A new cellular automata model for city traffic

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Abstract. We present a new cellular automata model of vehicular traffic in cities by combining ideas borrowed from the Biham-Middleton-Levine (BML) model of city traffic and the Nagel-Schreckenberg (NaSch) model of highway traffic. The model exhibits a dynamical phase transition to a completely jammed phase at a critical density which depends on the time periods of the synchronized signals.

1 Introduction

A one-dimensional cellular automata (CA) model of highway traffic and a two-dimensional CA model of city traffic were developed independently by Nagel and Schreckenberg (NaSch) [1] and Biham, Middleton and Levine (BML) [2], respectively[1]. Highway traffic becomes gradually more and more congested in the NaSch model with the increase of density. Traffic jams appear because of the *intrinsic stochasticity* of the dynamics but no jam persists for ever. On the other hand, a first order phase transition takes place in the BML model at a finite non-vanishing density, where the average velocity of the vehicles vanishes discontinuously signalling complete jamming. In the BML model, the randomness arises only from the *random initial conditions*, as the dynamical rule for the movement of the vehicles is fully deterministic [2].

In the BML model a square lattice models the network of the streets. Each site of the lattice represents a crossing of an east-bound and a north-bound street; it can be empty or occupied by a vehicle moving either to the east or to the north. The dynamics is controlled by signals of period 1 such that at odd time steps only the east-bound vehicles are updated whereas at even time steps only the north-bound vehicles are updated, both in parallel. During the updating a vehicle moves one site forward if the next site ahead is not occupied by any other vehicle. In this simplest version of the model lane changes (e.g. by turning) are not possible and the number of vehicles on each street is conserved separately.

In the NaSch model a highway is represented by a one-dimensional lattice of cells that can accomodate not more than one vehicle at a time. Each vehicle

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[1] For a review of the different approaches in modelling traffic flow we refer to [3] and references therein.

is characterized by a maximum velocity v_{max} and a randomization parameter p. The dynamics consists of four steps, each applied in parallel to all vehicles. In the first step all vehicles accelerate by 1 if they have not already reached the maximum velocity v_{max}. Step 2 is the interaction step. Vehicles which have d empty cells in front and a velocity $v > d$ reduce their velocity to $v = d$ in order to avoid a crash. In step 3 the velocity is reduced by one unit with probability p. In step 4 the vehicles move forward v cells where v is the new velocity after the randomization step 3.

2 Definition of the model

In the BML model the interplay between the vehicle dynamics and the time-scale set by the length of the signal period can not be studied. We therefore suggested a "unified" model [4] combining the BML model with the rules for the vehicle dynamics of the NaSch model. The lattice of our new model consists (in the simplest case) of N north-bound and N east-bound streets. The $N \times N$ crossings of these streets are arranged equidistantly. Between two consecutive crossings on a street there are $D - 1$ cells, i.e. each street has length $L = ND$ (see Fig. 1). The signals, installed at the crossings, are synchronized in such a way that all the signals remain green for the east-bound vehicles (and simultaneously, red for the north-bound vehicles) for a time interval T and then, simultaneously, all the signals turn red for the east-bound vehicles (and green for the north-bound vehicles) for the next T time steps before turning green again. This process is repeated so that there is a total time interval $2T$ between the beginning of two successive green (or red) phases of the signals.

As in the NaSch model the speed v of each vehicle can take one of the $v_{max} + 1$ integer values $v = 0, 1, ..., v_{max}$. Suppose, v_n is the speed of the n-th vehicle at time t while moving either towards east or towards north. In the initial state of the system, N_x (N_y) vehicles are distributed among the east-bound (north-bound) streets. Here we only consider the case $N_x = N_y = N_v/2$ where N_v is the total number of vehicles. Since in the initial configuration the occupation of a crossing is strictly avoided, the global density is defined by $\rho = N_v/N^2(2D-1)$. Also, suppose d_n is the distance to the next vehicle in front while s_n denotes the distance to the nearest crossing in front of it. At each *discrete time* step $t \to t+1$, the arrangement of vehicles is updated *in parallel* according to the following "rules":

– Step 1 (Acceleration):
   $v_n \to \min(v_n + 1, v_{max})$

– Step 2 (Deceleration due to other vehicles or signals):

   Case I: The signal is **red** for the n-th vehicle under consideration:
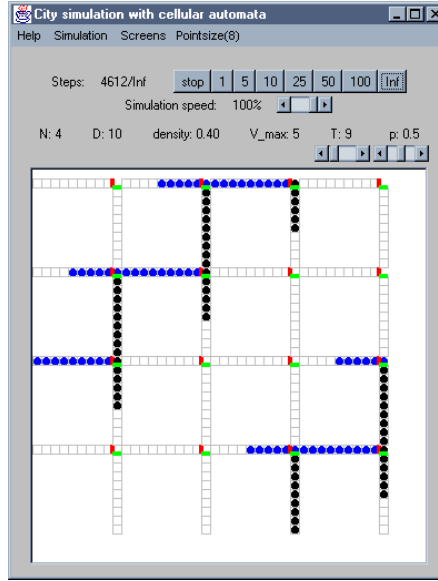 $v_n \to \min(v_n, d_n - 1, s_n - 1)$

Fig. 1. Structure of the underlying lattice for $N = 4$ and $D = 10$. Shown is a typical jammed configuration of the vehicles. The east-bound and north-bound vehicles are represented by the blue and black symbols, respectively.

    Case II: The signal is **green** for the n-th vehicle under consideration:
        If the signal is going to turn to red in the next timestep then
$$v_n \to \min(v_n, d_n - 1, s_n - 1)$$
        else $v_n \to \min(v_n, d_n - 1)$.

– Step 3 (Randomization):
$v_n \to \max(v_n - 1, 0)$ with probability p

– Step 4 (Movement):
$x_n \to x_n + v_n$.

    Note that we have simplified Case II of Step 2 in comparison to [4]. This simplification does not change the overall behaviour of the model [5].

    These rules are not merely a combination of the BML and the NaSch rules but also involve some modifications. For example, unlike all the earlier BML-type models, a vehicle approaching a crossing can keep moving, even when the signal is red, until it reaches a site immediately in front of which there is either a halting vehicle or a crossing. Moreover, if $p = 0$ every east-bound (north-bound) vehicle can adjust speed in the deceleration stage so as not to block the north-bound (east-bound) traffic when the signal is red for the east-bound (north-bound) vehicles.

3 Results

The variations of $\langle v_x \rangle$ and $\langle v_y \rangle$ with time (see Fig. 2) as well as with c, D, T and p in the flowing phase are certainly more realistic than in the BML model [4]. In the case of $\rho < \rho_c$ and for $v_{max} = 1$ the dynamics of the system can be described accurately by treating a single street with an improved version of the 2-cluster approximation [6], where the 2-cluster probabilities are equipped with a time and space dependence [5] (see Fig. 3) .

The fundamental diagram (Fig. 4) also shows a rather complex behaviour, at least for finite systems. E.g. the density corresponding to the maximum flux shifts to smaller densities with the decrease of T. Furthermore, the maximum throughput is a non-monotonic function of T in the "free-flowing" phase; this result may be of practical use in traffic engineering for maximizing the throughput.

A phase transition from the "free-flowing" dynamical phase to the completely "jammed" phase takes place in this model at a vehicle density ρ_c. The intrinsic stochasticity of the dynamics, which triggers the onset of jamming, is similar to that in the NaSch model, while the phenomenon of complete jamming through self-organization as well as the final jammed configurations (see Fig. 1) are similar to those in the BML model.

Due to the importance of finite-size and finite-time corrections it is not clear up to now how the critical density $\rho_c(D)$ depends on the dynamical parameters v_{max}, p and T. It is possible that in the thermodynamic limit $N \to \infty$ the density ρ_c is completely determined by the structure of the underlying lattice, i.e. by D, as long as $p > 0$ which is necessary for the jamming transition to occur. In that case ρ_c would be independent of v_{max}, p and T and the transition would be of 'geometrical' nature similar to the percolation transition. On the other hand, the transition could also be truly dynamical with ρ_c depending also on v_{max}, p or T. The data obtained so far from the computer simulations (see Fig. 5) do not conclusively rule out either of these two possible scenarios.

The "unified" model has been formulated intentionally to keep it as simple as possible and at the same time capture some of the interesting features of the NaSch model as well as the BML model. We believe that this model can be generalized (i) to allow traffic flow in both ways on each street which may consist of more than one lane, (ii) to make more realistic rules for the right-of-the-way at the crossings and turning of the vehicles, (iii) to implement different types of synchronization or staggering of traffic lights, e.g. green-waves [7].

Acknowledgment

References

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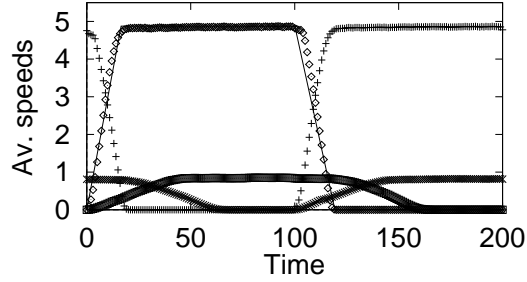7. P.M. Simon and K. Nagel: Phys. Rev. E **58**, 1286 (1998)

Fig. 2. Time-dependence of average speeds of vehicles. The symbols $+$, $\times$, $*$ and $\square$ correspond, respectively, to the average speeds $\langle v_x \rangle$, $\langle v_y \rangle$, and the fractions of vehicles with instantaneous speed $V = 0$, f_{x0} and f_{y0}, respectively. The common parameters are $v_{max} = 5, p = 0.1, D = 100, T = 100$ and $c = 0.1$. The continuous line has been obtained from heuristic arguments given in [4].
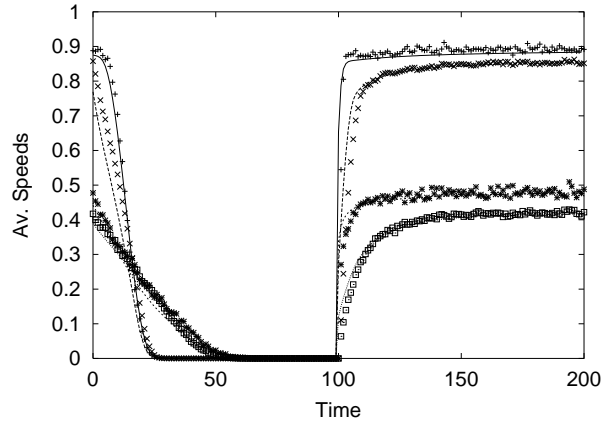

Fig. 3. Comparison between MC data and 2-cluster results. The common paramters are $v_{max} = 1$, $D = 25$, $T = 100$ and $N = 4$. The solid lines correspond to 2-cluster results. The symbols $+, \times, *, \square$ correspond to the ρ/p MC data sets $0.05/0.1, 0.25/0.1, 0.05/0.5, 0.25/0.5$ respectively.
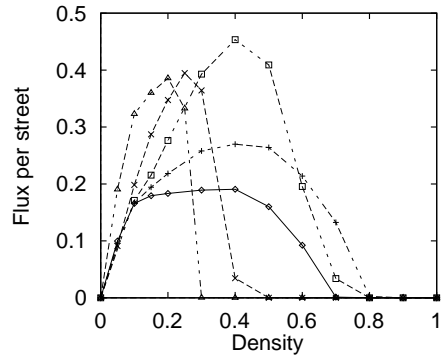
Fig. 4. Fundamental diagram for $v_{max} = 5, p = 0.5, L = 100$, and $D = 20$. The symbols $\diamond, +, \square, \times$ and $\triangle$ correspond, respectively, to $T = 100, 50, 20, 10, 4$.
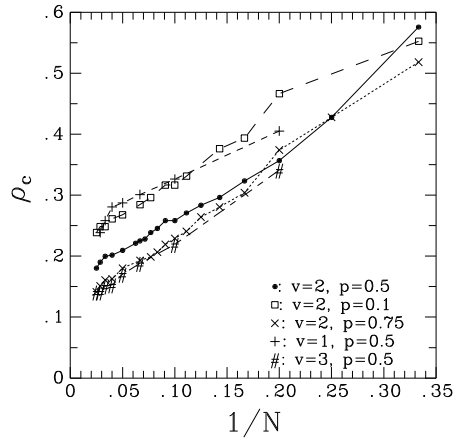

Fig. 5. Critical density ρ_c for different parameter combinations as function of the number of streets N. The common parameters are $D = 20$ and $T = 5$.