

Expansion of radio galaxies in a cosmologically evolving medium: possible implications for the cosmic star-formation history

Paramita Barai,^{1*} Gopal-Krishna,² M. Angela Osterman¹ and Paul J. Wiita¹

¹*Department of Physics & Astronomy, P.O. Box 4106, Georgia State University, Atlanta, GA 30302-4106, USA*

²*National Centre for Radio Astrophysics/TIFR, Post Bag No. 3, Pune University Campus, Pune 411 007, India*

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Abstract. We compare earlier estimates of the volumes filled by lobes of radio galaxies during the quasar era based upon non-evolving ambient media with new ones assuming a strong cosmological evolution of the ambient medium. If the sources remain active for over 10^8 years the volumes filled by them are found to be comparable for the two scenarios. This strengthens our earlier inference that much of the cosmic web of gaseous filaments, the site of galaxy formation, was probably permeated by radio lobes during the quasar era and this could have triggered extensive star formation and made large contributions to the spread of magnetic fields and metals through the universe by $z \sim 2$.

Keywords : galaxies: active – galaxies: jets – quasars: general – radio continuum: galaxies – stars: formation

1. Introduction

A steady decrease with redshift, z , of the physical size, D , of radio luminous quasars was first noted over three decades ago; with the parameterization $D \propto (1+z)^{-n}$, $n \sim 1$ was found (Legg, 1970; Miley, 1971). This same result was generalized by Kapahi (1975) to double-lobed radio sources of Fanaroff-Riley type II (Fanaroff & Riley, 1974), using the angular size–flux density plot derived from the Ooty lunar occultation survey

*e-mail:barai@chara.gsu.edu

(Swarup, 1975). Subsequent studies using deeper radio surveys indicated a steeper cosmic evolution, with $n \sim 2$ out to $z \sim 0.5$ (Kapahi, 1985) or even as high as $n \sim 3.5$ (Oort *et al.*, 1987). The cosmological evolution of linear size was first interpreted in terms of a systematic decrease in the ambient density due to the expansion of the universe ($\rho \propto (1+z)^3$) (e.g., Wardle & Miley, 1974). Following the discovery of X-ray emitting hot gaseous halos around massive elliptical galaxies, Baldwin (1982) considered a more realistic, power-law density profile of the ambient medium for the propagation of the jets.

The evolution of conical jets passing from such a power-law density distribution of the interstellar medium (ISM), and then into a cosmologically evolving intra-cluster medium (ICM), after crossing an ISM/ICM interface, was first considered analytically by Gopal-Krishna & Wiita (1987) and numerically by Rosen & Wiita (1988) and Wiita *et al.* (1990). These models were indeed able to account for the observed steep size–redshift evolution. However, were this outer confining gas to be identified with an all pervasive intergalactic medium (IGM), its assumed properties and redshift dependence would be inconsistent with limits on the Compton y parameter (e.g., Rosen & Wiita, 1991) set by the COBE microwave background radiation measurements (Smoot *et al.*, 1991).

A subsequent detailed analytical study also took into account the increase in the “radiative efficiency”, ϵ , of the synchrotron radio lobes surrounded by a denser medium (Gopal-Krishna & Wiita, 1991). There we showed that nearly half of the steep linear size evolution could be attributed to this ϵ factor, and the remainder could be explained in terms of a cosmological evolution of the King-type density profile of the hot gaseous halos of the massive ellipticals. However, later studies of the redshift–size–power relations indicated a somewhat weaker dependence of size on redshift (e.g., Neeser *et al.*, 1995). In a radical departure from this approach, Blundell & Rawlings (1999) more recently argued that the observed linear size evolution may well be an artefact of the “youth–redshift degeneracy” of radio sources, which is imposed by the steeply rising inverse Compton losses against the cosmic microwave background at earlier epochs (see Rees & Setti, 1968; Gopal-Krishna *et al.*, 1989), coupled with substantial adiabatic losses as the lobes expand.

Several of these interpretations rest on the simplifying assumptions that the shape and central density of the radial density profiles of the gaseous halos do not evolve with the cosmic epoch. In fact, Blundell *et al.* (1999, hereafter BRW) have claimed that there is no good evidence for a changing environment of most radio galaxies (e.g., Mulchaey & Zabludoff, 1998), even back to quite early epochs ($z \sim 3$). Nonetheless, several lines of observational evidence indicate changes in radio galaxy environment with redshift. At medium redshifts ($z \sim 0.5$), moderate to high power radio galaxies are found in environments richer by $\sim 2 - 3$ times, compared to their counterparts in the nearby universe ($z < 0.1$) (Hill & Lilly, 1991; also, Yates, Miller & Peacock, 1989). More distant ($z \sim 1$) radio galaxies in the 3CR catalogue are more frequently associated with moderately rich (proto-) clusters, whereas the nearby 3CR FR II galaxies are usually

found in more sparsely populated regions, implying that the cluster environments of the 3CR galaxies have changed with redshift (Best et al., 1998).

Good fits to the radio power, P , linear size, D , and z distributions of complete samples of radio galaxies selected at meter wavelengths were claimed by BRW, who assumed a non-evolving ambient medium and a constant radius for the hot-spots. Empirically derived radio source properties include a large active lifetime of the central engine, $\sim 5 \times 10^8$ yr (e.g., BRW; Barger et al., 2001; McLure & Dunlop, 2004) and a power-law beam-power function with a slope of about -2.6, based on matching complete samples of radio sources with essentially complete redshift data (BRW).

Employing the above results and assumptions, Gopal-Krishna & Wiita (2001, hereafter GKW01) showed that during the ‘quasar era’ ($z \sim 2-3$), much of the denser (proto-galactic) material in the universe (which was concentrated within the cosmic sheets and filaments) was directly impacted by the expanding lobes of the generations of radio galaxies born during that era. Denser clumps of gas scattered across those cosmic filaments could thus be compressed, yielding global starbursts, and the overpressured radio lobes could trigger, or at least, accelerate, the formation of entire new galaxies (GKW01; Gopal-Krishna and Wiita, 2003a, hereafter GKW03a; cf. Daly, 1990). They further argued that this picture of a radio lobe-filled early universe could not only explain the much higher star formation rate found at high redshifts (e.g., Archibald et al., 2001) but also readily account for the presence of magnetic fields in distant galaxies (GKW01; Gopal-Krishna et al., 2003 hereafter GKWO). Two entirely independent groups, approaching the problem from different directions, also concluded that QSOs were energetically capable of magnetizing the universe (Kronberg et al., 2001; Furlanetto & Loeb, 2001). Our scenario can also account for the widespread distribution of metals in the proto-galaxies seen at these high redshifts (Gopal-Krishna & Wiita, 2003b, hereafter GKW03b).

In this communication, we wish to revisit this issue by relaxing the simplifying assumption of a redshift-independent environment of powerful radio galaxies. Accordingly, we present a simple quantitative treatment of the beam propagation, roughly taking into account cosmic evolution of both the density and radial distribution of gas around radio-loud ellipticals. It is expected that this first-order analytic treatment will inspire detailed numerical simulations of this important problem.

2. Beam propagation in a cosmologically evolving medium

Recent studies of structure formation in the universe have led to a picture wherein the formation of stars and galaxies occurs inside a cosmic web of filaments containing the proto-galactic baryonic material (e.g., Cen et al., 2001). From these Λ CDM simulations, it is estimated that during the quasar era ($z \sim 2-3$), the filament contained $\sim 10\%$ of the total baryons but occupied only $\sim 3\%$ of the volume of the universe (e.g., Cen & Ostriker, 1999; Davé et al., 2001). The corresponding numbers for the present epoch are

estimated to be $\sim 50\%$ and $\sim 10\%$, respectively. Taken at face value, these estimates, together with the growth factor of the volume of the universe, $(1+z)^3$, suggest that the *mean* gas density in the filaments during the quasar era was roughly a factor of 30 (i.e., $[(0.1/0.5)(0.1/0.03)][1+2.5]^3$) higher than at the present epoch. This very different environment for RGs during that epoch is also indicated by the evidence cited above for intermediate z (Yates et al., 1991; Hill & Lilly, 1991; Best et al., 1998). We next consider the radial distribution of the ambient gas around the elliptical galaxy hosts of double radio sources. In all our computations of the volume attained by the radio lobes (GKW01), we have adopted a power-law (essentially King model) density profile for the hot gaseous halos: $n(r) = n_0(r/a_0)^{-\beta}$, with $n_0 = 1.0 \times 10^{-2} \text{cm}^{-3}$, $a_0 = 10 \text{ kpc}$, and $\beta = 1.5$, which is also used by BRW and is based on X-ray images of nearby massive ellipticals in groups (e.g., Sarazin, 1988; Mulchaey & Zabludoff, 1998). While such a typical radial density distribution is certainly appropriate for small redshifts, this may not be a good approximation at the high redshifts corresponding to the quasar era, which witnessed a $10^2 - 10^3$ times higher co-moving density of powerful radio-loud ellipticals (e.g., Jackson & Wall, 1999). At such early epochs, the cosmic filaments were accreting gas vigorously. The hot gas is likely to have been more uniformly distributed within the filaments, but was in the process of becoming increasingly non-uniform due to gravitational accretion onto the dark matter halos and galaxies existing or forming within the filaments (e.g., Cen et al., 2001; Viel et al., 2003).

For the gas distribution around the elliptical hosts at $z = 2 - 3$, we consider two models. Model 1 adopts the mean density distribution around the massive ellipticals found in the local universe (which has an essentially power-law profile; see above). However, given the lack of any reliable estimate for the degree of non-uniformity of the medium which had developed by such early epochs, we shall consider an alternative simplifying assumption as well. Thus, in our model 2, at $z \sim 2 - 3$, the galaxies within the filaments are taken to be immersed in an ambient gas of uniform density. These two gas density profiles adopted by us for the quasar era represent two extreme situations. The uniform density (for model 2) is $N(\sim 30)$ times higher than the mean density of the ambient gas in model 1, which was used in our previous analytical work (GKW01; GKW03a,b; GKWO), in which we had adopted an ambient density profile for $z = 2.5$ ellipticals identical to that found for the nearby massive ellipticals (and consistent with the inference of BRW).

To derive the mean gas density, we consider the mass of ambient gas contained within a sphere of radius R . From the mean density ratio (N) mentioned above, the mass within the spherical volume in the constant density case (model 2) is N times larger than in model 1, the case with a power-law density decline with radius. We average over the spherical volumes to calculate the mass contained in the sphere for model 1, and then after multiplying with the factor N , obtain the values of the constant density, ρ_2 , to be used in model 2. A few representative radii, $R = 3, 5$, and 10 Mpc are considered.

The temporal growth of the total extent of a radio source, assuming a cylindrical beam, as is roughly appropriate for large, powerful sources (e.g., Jeyakumar & Saikia,

2000) can be computed, and the total length of the cigar shaped radio cocoon for model 1 is given by (e.g., Kaiser et al., 1997)

$$D_1(t) = 3.6a_0 \left(\frac{t^3 Q_0}{a_0^5 \rho_0} \right)^{1/(5-\beta)}. \quad (1)$$

The cocoon length for the constant density case, model 2, is

$$D_2(t) = 3.6 \left(\frac{t^3 Q_0}{\rho_2} \right)^{1/5}. \quad (2)$$

Here $\rho_0 = m_p n_0$, t is the time for which the source has been expanding, Q_0 is the (assumed constant) power fed into each jet, and m_p is the mass of a proton. From the previous discussion we find that $\rho_2 = [3N\rho_0/(3-\beta)](a_0/R)^\beta$. We define

$$\eta(t) \equiv \frac{D_1(t)}{D_2(t)} = \left(\frac{t^3 Q_0}{a_0^5} \right)^{\beta/[5(5-\beta)]} \left(\frac{\rho_2^{1/5}}{\rho_0^{1/(5-\beta)}} \right). \quad (3)$$

For the two extreme scenarios, Fig. 1 shows the ratio, η , of the sizes attained at different ages of the radio source, taking $R = 5$ Mpc and considering several values of N . Even allowing for a wide range in both N and R we find that $0.2 < \eta < 2.0$. The total volumes filled by the lobes, for fixed N and R , but for three beam powers spanning most of the relevant range, are displayed in Fig. 2. These volumes were computed using the method of BRW (their Eq. 20). We see that for either model, volumes approaching or exceeding 1 Mpc^3 are found for all FR II radio source powers if the sources are (quasi-) continuously active for times exceeding 10^8 yr; the curves for $Q_0 = 5 \times 10^{44} \text{ erg s}^{-1}$ correspond to the weakest such sources (BRW; GKW01). Such large lobe volumes, when integrated over appropriate distributions in Q_0 and z (GKW01, GKW03a), imply that the radio lobes would fill substantial fractions of the cosmic web during the quasar era.

The total energy inputs into the radio lobes and the surrounding medium, $Q_0 t$, as well as the pressure-weighted volumes of the lobes, corresponding to the energies stored in the lobes, E (computed using Eqs. (9) and (20) of BRW), for the two models of ambient density profile are displayed in Fig. 3. This energy stored in the lobes has a more direct bearing on the efficacy of starbursts being triggered by the collapse of the denser and cooler gaseous clumps engulfed by the lobe. The comparable values of E_1 and E_2 (never different by more than a factor of 3) for the two models imply that the impact of the bow shocks on any clouds within the IGM will be similar under both of these scenarios. These results are not dramatically affected when we considered different values of N between 10 and 50 and R between 3 and 10 Mpc; those parameter values should roughly span the conditions prevailing during the quasar era.

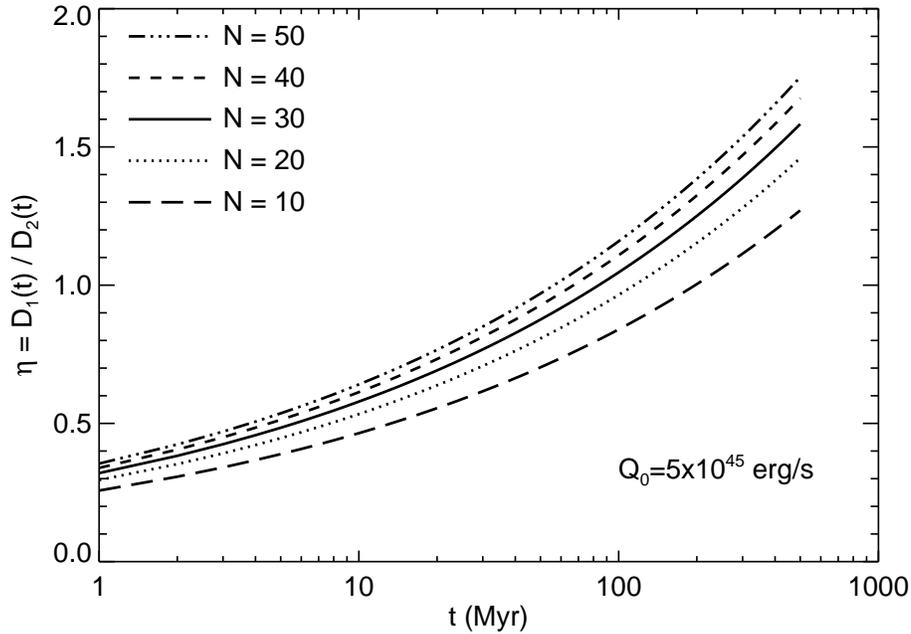


Figure 1. Distance ratios for the power-law model 1 to the enhanced constant density model 2 against source age for a typical FR II power, Q_0 and for several density enhancement factors, N ; $R = 5$ Mpc is assumed.

3. Discussion and conclusions

A rather unexpected result emerging from the above computation is that even if the average ambient density through which the lobes must penetrate is much greater during the quasar era, RGs can still grow to enormous sizes. The volumes that they can fill, while still significantly overpressured, can easily exceed 1 Mpc^3 . Furthermore, the total energy available to compress gaseous clouds in the IGM, and thereby trigger extensive star- and even galaxy-formation, is comparable for the models where jets propagate through unevolving galactic halos and those which assume propagation through a uniform ambient medium with mean density scaling roughly as $(1+z)^3$. Recent hydrodynamical simulations which include cooling of lobe-type shocks interacting with large clouds clearly reveal the formation of numerous dense cooling clumps which should continue to collapse into star clusters (Mellema et al., 2002; Fragile et al., 2004a,b). Observational support for this hypothesis comes from the evidence for a young stellar component in the extended optical emission revealed by the HST observations of high- z RGs (e.g. Best et al., 1996; Dey et al., 1997; Bicknell et al., 2000). In view of the above, our earlier inference concerning the role of radio galaxies in the cosmic star formation history (GKW01, GKW03b, GKWO) appears to hold considerable promise and needs further investigation.

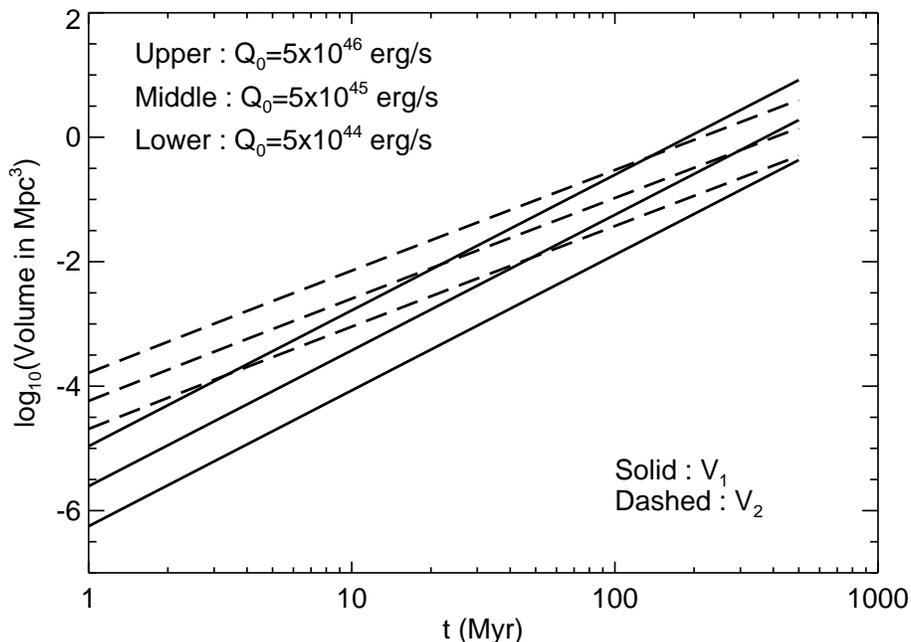


Figure 2. Total volumes filled by the lobes for the two different models as functions of time for several beam powers; $N = 30$ and $R = 5$ Mpc are assumed for model 2.

To summarize, our earlier key conclusion, that RG lobes could accelerate or even trigger the formation of many new galaxies during the quasar era (GKW01), appears to be fairly robust. In addition, the infusion of magnetic fields of significant strengths ($\sim 10^{-8}$ G, e.g., Ryu et al., 1998) into at least the cosmic web portion of the IGM certainly could have been caused by the lobes of radio galaxies (GKW01; Furlanetto & Loeb, 2001; Kronberg et al., 2001; GKWO). An alternative picture of magnetising the IGM comes from the superwinds driven by outflows from stars and galaxies (Kronberg et al., 1999). However, this situation would not naturally lead to preferential alignment between radio lobes and newly forming galaxies as has been observed (e.g., Best et al., 1996; Bicknell et al., 2000). Also, the radio loud AGNs outside clusters offer a potentially more energetic route for magnetisation of the wider IGM (Kronberg et al., 2001)

Finally, these expanding radio lobes could have swept out the metal-rich ISM of young galaxies they encounter (including that of the active hosts), thereby contributing substantially to the widespread metal pollution of the IGM (GKW03b). Recent evidence of super-solar metallicities in quasar nuclei at $z \sim 4$ (e.g., Dietrich et al., 2003) and substantial metallicity in even underdense regions of the IGM at similarly high z (e.g., Schaye et al., 2003) strongly hints at the need for an efficient means of spreading metals widely at early cosmic epochs. While the obvious sources for the production of metals detected in quasars are starbursts in the host galaxies, the possibility of nucleosynthesis

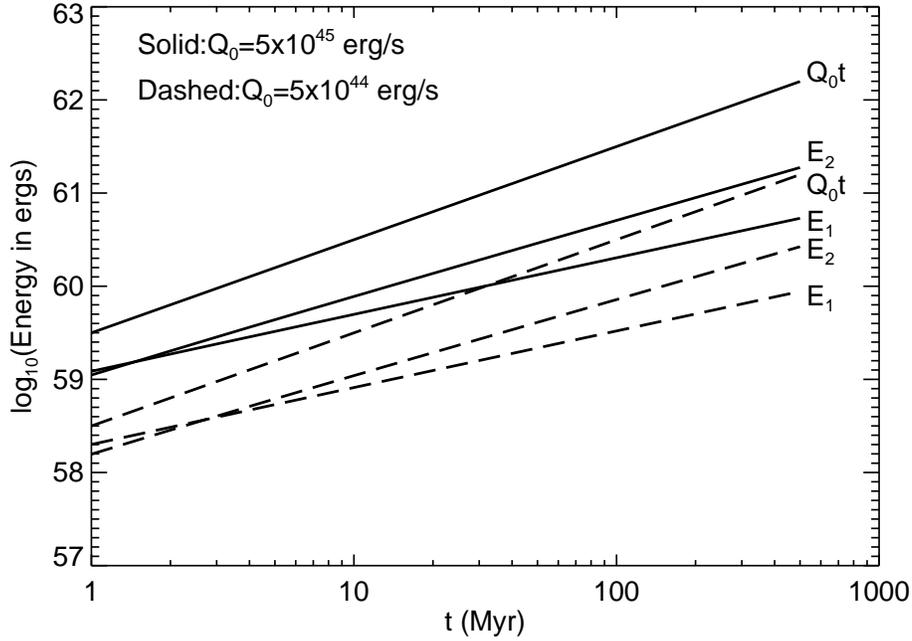


Figure 3. Injected energies ($Q_0 t$) and the energies stored in the lobes (E) at different ages, for the two models. $N = 30$ and $R = 5$ Mpc are adopted for model 2.

in the accretion disks feeding the central black holes (e.g., Mukhopadhyay & Chakrabarti, 2000; Kundt, 2002) also should be considered. It is possible that the superwind outflow model (Kronberg et al., 1999) would also contribute to the metalization of IGM, though that aspect of this scenario has not yet been investigated.

Here, we draw attention to a recent work by Rawlings & Jarvis (2004). While agreeing that RG lobes will penetrate much of the universe, they argue that this may often shut off star formation by expelling gas from protoclusters. However, unlike our picture (see, also, Rees 1989), they assume a single phase medium.

To test our picture, observation of Giant Radio Galaxies at low frequencies using the Giant Metrewave Radio Telescope (GMRT) (e.g., Konar et al., 2003), can be useful. Still more vital input is expected from upcoming low frequency radio telescope LOFAR (Low Frequency ARray), which holds the potential of observing high- z radio galaxies and, especially, the fading giant RG's at $z > 1$ (Röttgering, 2003).

In future work we will explore more sophisticated models where the jets first propagate through halos with evolving central densities and eventually enter the IGM, whose density will also be allowed to evolve. Models where the jets are conical (cf. Gopal-Krishna & Wiita, 1987, 1991), at least at early times (e.g., Jeyakumar & Saikia, 2000) will also be

considered. In addition, we will compare the resulting distributions of RG sizes, powers and redshifts against observational surveys so as to simultaneously constrain the range of evolutionary models for both the IGM and the radio source population.

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