables, by a factor of six. Another simplified form of FCM in this line is mentioned in reference (58). Before leaving this section, we mention about the work in reference (59) which defines the concept of fuzzy objects and describes algorithms for their extraction.

5.4 Contour Detection

Edge detection is also an image segmentation technique where the contours/boundaries of various regions are extracted based on the detection of discontinuity in grayness. Here we present a method for fuzzy edge detection using an edginess measure based on H^1 (eq. 2) which denotes an amount of difficulty in deciding whether a pixel can be called an edge or not[19]. Let N_{xy}^3 be a 3×3 neighbourhood of a pixel at (x,y). The edge-entropy H_{xy}^E of the pixel (x,y), giving a measure of edginess at (x,y), may be computed as follows. For every pixel (x,y), compute the average, maximum and minimum values of gray levels over N_{xy}^3. Let us denote the average, maximum and minimum values by *Avg. Max, Min* respectively. Now define the following parameters.

$$D = max\{Max - Avg, Avg - Min\} \qquad \ldots (28)$$

$$B = Avg \qquad \ldots (29)$$

$$A = B - D \qquad \ldots (30)$$

$$C = B + D \qquad \ldots (31)$$

A π-type membership function (Fig. 2) is then used to compute μ_{xy} for all $(x,y) \in N_{xy}^3$, such that $\mu(A)=\mu(C)=0.5$ and $\mu(B)=1$. It is to be noted that $\mu_{xy} \ge 0.5$. Such a μ_{xy}, therefore, characterises a fuzzy set *"pixel intensity close to its average value"*, averaged over N_{xy}^3 are either equal or close to each other (i.e., they are within the same region), such a transformation will make all $\mu_{xy}=1$ or close to 1. In other words, if there is no edge, pixel values will be close to each other and the μ values will be close to one (1); thus resulting in a low value of H^1. On the other hand, if there is an edge (dissimilarity in gray values over $N_{x,y}^3$), then the μ values will be more away from unity; thus resulting in a high value of H^1. Therefore, the entropy H^1

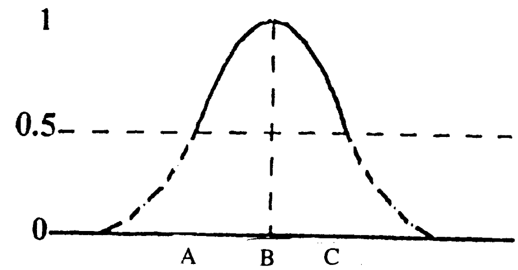

Fig. 2 π function for computing edge entropy over $N^3_{x,y}$ can be viewed as a measure of edginess ($H^E_{x,y}$) at the point (x,y). The higher the value of $H^E_{x,y}$, the stronger is the edge intensity and the easier is its detection. Such an entropy plane will represent the fuzzy edge detected version of the image.

The proposed entropic measure is less sensitive to noise because the use of a dynamic membership function based on a local neighbourhood. The method is also not sensitive to the direction of edges. Other edginess measures and algorithms based on fuzzy set theory are available elsewhere[10,21,37]

5.5 Fuzzy Skeleton Extraction

Let us now explain two methods for extracting the fuzzy skeleton (skeleton having ill-defined boundary) of an object from a gray tone image without getting involved into its (questionable) hard thresholding. The first one is based on minimization of the parameter *IOAC* (eq. 17) or compactness (eq. 11) with respect to α-cuts (α-cut of a fuzzy set A comprises all elements of X whose membership value is greater than or equal to α, $0 < \alpha \leq 1$) over a fuzzy '*core line*' (or skeleton) plane. The membership value of a pixel to the core line plane depends on its property of possessing maximum intensity, and property of occupying vertically and horizontally middle positions from the ε-edges (pixels beyond which the membership value in the fuzzy segmented image becomes less than or equal to $\varepsilon, \varepsilon > 0$) of the object[60]. If a non-fuzzy (or crisp) single pixel width skeleton is deserved, it can be obtained by a contour tracing algorithm[61] which takes into account the direction of contour. Note that the original image can not be reconstructed, like the other conventional techniques of gray skeleton extraction[2,62,63] from the fuzzy skeleton obtained here.

The second method is based on fuzzy medial axis transformation (*FMAT*)[28] using the concept of fuzzy disks. A fuzzy disk with center P is a fuzzy set in which membership depends only on the distance from P. For any fuzzy set f, there is a maximal fuzzy disk $gP^f \leq f$ centered at every point P, and f is the *sup* of the gP^fs. (Moreover, if f is fuzzy convex, so is very gP^f, but *not conversely.) Let us call a set S_f of points* f-sufficient if every $gP^f \leq gQ^f$ for some set of Q in S_f; evidently f is then the *sup* of the *gQ^f's*. In particular, in a digital image, the set of Q's at which gf is a (non-strict) local maximum is f-sufficient. This set is called the fuzzy medial axis of f, and the set of *gQ^f's* is called the fuzzy medial axis transformation (FMAT) of f. These definitions reduce to the standard one, if f is a crisp set.

For a gray tone image X (denoting the non-normalized fuzzy "bright image" plane), the *FMAT* algorithm computes, first of all, various fuzzy disks centered at the pixels and then retains a few (as small as possible) of them, as designated by gQ's, so that their union can represent the entire image X. That is, the pixel value at any point t can be obtained from a union operation, as t has membership value equal to its own gray value (i.e., equal to its non-normalized membership value to the bright image plane) in one of those retained disks.

Note that the above representation is redundant i.e., some more disks can further be deleted without affecting the reconstruction. The redundancy in pixels (fuzzy disks) from the fuzzy medial axis output can be reduced by considering the criterion $gP^f(t) \leq \sup gQ^i_f(t)$, i=1,2...instead of $gP^f(t) \leq gQ^f(t)$. In other words, elimiante many other gP^f's for which there exists a set of gQ^f's whose sup is greater than or equal to gP^f.

Let *REMAT* denote the *FMAT* after reducing its redundancy. The fuzzy medial axis is seen to provide a good skeleton of the darker (higher intensity) pixels in an image apart from its exact representation. *FMAT* of an image can be considered as its core (prototype) version for the purpose of image matching. It is to be mentioned here that such a representation may not be economical in a practical situation. The details on this feature and the possible approximation in order to make it practically feasible are available in reference (64).

Note that the membership values of the disks contain the information of image statistics. For example, if the image is smooth the disk will not have abrupt change in its values. On the other hand, it will have abrupt change in case the image has salt and pepper noise or edginess. The concept of fuzzy *MAT* can therefore be used as spatial filtering (both high pass and low pass) of an image by manipulating the disk values to the extent desired and then putting them back while reconstructing the processed image. A gray scale thinning algorithm is described in reference (50) and (65) based on the concept of fuzzy connectedness between two pixels; the dark regions can be thinned without ever being explicitly segmented.

6 Some Applications

Here we provide a few applications of the methodologies and tools described before.

6.1 Motion Frame Analysis and Scene Abstraction

With rapid advancements in multimedia technology, it is increasingly common to have time varied data like video as computer data types. Existing data base systems do not have the capability of search within such information. It is a difficult problem to automatically determine one scene from another because there are no precise markers that identify where they begin and end. Moreover, divisions of scenes can be subjective, especially if transitions are subtle. One way to estimate scene transitions is to approximate the change of information between each of two successive frames by computing the distance between their discriminatory properties.

A solution is provided in reference (66) to the problem of scene estimation/abstraction of motion video data in the fuzzy set theoretic framework. Using various fuzzy geometrical and information measures (Section 3) as image features, an algorithm is developed to compute the change of information in each of two successive frames to classify scenes/frames. Frame similarity is measured in terms of weighted distance in fuzzy feature space. This categorization process of raw input visual data can be used to establish structure for correlation. The investigation not only attempts to determine the discrimination ability of the fuzziness measures for classifying scenes, but also enhances the capability of nonlinear, frame-accurate access to video data for applications such as video editing and visual document archival retrieval systems in multimedia environments. Such an investigation is recently carried out in NASA Johnson Space Center, TX[66].

A set of digitised videos of previous space shuttle missions obtained from NASA/JSC was used (Fig. 3). The scenes were named payload deployment, onboard astronaut, remote manipulator arm, and mission control room. Experiments were conducted for various combinations of uncertainty, orientation, and shape measures. As an illustration, Fig. 4 shows a result when {entropy, compactness, length/height} was considered as a feature set for computing distance between two successive frames. Here the abscissa represents the total number of frame distances, in the sampled time series, and the ordinate is the compound distance value between two successive images.

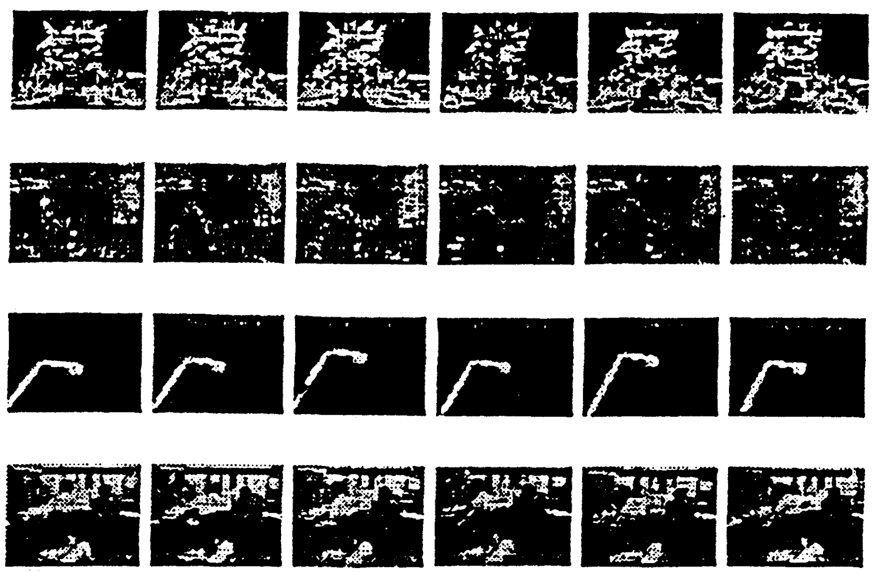

Fig. 3 A payload deployment sequence of four scenes as input data

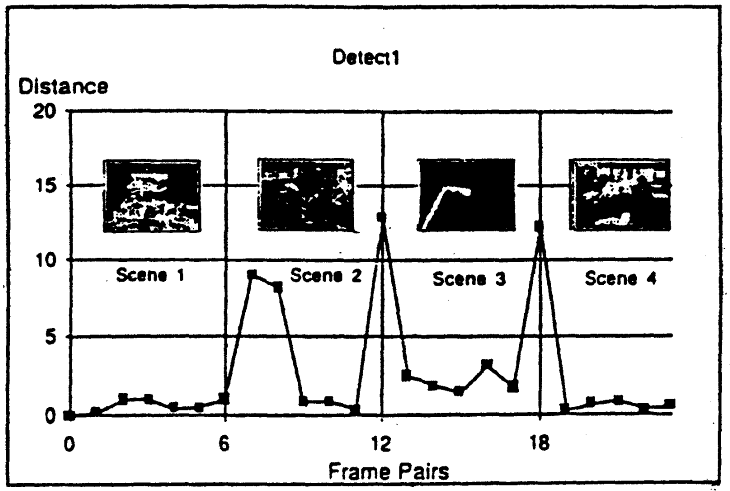

Fig. 4 Distance between successive frames with feature set (entropy, compactness, length/height)

Each scene consists of six frames. Therefore, there is a change of scene at every sixth index on the abscissa. The scene separation is denoted with vertical grid lines. The effectiveness of the aforesaid fuzzy geometrical parameter is also demonstrated recently[67] for recognising over-lapping finger prints with a multilayer perceptron.

6.2 Hand-written Character Recognition

Hand-written characters, like all patterns of human origin, are examples of ill-defined patterns. Hence the recognition of hand-written characters is a very promising field for the application of pattern recognition techniques using the fuzzy approach. It has been claimed that the concept of vagueness unerlying fuzzy theory is more appropriate for describing the inherent variability of such systems than the probabilistic concept of randomness. An important application of handwriting recognition is to build efficient man-machine interface for communicating with the computer by human beings. There are several attempts made for handwritten character recognition in different languages. Here we mention a pioneeing contribution of Kickert and Koppelaar[68], the subsequent developments based on this and then a recent attempt made for fuzzy feature description in this context.

The 26 capital letters of the English alphabet constituting the set

$L = \{H_k \mid k = 1,2,\ldots\ldots, 26\}$

$= \{A, B, C,\ldots\ldots, Y, Z\}$

are seen to be composed of the elements of the following set of 'ideal' elements[68]

$V_T=\{a_i/i=1, 2, \ldots., 7\}= \{/, |, \backslash, -, (,), \in\}$

Where $\in$ is a null segment whose use will be explained shortly. Also, there is a set P of eleven ordered recognition routines capable of recognizing the 'ideal' segments. Each element of P can be considered as a portion of a context-free grammar having productions of the form

$A \to a_iB$ *or* $A \to a_i$, where $a_i \in V_T$, $A, B \in V_n^*$,

V_n being the non-terminal elements of the grammer.

Each of the eleven recognition routines is applied sequentially to any unknown pattern S to be recognized as one of the members of L. Each routine attempts to recognize a given segment in a given structural context. If successful, the application of the rules in P results in a parsing of S as a vector of segments $S = (x_1, x_2,\ldots.., x_n)$, where $x_i \in V_T$.

Each letter, then, is defined by its vector of segments. Let us assume that the vectors are padded out with null segments $\in$ so that all letters are defined by vectors of equal length. Each letter, therefore, can be defined as follows:

$H_k=(bk_1, \ldots\ldots\ldots\ldots, bk_n)$, $k=1,2 \ldots\ldots\ldots\ldots, 26$
where $bk_j=a_i$ for some i, $i=1, 2, \ldots\ldots\ldots\ldots\ldots, 7$
$=j$ th segment of the k th letter.

The element of fuzziness is introduced by associating with each segment $a_i \in V_T$ a fuzzy set on the actual pattern space With each a_i is associated a fuzzy membership function μ_{ai} so that, given a segment x_i of a pattern S, $\mu_{aj}(x_i)$ is a measure of the degree to which the segment x_i corresponds to the ideal segment a_j.

The recognition procedure is now simply explained. The sequence of recognition rules is executed, evaluating all possible parsings of the input pattern. For each H_k for which a parse can be made, the result is a sequence $(x_1, x_2, \ldots\ldots\ldots\ldots x_n)$ of segments. The membership of S in H_k is the intersection in the sense of fuzzy sets of the memberships of the segments x_i

$$\mu H_k(S)=min\ [\mu bk_1(x_1), \ldots\ldots\ldots\ldots, \mu bk_n(x_n)]$$

Finally, the pattern is recognized as letter H_m if

$$\mu H_\mu(\xi)=\mu\alpha\xi_\kappa\mu H_k(S)$$

This approach was criticised by Stallings[69] who developed a Bayesian hypothesis-testing scheme for the same problem. Given a pattern S, hypothesis H_k is that the writer intended letter H_k. Associated with each decision is a cost C_{ij} which is the cost of choosing H_i when H_j is true. The parsing of the pattern is performed as before. Only a probability is associated with each segment for a given letter. Regarding unknown densities the author[69] suggests the use of maximum likelihood tests. Since both membership function and probability density functions are maps into the interval [0,1], the only difference is the use of min/max operators, where, the author argues, the '*min*' operator loses a lot of information and is drastically affected by one low value. The author claims that though frequentistic probability is not appropriate in dealing with pattern variability, subjective probability is perfectly suitable and more intuitively obvious than "grade of membership'.

In a rejoinder[70], it is argued that fuzzy set theory is more flexible than is assumed in reference (69) where all arguments are directed against a particular case[68]. Recalling the idea of collectives (from property sets), where the arithmetic average replaces '*min*', there remains little difference between the schemes in references (68) and (69). In a reply, Stalling insisted that the Bayesian approach is superior since it offers a convenient way for assignment of costs to errors and gains to correct answers. For the recognition of hand-written English capital letters, the readers may also refer to the work described in reference (71).

Existing computational recognition methods use feature extraction to assign a pattern to a prototype class. Therefore, the recognition ability depends on the selection procedure. To handle with the inherent uncertainties/imprecision in handwritten characters, Malaviya and Peters[72] have introduced recently a fuzziness factor in the definition of selected pattern features. The fuzzy features are confined to their meaningfulness with the help of a multi-stage feature aggregation. These can be combined in a set of linguistic rules, which form the fuzzy rule-base for handwritten information recognition. Note that the concept of introducing fuzziness in the definition and extraction of features and in their relations is not new. A detailed discussion is available in Pal and others[61,73] for extraction of primitives for X-ray identification and character recognition in terms of gentle, fair and sharp curves. A similar interpretation of the shape parameters of triangle, rectangle and quadrangle in terms of membership for "approximate isosceles triangles", "approximate equilateral triangles" and "approximate right triangle" and so on has also been made[74] for their classification in a colour image. However, the work in[72] is significant from the point that it has described many global, positional and geometrical features to account for the variabilities in patterns and these are supported with experimental results.

In order to represent the uncertainty in physical relations among the primitives, the production rules of a formal grammar are fuzzified to account for the fuzziness in relation among the primitives, thereby increasing the generative power of a grammar. Such a grammar is called fuzzy grammar[75-77].

It has been observed[73] that the incorporation of the element of fuzziness in defining 'sharp', 'fair' and 'gentle' curves in the grammars enables one to work with a much smaller number of primitives. By introducing fuzziness in the physical relations among the primitives, it was also possible to use the same set of production rules and non-terminals at each stage. This is expected to reduce, to some extent, the time required for parsing in the sense that parsing needs to be done only once at each stage, unlike the case of the non-fuzzy approach, where each string has to be parsed more than once, in general, at each stage. However, this merit has to be balanced against the fact that the fuzzy grammars are not as simple as the corresponding nonfuzzy grammars.

6.3 Detecting Man-Made Objects from Remote Sensing Images

In a remotely sensed image, the regions (objects) are usually ill-defined because of both grayness and spatial ambiguities. Moreover, the gray value assigned to a particular pixel of a remotely sensed image is the average reflectance of different types of ground covers present in the corresponding pixel area [36.25 m-36.25 m for the Indian Remote Sensing (IRS) imagery]. Therefore, a pixel may represent more than one class with a varying degree of belonging.

A multivalued recognition system[78,79] formulated based on the concept of fuzzy sets has been used recently for detecting curved structure from IRS images[80]. The system is capable of handling various imprecise inputs and in providing multiple class choices corresponding to any input. Depending on the geometric complexity[8,9] and the relative positions of the pattern classes in the feature space, the entire feature space is decomposed into some overlapping regions. The system uses Zadeh's compositional rule of inference[81] in order to recognize the samples. The recognition system is initially applied on an IRS image to classify (based on the spectral knowledge of the image) its pixels into six classes corresponding to six land cover types namely, *pond water, turbid water, concrete structure, habitation, vegetation* and *open space*. The green and infrared band information, being sensitive than other band images to discriminate various land cover types, are used for the classification.

The clustered images are then processed for detecting the narrow concrete structure curves. These curves include basically the roads and railway tracks. The width of such attributes has an upper bound which was considered there to be 3 pixels for practical reasons. So all the pixels lying on the concrete structure curves with width not more than 3 pixels were initially considered as the candidate set for the narrow curves. Because of the low pixel resolutions (36.25 *m*-36.25 *m* for IRS imagery) of the remotely sensed images, all existing portions of such real curve segments may not be reflected as concrete structures and as a result, the candidate pixel set may constitute some broken curve segments. In order to identify the curves in a better extent, a traversal through the candidate pixels was used. Before traversing process, one also needs to thin the candidate curve patterns so that a unique traversal can be made through the existing curve segments with candidate pixels. Thus, the total procedure to find the narrow concrete structure curves consists of three parts (i) selecting the candidate pixels for such curves, (ii) thinning the candidate curve patterns and (iii) traversing the thinned patterns to make some obvious connections between different isolated curve segments. The multiple choices provided by the classifier in making a decision are utilized to a great extent in the traversal algorithm. Some of the movements are governed by only the second and combined choices.

After the traversal, the noisy curve segments (i.e., with insignificant lengths) are discarded from the curve patterns. The residual curve segments represent the skeleton version of the curve patterns. To complete the curve pattern, the concrete structure pixels lying in the 8 neighbouring positions corresponding to the pixels on the above obtained narrow curve patterns are now put back. The resultant image represents the narrow concrete structure curves corresponding to an image frame[80].

The results are found to agree well with the ground truths. The classification accuracy of the recognition system[79,80] is not only found to be good, but also its ability of providing multiple choices in making *decisions* is found to be very effective in detecting the road like structures from IRS images.

7 Discussion

The problem image processing and recognition under fuzziness and uncertainty has been considered. The role of fuzzy logic in representing and managing the uncertainties in these tasks was explained. Various fuzzy set theoretic tools for measuring information on grayness ambiguity and spatial ambiguity in an image were listed along with their characteristics. Some examples of image processing operations (e.g., segmentation, skeleton extraction and edge detection), whose outputs are responsible for the overall performance of a recognition (vision) system, were considered in order to demonstrate the effectiveness of these tools in providing both soft and hard decisions. The significance of retaining the gray information in the form of class membership for soft decision is evident. Uncertainty in determining a membership function in this regard and the tools for its management were also stated. Finally a few real life applications of these methodologies are described.

8 Conclusion

Gray information is expensive and informative. Once it is thrown away, there is no way to get it back. Therefore one should try to retain this information as long as possible throughout the decision making tasks for its full use. When it is required to make a crisp decision at the highest level one can always throw away or ignore this information.

Most of the algorithms and tools described here were developed by the author with his colleagues. Processing of colour images has not been considered here. Some significant results on colour image information and processing in the notion of fuzzy logic are available in references (82) to (84).

Note that fuzzy set theory has led to the development of the concept of soft computing as a foundation for the conception and design of high Machine IQ (MIQ) system. Recently, the merits of fuzzy set theory are being integrated with those of other soft computing tools e.g., artificial neural networks, genetic algorithms and rough sets with a hope of building more efficient recognition systems.

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