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Local field distribution revisited in the light of percolation model of spin glass transition

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Abstract. The existence of a 'hole' in the local field distribution $P(H)$ in canonical spin glasses is proved in the framework of Mookerjee and Chowdhury's percolation model of spin glass transition.

Keywords. Local field distribution; percolation model; spin glass transition.

1. Introduction

The local field distribution (LFD) $P(H)$ in spin glasses is a very important quantity because the configurationally averaged value A_{av} of any observable A is defined by

$$[A]_{av} = \int A[\{H\}]\,P[\{H\}]\,d[\{H\}].$$

This is based on the belief that the configurationally disordered system under consideration is spatially ergodic.

The existence (or, non-existence) of a 'hole' in $P(H)$ in a spin glass is a very controversial matter. Palmer and Pond (1979) showed that for Ising spin glasses with infinitely weak infinite range interaction (*i.e.* the Sherrington-Kirkpatrick (SK) model 1975) the local field distribution $P(H) = 0$ at $H = 0$ but $P(H) \propto H^{\alpha} (\alpha \geqslant 1)$ for small $|H|$. This was consistent with the Monte-Carlo simulation on the SK model (Palmer and Pond 1979; Kirkpatrick and Sherrington 1978). On the other hand, Schowalter and Klein (1979) showed that $P(H) = 0$ for $|H| < \delta$ and that there is a *square* hole in $P(H)$. The reason for this contradiction will become clear from the following arguments.

The original solution (Sherrington and Kirkpatrick 1975) of the SK model led to

$$(1-q) \sim \alpha T, \tag{1}$$

whereas the TAP solution (Thouless *et al* 1977) yields

$$(1-q) \sim \alpha' T^2, \tag{2}$$

where q is the Edwards-Anderson (EA) order parameter and α and α' are two constants. The choice of (1) rather than (2) is very crucial because the latter would predict no 'hole' in the LFD. Palmer and Pond (1979) used (2) whereas Schowalter and Klein (1979) used (1) and hence the contradiction in their results. To our knowledge, there is no convincing justification for using (1) rather than (2) for the SK model. However, this problem of choice between (1) and (2) will not arise in our approach here. The treatment presented in this paper will be based on our recent percolation model of spin glass transition which is applicable to realistic systems (Mookerjee and Chowdhury (MC) 1983).

In contrast with the infinitesimally weak infinitely long-ranged exchange interaction in the SK model, the interaction in MC's model is effectively finite-ranged, *e.g.* damped RKKY (Mookerjee and Chowdhury 1983; Abrikosov 1978, 1980). In this model the spin glass transition is modelled as a percolation of dynamically-correlated spins (see MC for details). The relation between this percolation model and some other models have been clarified (Chowdhury and Mookerjee 1983a). Besides, it leads to logarithmic relaxation of magnetization both in the absence as well as in the presence of external magnetic field (Chowdhury and Mookerjee 1983b, 1984a) and hence reveals a deeper connection between the $1/f$ noise spectrum in spin glasses and that in many other physical systems (Chowdhury and Mookerjee 1983c). The aim of this paper is to prove the *existence* of a 'hole' in the LFD in an Ising spin glass with realistic interaction. This is the first such proof because all the earlier proofs were developed for infinite ranged models.

2. Hole in LFD

Theorem:
In any Ising spin glass there exists a field δ such that $P(H) = 0$ for $-\delta \leqslant H \leqslant \delta$ at sufficiently low temperatures.
Proof:
To prove this theorem one has to go beyond the simple-minded Weiss mean field approximation and take the reaction field into account. We stress that one does not necessarily have to work with the TAP solution of the SK model (or, more generally, with any cooperative phase transition model) in order to take the effect of the reaction field into account. The physical origin of the reaction field is much more general and can be derived as follows: the local field produced at the jth site by the ith spin is $J_{ij}\, m_i$. This field induces a local moment $J_{ij}\chi_j\, m_i$ at the jth site, where χ_j is the local susceptibility at the jth site. This induced moment again produces a local field $J_{ij}^2\chi_j\, m_i$ at the ith site. Hence the reaction field produced at the ith site by all other spins $j (j \neq i)$ is $\sum_{j\neq i} J_{ij}^2\chi_j\, m_i$.

If the cavity field at the ith site (*i.e.* the local field at the ith site in the absence of the ith spin) is h_i, then

$$h_i = \sum J_{ij}\, m_j - \sum J_{ij}^2 \chi_j\, m_i,$$

or

$$H_i = \sum J_{ij}\, m_j = h_i + \sum J_{ij}^2\chi_j\, m_i. \tag{3}$$

Notice that so far we have not assumed anything about the nature of the spin glass transition, *i.e.* whether it is a cooperative phase transition or a gradual freezing process. Let us define the spatial average of a local property P_i by $\overline{P} = (1/N)\sum_i P_i$. We now *assume* that J_{ij} and χ_j can be averaged as independent random variables. Then

$$\overline{J_{ij}^2\chi_j} = A\chi, \tag{4}$$

where A is a temperature-independent constant and χ is average susceptibility. The right side of (4) is also independent of i because of statistical homogeneity in the thermodynamic limit.

EA cooperative phase transition model yields

$$\chi \propto (1-q)\beta, \tag{5}$$

whereas in the percolation model (MC) $\chi \propto (1-\phi q)\beta$ where $\phi(T)$ is the fraction of spins included in the infinite cluster at temperature T. This is so because in the percolation model only the fraction ϕ of the spins have finite value of long-time ($t \to \infty$) autocorrelation whereas in any cooperative phase transition model all the spins have finite value of long-time autocorrelation. Recently we have shown (Chowdhury and Mookerjee 1984b) that predictions of our percolation model will be consistent with experimental data on magnetic susceptibilities, both field cooled (*i.e.* equilibrium) and zero field cooled (*i.e.* non-equilibrium), carried out on metallic spin glasses by Yeshurun and Sompolinsky, provided

$$1-q = \alpha/\beta^2, \tag{6}$$

and

$$1-\phi q = \gamma/\beta, \tag{7}$$

at sufficiently low temperatures; α and γ being two constants independent of temperatures. Equations (6) and (7) are consistent with each other because $\phi < 1$ for all $T > 0$.

We also know that

$$m_i = \tanh(\beta h_i), \tag{8}$$

because local magnetisation is determined by the local cavity field h_i rather than the total Weiss field H_i. Therefore as $T \to 0$

$$m_i \to \text{sign}(h_i). \tag{9}$$

Substituting (7) and (9) into (3) we get

$$H_i = h_i + \gamma A \,\text{sign}(h_i), \text{ or, } |H_i| = |h_i| + |\delta| \tag{10}$$

where $|\delta| = \gamma A$.

The second term on the right side of (10) is a constant characteristic of the system and does not depend on field. Therefore, the magnitude of the local field *i.e.* $|H_i|$ can never be zero; in other words, the probability that $H_i = 0$ is zero. This proves the theorem.

Next let us study the detailed shape of the $P(H)$ curve. Extending Mookerjee's (1978) approach so as to include the effects of the reaction field we get

$$\begin{aligned} P(H) &= 1/(2\pi cqJ_1^2)^{\frac{1}{2}} \exp[-(H-\delta)^2/(2cJ_1^2 q)] \text{ for } H > \delta \\ &= 0 \text{ for } -\delta < H < \delta \\ &= 1/(2\pi cqJ_1^2)^{\frac{1}{2}} \exp[-(H+\delta)^2/(2c\, J_1^2 q)] \text{ for } H < -\delta, \end{aligned}$$

where c is the concentration of the magnetic constituent $J_0 = \sum_{\mathbf{R}} J(\mathbf{R})$ and $J_1^2 = \sum_{\mathbf{R}} J^2(\mathbf{R})$, $J(\mathbf{R})$ being the RKKY interaction. In other words, there is a square hole in the LFD in our percolation model. On the other hand, linear hole is observed in the Monte-Carlo simulation (Palmer and Pond 1979; Walker and Walstedt 1977). The reason for this disagreement is the same as that suggested by Schowalter and Klein (1979). However, we repeat the argument here for completeness. While writing (4) we assumed that J_{ij} and χ_j can be averaged as independent random variables. If the correlations between J_{ij} and χ_j could be incorporated (which is not possible at the present level of sophistication of our analytical arguments) the square hole might reduce to a linear hole. The correlations, of course, can be taken into account numerically. Strictly speaking, there is no rigorous proof of a linear rise of $P(H)$ with H for small H. From the stability analysis of the ground state of the SK model Palmer and Pond (1979) concluded that "$P(H) = O(H)$ for small H, but is entirely consistent with an H^2 or even an exponential approach to zero". However, the results of computer simulation on the SK model (Palmer and Pond 1979) is not necessarily applicable to our percolation model because of the difference in the nature of the corresponding interactions. Computer simulation with damped RKKY interaction should be performed to settle the question of the existence and the shape of the local field distribution in realistic systems.

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