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Kinematical Diagrams for Conical Relativistic Jets

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Abstract. We present diagrams depicting the expected inter-dependences of two key kinematical parameters of radio knots in the parsec-scale jets of blazars, deduced from VLBI observations. The two parameters are the apparent speed ($v_{\text{app}} = c\beta_{\text{app}}$) and the *effective* Doppler boosting factor (δ_{eff}) of the relativistically moving radio knot. A novel aspect of these analytical computations of β–δ diagrams is that they are made for parsec-scale jets having a *conical* shape, with modest opening angles (ω up to 10°), in accord with the VLBI observations of the nuclei of the nearest radio galaxies. Another motivating factor is the recent finding that consideration of a conical geometry can have important implications for the interpretation of a variety of radio observations of blazar jets. In addition to uniform jet flows (i.e., those having a uniform bulk Lorentz factor, Γ), computational results are also presented for stratified jets where an ultra-relativistic central spine along the jet axis is surrounded by a slower moving sheath, possibly arising from a velocity shear.

Key words. Blazars: general—galaxies: active, quasars, jets, nuclei, radio continuum.

1. Introduction

Ejection of a pair of jets of relativistic material is the key manifestation of the radio-loud phase of Active Galactic Nuclei (AGN) and micro-quasars. On parsec scale, two basic attributes of relativistic jets of AGN, when directed appropriately close towards the observer, are: **(i)** an apparent superluminal motion and **(ii)** a strong Doppler boosting of the radiation received. Attempts to use these effects for probing the physics of AGN have been reviewed extensively in the literature (e.g., Begelman *et al.* 1984; Phinney 1985; Barthel 1994; Urry & Padovani 1995; Gopal-Krishna 1995; Scheuer 1996; Zensus 1997; Livio 1997; Ghisellini 2004; Giovannini 2004; Wiita 2006; Collin-Souffrin 2006). Until recently, quantitative discussion of both these properties was limited essentially to "collimated" jets (i.e., jets whose full opening angle, $\omega = 0$). However, recent VLBI observations provide good evidence for conical shape of AGN jets, with typical opening angle of several degrees on parsec scale. Salient examples are the nuclear radio jets of the nearest radio galaxies M87 (Biretta *et al.* 2002; Ly *et al.* 2004; Dodson *et al.* 2006) and Centaurus A (Horiuchi *et al.* 2006). Additional evidence comes from the studies by Tavecchio *et al.* (2004) and Jorstad *et al.* (2005). Conical geometry on parsec scale is also expected in certain theoretical models of jet formation (e.g., Meier *et al.* 2001). Thus, a fresh evaluation of the jet kinematics, taking into account the conical geometry of the jets, is warranted. This would also have a direct bearing on the schemes to relate the VLBI radio observations of blazar nuclei to their observed properties in the x-ray and γ-ray bands (e.g., Tavecchio *et al.* 2000; Celotti 2001; Kellermann *et al.* 2004) and to the energetics of the jet flow on parsec-scale (e.g., Sikora *et al.* 2005; Henri & Saugé 2006).

It has become evident from recent studies that considering a conical geometry of jets, even with modest opening angles, can have a major influence on the interpretation of several observed kinematical properties of the radio knots in the parsec-scale AGN jets, which usually remain largely unresolved in the VLBI observations (see Gopal-Krishna *et al.* 2004, 2006, hereafter referred to as Paper I and Paper II). This is particularly so when the jet's bulk speed is extremely relativistic ($\Gamma = 30$–100). Such extremely high bulk Lorentz factors of jets have in fact been inferred in a number of analyses of the rapid flux variability of blazars in the TeV range (e.g., Krawczynski *et al.* 2001; Konopelko *et al.* 2003; Henri & Saugé 2006) and at centimetre wavelengths (e.g., Qian *et al.* 1991; Wagner & Witzel 1995; Blandford 2001; Rickett *et al.* 2002; Macquart & de Bryun 2006). For the extensively observed jet of the quasar 3C273, the jet's bulk Lorentz factor, required to explain the radio and optical data is $\Gamma =$ 50–100 (Jester *et al.* 2006). Additional evidence for such ultra-relativistic jets comes from the detection of extremely superluminal radio knots from VLBI observations of several blazars, suggesting Γ to be above 20 to 60, or even more (e.g., Fujisawa *et al.* 1999; Jorstad *et al.* 2004, 2005; Piner & Edwards 2005). Note that as in Paper I & II, the conical jet flow has been assumed here to be *ballistic*, as also considered in many other studies (e.g., Falcke & Biermann 1996; Lobanov & Zensus 2001; Kaiser 2006).

Briefly, the main conclusions resulting from the consideration of modest opening angles of AGN jets ($\omega \sim 5$–10°, Papers I & II) are:

- The relatively slow (often subluminal) apparent motions of the VLBI radio knots of TeV blazars (e.g., Marscher & Marchenko 1999; Piner *et al.* 1999; Piner & Edwards 2004; Giroletti *et al.* 2004) can be reconciled with the ultra-relativistic jet speeds ($\Gamma = 30$–100) inferred from the flux variations mentioned above, without recourse to postulating phenomena like "pattern speed" (e.g., Lind & Blandford 1985; Vermeulen & Cohen 1994), or a spine-sheath jet flow (section 2).
- The viewing angle of an ultra-relativistic jet from the line-of-sight can be substantially larger than that inferred typically by combining the flux variability and proper motion measurements of the VLBI radio knots. This would normally imply not only an increased probability of the jet being observed, but also a smaller (and hence less uncomfortable) correction for foreshortening of the jet due to projection (e.g., see Schilizzi & de Bryun 1983).

2. Kinematical diagrams for uniform Lorentz factor jets

Very likely, the radio knots are manifestations of the relativistic shocks excited across the jet flow, e.g., due to instabilities in the flow speed, or some external perturbations (e.g., Marscher 1980; Laing 1980; Marscher & Gear 1985; Hughes *et al.* 1985; Laing 1993; Dermer & Chiang 1998; Spada *et al.* 2001). A useful conventional way to encapsulate the kinematical behaviour of the radio knots occuring in a fully collimated ($\omega = 0°$) relativistic jet is to display the Doppler factor (δ) against the apparent speed ($v_{\text{app}} = c\beta_{\text{app}}$), for a range of bulk Lorentz factors, Γ, and viewing angle, θ, of the jet (e.g., Nesci *et al.* 2005). Here we present numerical computations of the kinematics of the radio knots occuring in *conical* relativistic jets; elsewhere we have discussed in detail some implications of these results concerning the effect of the *conical* jet geometry on various estimated jet parameters (Gopal-Krishna *et al.* 2007). In accordance with the canonical picture, we assume the knots to coincide with the (relatively thin) transverse shocks in the jet flow, such that each knot is taken to have the shape of a circular disk of uniform intrinsic emissivity. Further, each surface element of the knot is assumed to move ballistically, with the bulk speed of the jet, thus maintaining the *conical* shape of the jet. The two sets of computations reported here correspond to the cases of (i) a uniform Γ across the radio knot (approximated with a transverse shock disc) and (ii) a radially decreasing Γ from the centre of the radio knot (i.e., the jet axis). This latter version approximates a transverse shock formed in a jet described by the currently popular "fast spine + slow sheath" (i.e., stratified) jet configuration of AGN jets, as considered by numerous authors (e.g., Baan 1980; Bicknell 1985; Sol *et al.* 1989; Pelletier & Roland 1989; Komissarov 1990; Laing 1993; Begelman *et al.* 1994; Swain *et al.* 1998; Hardcastle *et al.* 1999; Attridge *et al.* 1999; Chiaberge *et al.* 2000; Giovannini *et al.* 2001; Meier 2003; Giroletti *et al.* 2004; Ghisellini *et al.* 2005). Clearly, due to velocity shear, such knots would soon begin to deviate from a laminar flow and would therefore be more prone to distortion/disruption. However, consideration of such stages is beyond the scope of the present exploratory study.

Details of the analytical prescription employed here are available in Papers I & II. The basic equations for a discrete component moving in a jet with a bulk speed v directed at an angle θ from the line of sight, are:

$$\delta = \frac{1}{\Gamma[1 - \beta\cos(\theta)]} \tag{1}$$

where,

$$\beta = \frac{v}{c} \quad \text{and} \quad \Gamma = (1 - \beta^2)^{-1/2}, \tag{2}$$

$$\beta_{\text{app}} = \frac{\beta\sin(\theta)}{1 - \beta\cos(\theta)}, \tag{3}$$

$$S_{\text{obs}} = \int_{\Omega} \delta^p(\Omega')S_{\text{em}}(\Omega')d\Omega \equiv A(\theta)S_{\text{em}}, \tag{4}$$

$$\delta_{\text{eff}} = A^{1/p}(\theta), \tag{5}$$

$$\vec{\beta}_{\text{app,eff}} = \frac{1}{S_{\text{obs}}}\int_{\Omega} \vec{\beta}(\Omega')\delta^p(\Omega')S_{\text{em}}(\Omega')d\Omega. \tag{6}$$

Here, $S_{\rm em}$ and $S_{\rm obs}$ are the emitted and the (Doppler boosted) observed flux density and $A(\theta)$ is the flux boosting factor averaged over the (circular) radio knot. The viewing angle, θ of the jet measures the angular offset of the radio disc's center (i.e., the jet axis) from the direction of the AGN core. The parameter $p = n - \alpha$, where α is the spectral index (flux density $\propto$ frequency$^{\alpha}$). The term n is equal to 2 if the emission arises from a continuous jet and $n = 3$ if discrete knots are the dominant emitters (e.g., Scheuer & Readhead 1979; Phinney 1985; Lind & Blandford 1985; also, Ryle & Longair 1966). For the present purpose of compact radio sources, we shall adopt $\alpha = 0$.

The numerical integration was carried out by dividing the (circular) radio knot of diameter ω into small pixels, each being 1% of the disc's diameter. For each pixel (i, j), angular offset from the AGN core was computed and combined with the jet's bulk Lorentz factor (see below), to find: the Doppler factor, $\delta_{i,j}$, the vector $\vec{\beta}_{{\rm app}(i,j)}$; the flux boosting factor, $A_{i,j}$, taken to be $\delta^{p}_{i,j}$; and the product of the last two terms (which amounts to apparent velocity of the pixel, weighted by its apparent flux). Finally, the average values of each of these parameters computed for all the pixels across the radio disc were determined (e.g., $A_{\rm ave}$) and these are taken to be the *effective* values for the entire radio knot. Note that the effective δ for the knot is then $\delta_{\rm eff} = A^{1/p}_{\rm ave}$, where we have taken two values for p (= 2 and 3), widely employed in the literature for compact radio sources (see above). Further details of the method are available in Papers I and II.

2.1 *Kinematical diagrams for the 'stratified' (spine-sheath) jets*

To quantify the *spine-sheath* type stratification in the jet, we approximate the cross-section of the jet flow with a radially decreasing bulk Lorentz factor, $\Gamma(r)$, according to an exponential law with e-folding length equal to $\omega/2q$ [$\Rightarrow \Gamma(r) = \Gamma_{(r=0)} e^{(-2rq/\omega)}$], where r is the transverse angular separation from the centre of the radio disc/knot (i.e., from the jet axis). Two representative values for q (= 1 and 2) were chosen. (The choice of minimum $\Gamma_{(r=0)}$ is constrained by the requirement that the corresponding Γ at the jet's surface ($r' = \omega/2$) does not drop below the physical limit of $\Gamma = 1$.) Note that while the choice of an exponential form for $\Gamma(r)$ is admittedly arbitrary, it should be reasonable enough for the illustrative calculations presented here, particularly since no specific form of $\Gamma(r)$ has yet been established.

3. Results

The first set of kinematical diagrams, shown in the left column of Figs. 1 & 2 display the *effective* values of δ versus $\beta_{\rm app}$ of a VLBI radio knot, computed for a range of Γ and θ and taking three representative values of ω (= $1^\circ, 5^\circ$ and 10°) and two values of p (= 2 and 3), as mentioned above. The plots in the right column have been extracted from the corresponding plots in the left column, by zooming on the limited (but more commonly used) range of Γ up to 20. Note that for all these plots, Γ was assumed to remain constant across the radio emitting disc/knot (i.e., $q = 0$ case). In Figs. 3–6 we show the corresponding diagrams for the cases of transverse Γ gradient (i.e., Γ decreasing away from the jet axis), taking two values for the (exponential) scale length ($q = 1$ and 2) (section 2.1). All these β–δ diagrams provide a generalization to *conical* jets and spine-sheath geometries of the simple kinematical diagrams presented

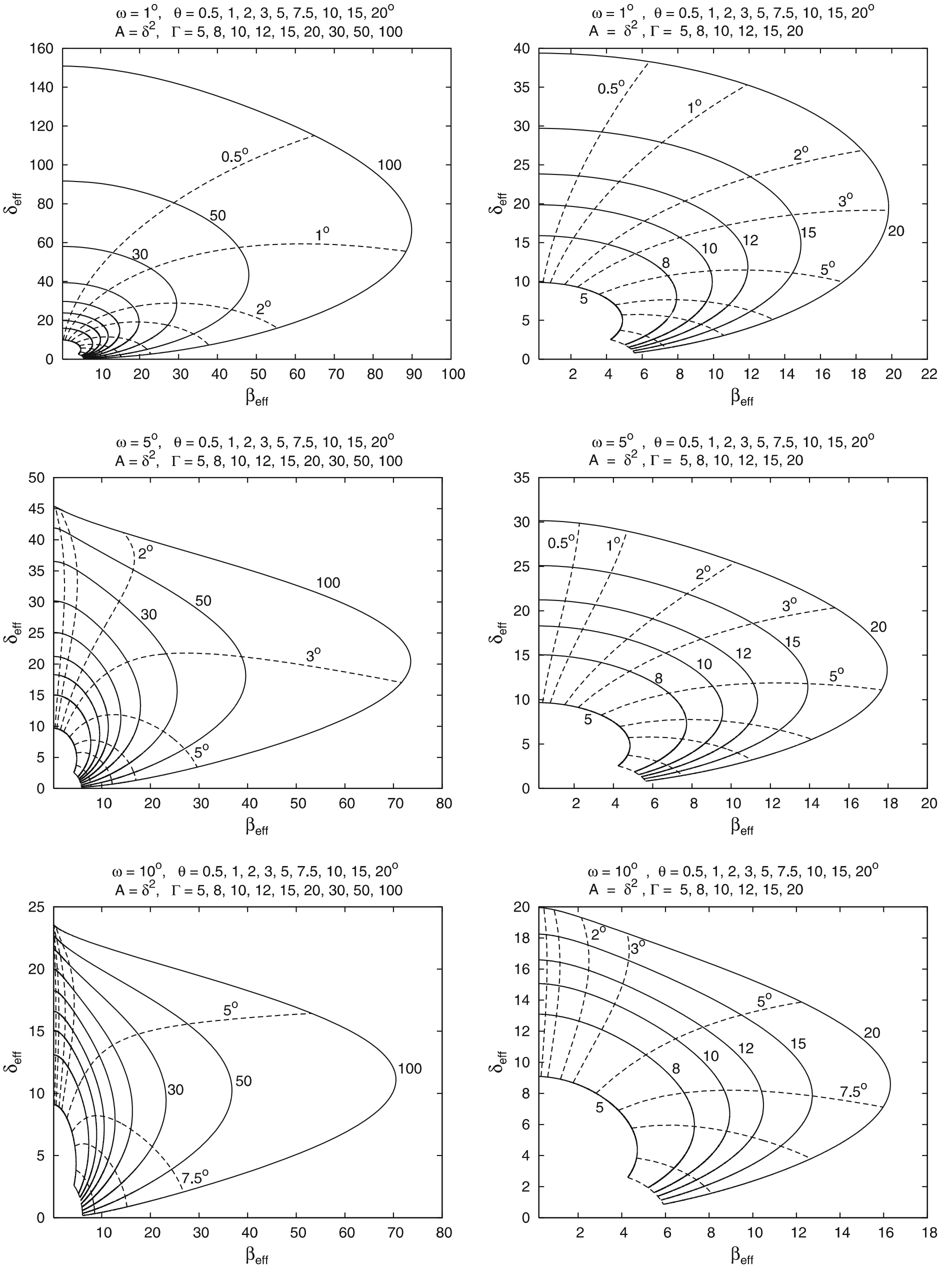

Figure 1. The curves show for a knot in a relativistic jet the computed *effective* values of Doppler factor (δ) and the apparent speed (in units of c), $\beta_{\rm app}$. Each solid curve is drawn for fixed bulk Lorentz factor (Γ) of the jet, whose value is shown besides the curve (when possible) and also at the top of each panel. The broken curves are for different values of the jet viewing angle (θ), as shown at the top of each panel. The panels on the right side show, for clarity, the zoomed versions of the inner parts of the corresponding panels to the left (section 3). All these panels refer to a Doppler boosting index $p = 2$ and a quotient $q = 0$ for the transverse gradient of the jet's bulk Lorentz factor (i.e., a uniform Γ case; section 2). The full opening angle of the jet, $\omega = 1^\circ$ for the top panels, 5° for the middle panels and 10° for the lower panels, as shown.

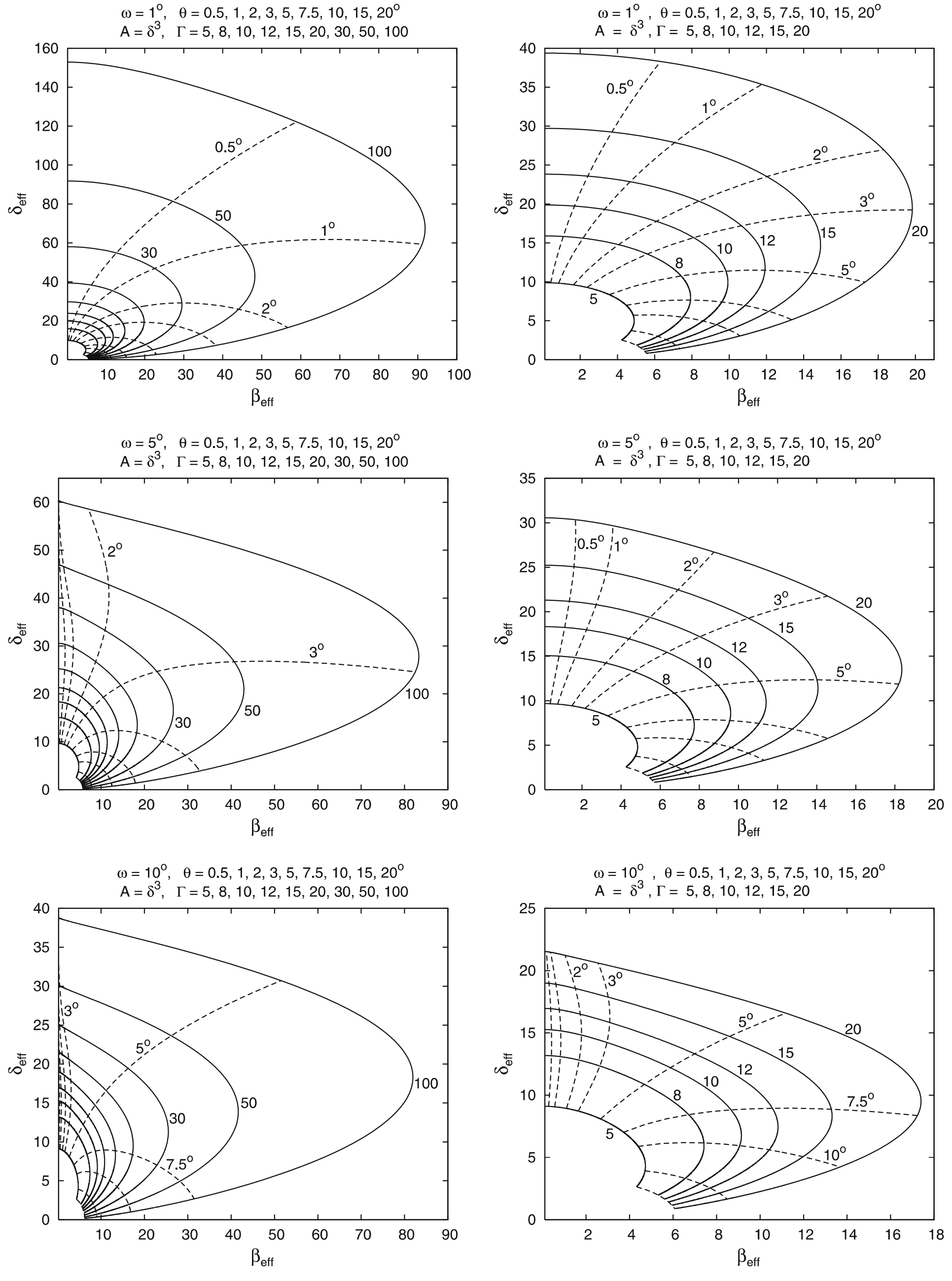

Figure 2. Same as Fig. 1, except that $p = 3$ and $q = 0$ (see the top of each panel).

by Nesci *et al.* (2005) for the case of fully collimated jets ($\omega = 0$). Hence, these generalized β–δ diagrams are likely to prove useful in quantitative interpretation of the VLBI and related multiband observations of blazars (see also Gopal-Krishna *et al.* 2007).

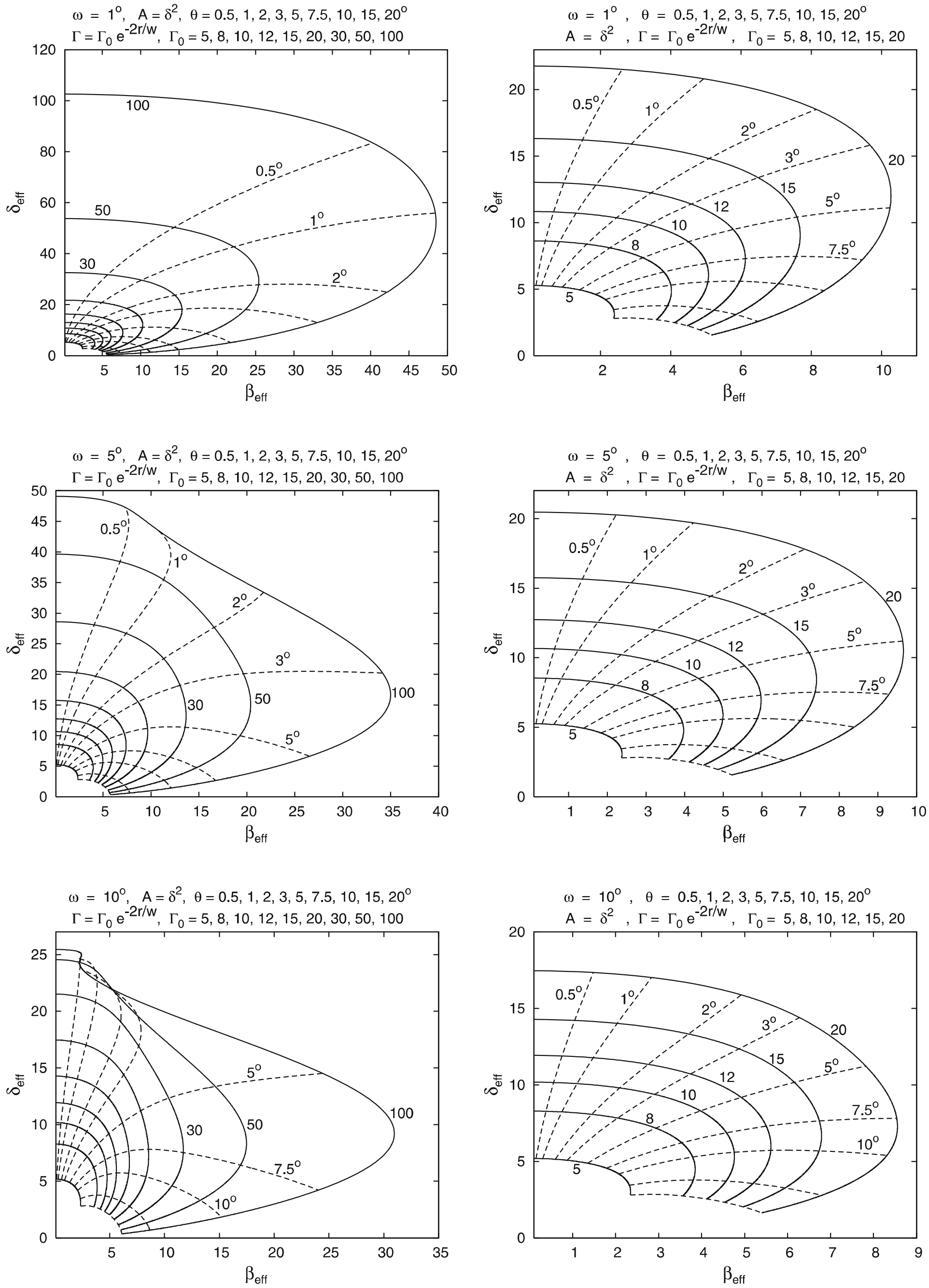

Figure 3. Same as Fig. 1, except that $p = 2$ and $q = 1$ (see the top of each panel). This is the case of a mildly stratified jet flow.

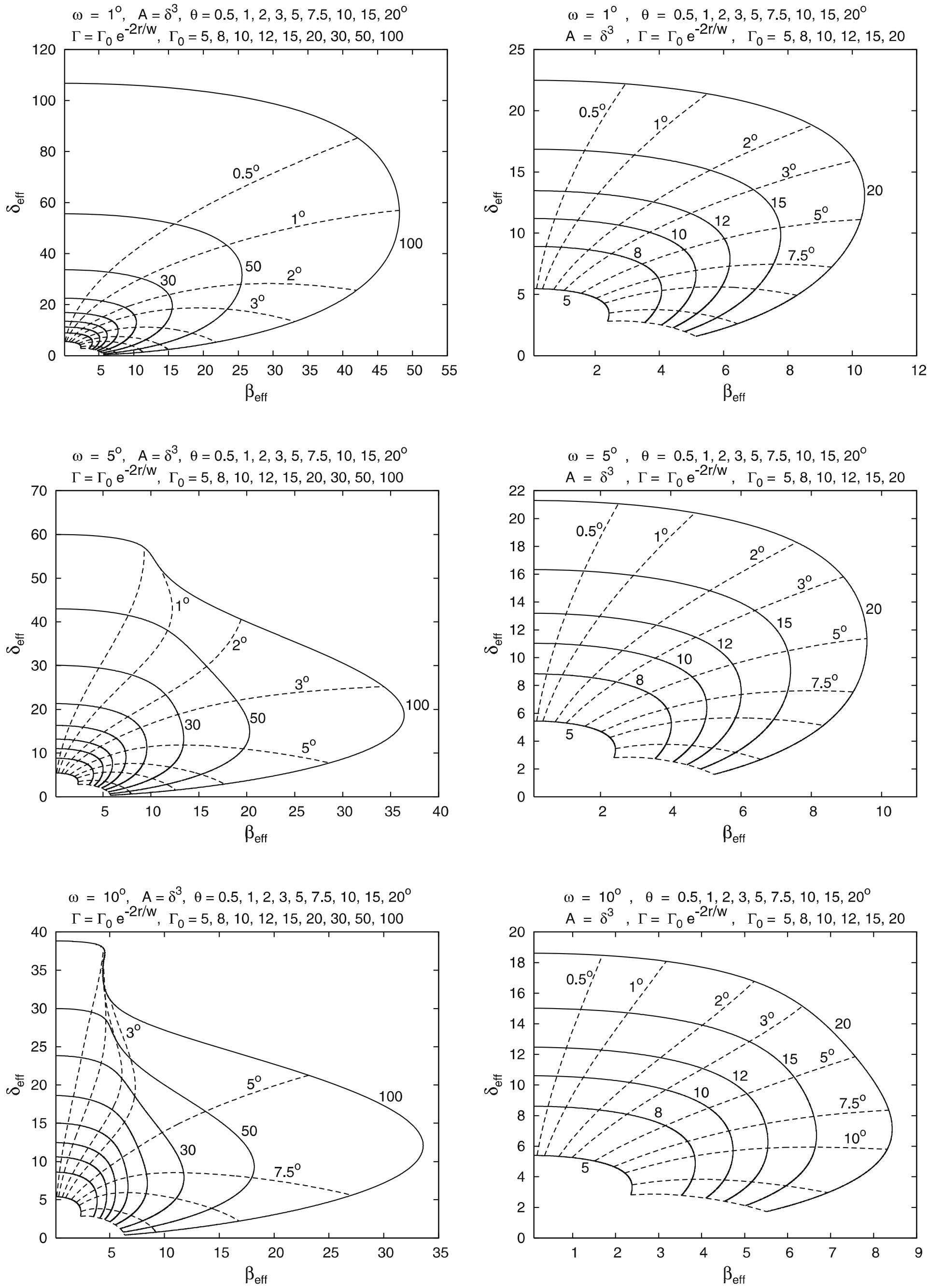

Figure 4. Same as Fig. 1, except that $p = 3$ and $q = 1$ (see the top of each panel). This is the case of a mildly stratified jet flow.

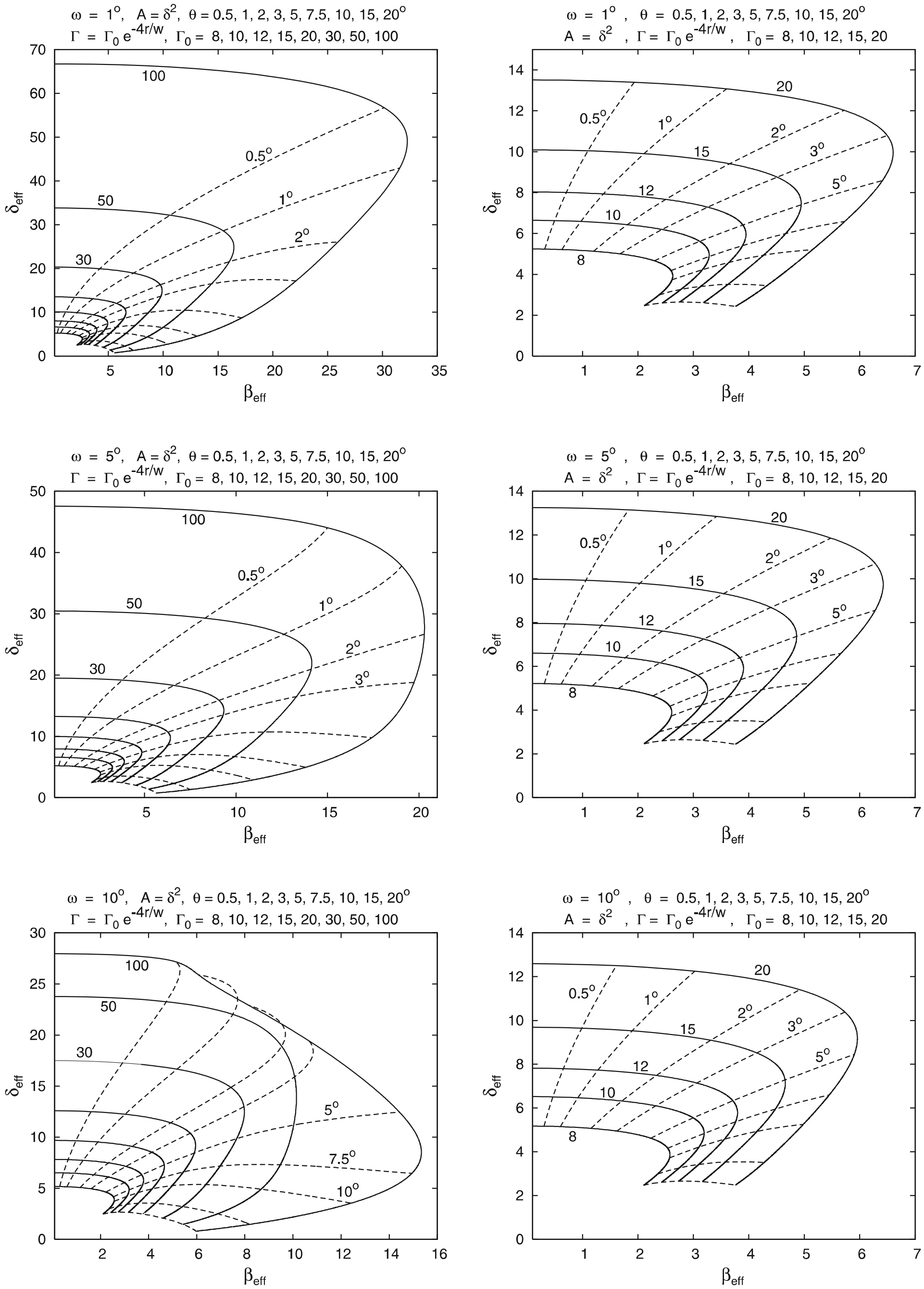

Figure 5. Same as Fig. 1, except that $p = 2$ and $q = 2$ (see the top of each panel). This is the case of a sharply stratified jet flow.

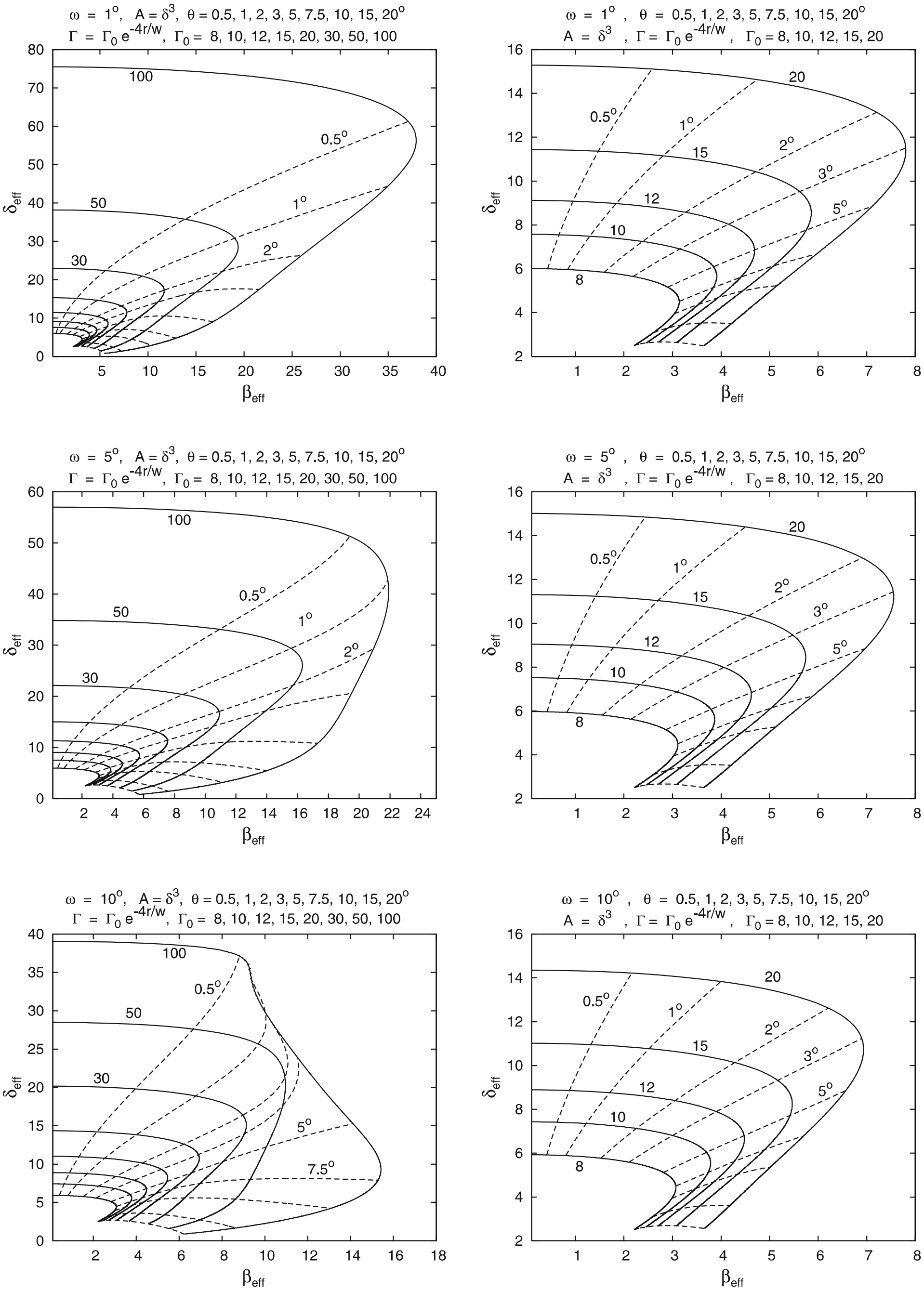

Figure 6. Same as Fig. 1, except that $p = 3$ and $q = 2$ (see the top of each panel). This is the case of a sharply stratified jet flow.

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