EXPERIMENTS ON THE RADIAL VARIATION OF CRITICAL VELOCITY OF WATER IN TUBES OF CIRCULAR CROSS-SECTION

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The present investigation was planned with a view of determining, if possible, at what point in the cross-section of a parallel tube through which water flows with stream-line motion, this motion first breaks down and becomes turbulent as the velocity is increased.

It was thought that this information might be of value as possibly affording some indication of the reason for the change from stream-line to turbulent motion at the critical velocity.

The Apparatus

The apparatus used in the tests was a modification of that on which Osborne Reynolds carried out his classical experiments on critical velocities. It consists of a large glass-sided tank 5–10½″ long by 18″ wide, fitted with baffles for steadying the flow from the inlet pipe, from which water was allowed to flow through a glass tube. The rate of flow was regulated by a valve at the outlet from the tube. The inlet end of the tube was fitted with a carefully finished bell mouthpiece.

The motion in the tube was made visible by a colour band of aniline dye, supplied from a tank and discharged through a fine capillary tube at the entrance to the bell mouthpiece. The position of this capillary tube could be regulated with precision both in a horizontal and a vertical direction by means of adjusting screens on its carrier.

At the point of observation of the coloured filament, the tube was enclosed in a glass-sided box, having vertical sides of parallel plate glass. The box had an open top and was filled with water.

In order to obtain the distance of the filament from the centre of the tube, a graduated scale was mounted on the vertical glass front of the box surrounding the tube, and a similar scale in the same plane across the open top of the box.

Above the box a plane mirror was mounted at 45° to the horizontal.

For observing the filament, two telescopes were mounted horizontally, one above the other, in the same vertical plane, *i.e.*, the plane containing the graduated scales. The upper one was focussed on the scale and on the filament, which was observed from above through the mirror, while the lower one was focussed on the filament as seen through the side of the box.

In this way the position of the filament at the section in question, with reference to each of the scales, and therefore with reference to the axis of the tube was obtained.

In order to correct for refraction, a thin graduated scale was made to fit the internal diameter of the tube. This was inserted into the tube, which was filled with water as when carrying out an experiment, and the readings of the internal and external scales were compared over the whole radius. The experiments were carried out on tubes of four diameters, *viz.*, 0·5″; 1·0″; 1·25″; and 1·5″.

Method of Carrying out Experiments

In carrying out an experiment the supply tank was filled and allowed to stand for some time in order to allow any initial disturbance of the water to die out. Some definite cross-section of the tube, at which to take observations, was then selected and the telescopes and gauges were brought into this plane. The outlet valve was then slightly opened and the rate of flow of the dye was regulated so as to give a fine colour band.

The opening of the outlet valve was then gradually increased until a slight flicker of the colour band was noticed near the outlet end of the tube. This indicates that the velocity was approaching the critical value.

A very slight increase in the velocity caused definite but intermittent breakdown into eddy formation at the end of the tube. A further slight increase in the velocity caused the eddies to become permanent and brought the point of breakdown nearer to the tube entrance. In this way by adjusting the velocity the point of breakdown with the filament at any required radius, could be brought into the plane of observation.

The exact radial position of the filament at this section was then observed and recorded, after which the mean velocity in the tube was determined by observing the time for the level in the supply tank to fall through a given distance when discharging under the same head with the inlet valve closed. The inverse of this time was proportional to the mean velocity of flow. With each tube the experiments were carried out for a series of radial positions of the filament, extending from the centre to a point near the walls.

Experimental Data

(1) *Experiments on tube* 1·5″ *diameter.*—This tube was 5·0′ long and the observations were made at a point distant 2·0 ft. from the inlet end.

The co-ordinates of the centre of the tube on the horizontal and vertical scale were:

Horizontal scale	4·38″
Vertical ,, ...	1·32″

The following table, which represents a typical set of observations, shows the horizontal and vertical co-ordinates of the filament at the instant of break-down; its radial distance from the centre of the tube; and the corresponding time for the level in the supply tank to fall through a distance of 1 inch.

Position of Colour Band		Distance from Centre	Time (Secs.) for Level in Tank to fall 1·0″
Horizontal	Vertical		
4·38	1·33	·01	42·5
4·30	1·38	·1	44·0
4·25	1·32	·230	48·2
4·15	1·49	·285	51·2
4·24	1·06	·295	52·2
4·63	1·04	·38	58·5
3·00	1·34	·38	58·0
4·00	1·35	·39	59·1
4·11	1·65	·427	62·2
4·50	0·90	·438	63·0
4·51	0·90	·44	63·2
4·00	1·10	·44	63·2
3·96	1·16	·45	64·0
4·81	1·40	·50	66·0
4·02	1·70	·53	62·8
3·88	1·52	·538	62·5
4·45	1·84	·556	59·8
4·07	1·73	·515	66·0
3·80	1·32	·58	55·5
3·90	1·00	·583	55·0
4·05	1·82	·60	49·9
4·85	0·95	·632	45·0
4·01	1·83	·632	45·0

These times are plotted on a base showing the distance from the centre of the tube of the corresponding filament. From this it will be seen that as the distance of the filament from the centre of the tube increases, the velocity of flow necessary to produce break-down into turbulent motion diminishes and attains a minimum value at a radius of approximately 0·5″ after which it increases with a further increase in radius.

This indicates that when, owing to increasing the velocity motion is made to break down at this section, the breakdown first occurs at a radius of approximately 0·475, or at a distance from the centre equal to 0·63 of the radius of the tube.

(2) *Experiments on tube* 1·25″ *diameter.*—This tube was 4·33′ long and the observations were made at a point distant 2·5′ from the inlet end.

The co-ordinates of the centre of the tube were:

Horizontal scale .. .. .. 2·48″
Vertical „ .. .. .. 4·87″

The following table shows a set of experimental observations on the tube.

Position of Colour Band		Distance from Centre inches	Time from 12″ to 11″ (Secs.)
Horiz. Scale	Vert. Scale		
2·49	4·91	·04	62·8
2·47	4·80	·07	64·2
2·50	4·78	·092	55·6
2·60	4·71	·20	57·0
2·50	4·62	·25	61·2
2·48	4·62	·25	60·2
2·49	4·56	·31	75·0
2·49	4·51	·36	87·2
2·47	4·48	·39	86·2
2·49	4·44	·43	79·0
2·49	4·42	·45	72·5
2·50	4·40	·47	62·6
2·48	4·30	·57	75·0

As in the tests on the 1·5″ tube, the velocity necessary to cause breakdown at this section diminishes with the distance of the filament from the centre of the tube and attains a minimum at a radius of 0·37″ or 0·595 of the radius of the tube; afterwards increasing as the radius of the filament is increased.

(3) *Experiments on tube* 1·0″ *diameter.*—In order, if possible, to ensure that the position of the radius at which breakdown first occurs should not be affected by the stabilising effect of the bell-mouthed entrance to the tube, a larger tube—10′ in length—was obtained for the second series of experiments. This tube was 1·0″ in diameter, and the observations were made at a section distant 5·0′ or sixty pipe diameters from the inlet.

The co-ordinates of the centre of this tube were:

Horizontal scale .. .. .. 4·08″

Vertical ,, ... 1·66″

The following table shows a typical set of experimental observations on this tube.

Experiments on 1″ *tube, Length* 10′

Co-ordinates of Centre (1.66 : 4.08)

Position of Colour Band		Distance from Centre	Time (Secs.) for Level in Tank to fall 1·0″
Vertical	Horizontal		
1·66	4·08	·0	75·6
1·73	4·10	·071	79·0
1·56	4·08	·10	80·1
1·59	4·00	·106	82·3
1·70	3·94	·146	86·1
1·60	3·89	·199	90·3
1·73	3·88	·212	100·8
1·70	3·80	·282	108·1
1·70	3·75	·331	112·7
1·90	3·77	·388	108·0
1·68	3·69	·395	90·7
1·63	3·68	·410	81·5

It will be seen that, as in the tests on the 1·5″ tube, the velocity necessary to cause breakdown at this section diminishes with the distance of the filament from the centre of the tube, and attains a minimum at a point whose radius (0·33″) is 0·66 of the radius of the tube, afterwards increasing with an increase in the radius.

(4) *Experiments on tube* 0·5″ *diameter.*—This tube was 7·0′ long, and observations were made in this case at three sections; Section A was at a distance of 19″ from the entrance; Section B, 43″ from the entrance; and Section C, 72″ from the entrance.

The experimental data are shown in the following tables.

On comparing these, it appears that the radius of the tube at which the minimum velocity is required to cause breakdown is sensibly the same in each case, namely, 0·165″ or 0·66 of the radius of the tube.

The mean velocities required to cause this breakdown, however, diminish very appreciably with the distance from the entrance. If the velocity required to breakdown at Section C be taken as unity, that at Section B is 1·17, and that at Section A is 1·91.

SECTION A

Distance 1′–7″ from entrance

Centre co-ordinates (3·87 : 1·02)

Position of Colour Band		Distance from Centre	Time (Secs.) for Level in Tank to fall 1·0″
Horizontal	Vertical		
3·87	1·025	·005	60·1
3·88	1·02	·01	64·0
3·87	1·03	·01	63·6
3·87	1·07	·05	69·0
3·81	1·02	·06	71·0
3·95	1·03	·081	78·0
4·00	1·02	·13	96·0
3·87	1·165	·145	103·5
4·01	1·09	·16	107·0
4·01	1·10	·166	109·4
3·88	1·20	·180	75·
3·74	1·16	·189	94·1
4·09	1·02	·22	75·1
4·07	1·16	·248	65·0

SECTION B

Centre co-ordinates (3·875 : 0·935)

Position of Colour Band		Distance from Centre	Time (Secs.) for Level in Tank to fall 1·0″
Horizontal	Vertical		
3·87	·96	·030	157·9
3·91	·93	·04	153·5
3·90	·90	·042	161·1
3·94	·93	·070	159·0
3·90	1·00	·076	153·8
3·84	1·04	·114	171·2
3·99	·93	·120	159·0
3·91	·80	·136	177·0
4·02	·93	·150	178·5
3·93	·78	·158	179·1
4·00	·83	·164	180·8
4·05	·84	·175	179·4
4·03	1·03	·186	173·5
4·04	·83	·197	168·5
4·08	·950	·212	161·1
4·08	·860	·22	157·5
4·08	·860	·22	155·0

This is due to the stabilising effect of the bell-mouth entrance, which is evidently still slightly felt at Section B which is 86 tube diameters from the entrance.

SECTION C

Taken at 6′ from entrance

Centre co-ordinates (3·89 : ·92)

Poaition of Colour Band		Distance from Centre	Time (Secs.) for Level in Tank to fall 1·0″
Horizontol	Vertical		
3·90	·92	·01	194·5
3·90	·933	·016	195·8
3·91	·933	·024	196·1
3·89	·95	·03	194·0
3·89	1·00	·18	196·0
3·89	1·02	·10	198·1
3·99	1·00	·127	202·5
3·89	1·06	·140	200·8
3·99	1·05	·15	205·1
4·04	·99	·158	208·3
4·02	1·01	·166	210·3
4·01	1·06	·189	206·3
4·19	·92	·20	200·1

Careful measurements of the diameter of this 0·5″ tube showed that it varies slightly, to the extent of 0·06″ in its length of 7′. The experiments already described had been carried out with the smaller end at the entrance, and in order to determine the effect of this, the tube was reversed and the experiment repeated, observations being made at a point (Section D) corresponding to Section C, *i.e.*, 72 ins. from the entrance.

The results of this experiment are shown in the following table.

SECTION D

(*Tube reversed*)

Taken at 6′ from the entrance end

Centre co-ordinates (3·86 : ·975)

Position of Colour Band		Distance from Centre	Time (Secs.) for Level in Tank to fall 1·0″
Horizontal	Vertical		
0·97	3·86	·005	121·1
1·00	3·89	·042	122·3
0·94	3·90	·050	122·9
0·90	3·90	·080	125·0
0·97	3·95	·090	127·3
1·03	3·93	·093	127·9
1·10	3·83	·133	136·1
1·03	3·98	·136	137·0
1·05	3·98	·144	137·8
1·12	3·86	·150	138·0
1·05	4·00	·161	139·1
1·13	3·92	·172	139·0
1·14	3·99	·214	133·1

The results show that while the radius of maximum instability is at the same point as before (at 0·66 of the radius of the tube) the velocity necessary to cause breakdown at this radius is 1·5 times as great as with flow in the opposite direction.

Ratio of Velocities Required to Cause Breakdown of Motion at Centre and at Radius of Maximum Instability

From the various graphs, the ratio of the velocities required to cause breakdown of the motion at the radius of maximum instability and at the centre, can be obtained. These are as follows:

	Diam of tube "	Ratio
	1·5	0·65
	1·25	0·65
	1·0	0·70
	0·5	..
Slightly divergent .. 19" from entrance ..	..	0·56
43 ,, ,, ..	..	0·88
72 ,, ,, ..	..	0·92
Slightly convergent .. 72 ,, ,, ..	..	0·88

These figures show that this ratio is greatly affected by the proximity to the bell-mouthpiece. On the whole, it would appear that the ratio increases slightly as the pipe diameter is decreased.

Critical Velocity near the Wall of a Tube

In the experiments an observation very near the wall of the tube was found to be very difficult. If the tube supplying the colour band was so adjusted that the colour band was very near the wall, it almost invariably touched the wall before observations could be made, and remained in contact with the wall.

If, however, the curves be produced to a radius corresponding to that of the respective tubes, it appears that the velocity required to produce breakdown of motion near the wall is much greater than that required at any other point in the cross-section. Taking the curves as a whole, it appears that the critical velocity is the same as at the centre, at a point whose radius is approximately 0·85 of the radius of the tube.

Conclusions and Deductions

All the experiments indicate that when breakdown of motion initially streamline into turbulent motion occurs during flow through a tube, the breakdown first occurs at a distance from the centre of the pipe, the experimental values ranging from 0·60 to 0·67 in the different tubes. From this point the turbulence spread inwards to the centre and outwards to the walls.

The exact reason for the breakdown or the mechanism producing it is not as yet understood. It is known to be due to the presence of the pipe walls, and to be independent of the roughness of the walls so long as the roughness is small. It is not due to the attainment of a limiting shear stress in the fluid, since stream-line motion is possible in a small tube with shear stresses much greater than that obtaining at the critical velocity in a larger tube.

An examination of the problem in the light of the experimental data of the present investigation, suggests that the breakdown may be related to the rate of variation of energy across a diameter of the tube. Since the pressure across a section of a parallel tube in which the flow is stream-line is constant, the rate of variation of the energy per unit mass along a radius is the same as the rate of change of the kinetic energy and therefore of v^2.

Assuming that some slight deviation of the particles from linear axial flow to be produced in any way, this will have its maximum disturbing effect it if occurs at a radius where the radial rate of change of energy is a maximum and therefore where $\frac{dv^2}{dr}$ is a maximum.

But in stream-line flow through a tube of radius a,

$$v \propto (a^2 - r^2)$$

$$\therefore \quad \frac{dv^2}{dr} \propto (a^2 - r^2)\, r$$

which is a maximum when $r = \frac{a}{\sqrt{3}} = 0\cdot58\, a$.

It is suggestive that this is very nearly the radius at which breakdown occurred in the experiments. The fact that the experimental value is somewhat greater than this is possibly due to the fact that in a tube whose walls are not perfectly smooth, those filaments nearest the walls suffer some slight lateral displacement due to the roughness, and thereby suffer a reduction in stability causing breakdown to occur somewhat nearer the walls than would be the case in an ideally smooth tube. That this is a possible explanation is indicated by the fact that the radius of primary breakdown in the experiments was smallest in the two larger tubes, having a mean value of 0·615 a in these tubes, as compared with 0·66 a in the smaller tubes. As the surface finish of all the tubes was as nearly as could be determined the same, the relative roughness would be smaller in the larger tubes.

It is to be noted that at the radius $(a/\sqrt{3})$, the value of $\frac{dv^2}{dr}$ is proportional to $a^3 \frac{dp}{dl}$ where $\frac{dp}{dl}$ is the pressure drop per unit length of the tube.

But in the similar tubes the product $a\bar{v}$, where $\bar{v}$ is the mean velocity, is constant at the critical velocity, while $\frac{dp}{dl} \propto \frac{\bar{v}}{a^2}$ $\therefore a^3 \frac{dp}{dl}$ or $\frac{dv^2}{dr}$ is a constant at the critical velocity in tubes of different diameter.

From this it would appear that in any tube the breakdown from stream-line to turbulent motion, occurs when the gradient of energy along the radius exceeds a certain definite value, and that this factor is the criterion of such a breakdown.