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Dynamical scaling for spin waves in dilute ferromagnets

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Abstract. The ambiguity in the application of the principle of dynamic scaling to spin waves in dilute ferromagnets is resolved by taking account of the fractal nature of the infinite percolation cluster.

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The study of dynamical processes in dilute magnetic systems near percolation threshold, p_c, has received much theoretical and experimental attention in recent years (Cowley and Buyers 1972; Kirkpatrick 1973; Harris and Kirkpatrick 1977; Buyers *et al* 1982; Lewis and Stinchcombe 1984). Near the percolation threshold, the conventional multiple scattering methods are not useful as the disorder becomes very strong and it occurs at large length scales at which the multiple scattering methods break down. For such situations, the principle of dynamic scaling (Halperin and Hohenberg 1969), originally developed to deal with the dynamics of critical fluctuations, has proved very useful. Shender (1976) and Stauffer (1976) used this principle to arrive at some important results regarding spin waves and phase transitions in dilute ferromagnets. A study of the work of these authors shows that there is some ambiguity in the way the dynamical scaling can be used in this context. For example, Shender (1976) derives his results by matching the density of states (DOS) at small and high frequencies, while Stauffer (1976) derives his results by matching the dispersion relation at small and large wavevectors. These two procedures yield somewhat different results.

The purpose of this note is to show that these two procedures can be reconciled if an account of the fractal nature (Kirkpatrick 1979; Mandelbrot 1982) of the infinite percolation cluster is taken. The application of dynamic scaling to the spin wave problem rests on the fact that in disordered systems, there are two distinct wavevector regimes, in which the nature of excitations is different. These regimes and the nature of excitations depend upon a single parameter, which is the percolation correlation length, to be denoted by ξ_p. ξ_p is the measure of the length scale at which the IPC is homogenous. When the wavevector $\mathbf{k} \ll \xi_p^{-1}$, we have the spin-wave modes with the dispersion

$$\omega(\mathbf{k}) = D(p)k^2, \tag{1}$$

where p denotes the concentration of the magnetic atoms and $D(p)$ is the concentration dependent spin wave stiffness, as $p \to p_{c+}$

$$D(p) = D_0(p-p_c)^{\mu} \tag{2}$$

Due to inter-relationship between the spin wave problem and the problem of conductance of a randomly diluted network (Brenig *et al* 1971), $D(p)$ is related to the conductance $\Sigma(p)$, through the relation

$$\Sigma(p) = P_\infty(p)D(p), \tag{3}$$

where $P_\infty(p)$ denotes the fraction of occupied sites belonging to the infinite cluster (IPC). As $p \to p_{c+}$, $\Sigma(p)$ vanishes like $(p-p_c)^t$, then (2) implies that $\mu = t - \beta_p$, where β_p is the usual percolation index. Equation (1) implies the following DOS on the infinite cluster in d-dimensions

$$\rho_{\text{inf}}(\omega) \propto \frac{\omega^{d/2-1}}{D_0^{d/2}}(p-p_c)^{-d(t-\beta_p)/2}. \tag{4}$$

The above form is obviously valid only for $\omega \ll \omega_{c0} = D\xi_p^{-2}$. The excitations with $\omega \gg \omega_{c0}$ correspond to wavelengths much smaller than ξ_p. At these length scales, the cluster is highly inhomogeneous and according to Shender (1976), the excitations have a quasi-localized character, so that $\rho_{\text{inf}}(\omega)$ does not depend upon ξ_p or $p-p_c$ apart from the trivial geometric factor of $P_\infty(p)$. Thus Shender proposed for $\omega \gg \omega_{c0}$

$$\rho_{\text{inf}}(\omega) = P_\infty(p)f(\omega/J), \tag{5}$$

where J is the exchange constant. Now according to the continuity implied in dynamic scaling, (4) and (5) must match at $\omega = \omega_{c0}$. Matching these expressions, Shender finds

$$f(\omega/J) \propto (J/\omega)^x, \tag{6}$$

with

$$x = \frac{t-(d-2)\nu_p}{t+2\nu_p-\beta_p}, \tag{7}$$

where ν_p again denotes the usual critical exponent corresponding to percolation transition.

Stauffer (1976) on the other hand proposed a different procedure for matching the two frequency regimes. He wrote a scaling form for the dispersion relation in the following form

$$\omega = k^z f(k\xi_p), \tag{8}$$

with

$$f(y) \propto y^{2-z}, \quad \text{as} \quad y \to 0 \tag{9a}$$

$$\propto \text{constant} \quad \text{as} \quad y \to \infty \tag{9b}$$

This yields

$$\omega \propto \xi_p^{2-z}k^2 \qquad k\xi_p \ll 1 \tag{10a}$$

$$\omega \propto k^z \qquad k\xi_p \gg 1 \tag{10b}$$

From (10a), (1) and (2), one finds

$$z = 2 + (t-\beta_p)/\nu_p. \tag{11}$$

Stauffer's procedure is more in keeping with the original formulation of Halperin and Hohenberg (1969). Let us see now what (10b) implies for DOS at $\omega \gg \omega_{c0}$. The usual procedure gives

$$\rho_{\text{inf}}(\omega)\,\mathrm{d}w \propto k^{d-1}\,\mathrm{d}k, \tag{12}$$

$$\propto \omega^{d/z-1}\,\mathrm{d}\omega. \tag{13}$$

Substituting the value of z from (11), (12) gives

$$x = \frac{t-\beta_p-(d-2)\nu_p}{t-\beta_p+2\nu_p}, \tag{14}$$

which is different from (7).

The cause of the discrepancy lies in the use of (12). Many authors (Kirkpatrick 1979; Mandelbrot 1982) have recently pointed out, and numerically verified that the IPC is a fractal at length scales smaller than ξ_p, with the fractal dimension $\bar{d} = d-\beta_p/\nu_p$. The density of states on a fractal can be calculated by use of a scaling relation (Kumar 1982; Rammal and Toulouse 1983). For spin waves, this relation takes the form

$$\rho_{\text{inf}}(\omega) = b^{2-\bar{d}}\rho(b^2\omega). \tag{15}$$

For excitations having a dispersion relation given by (10b), one can generalize, (15) in the following way (Kumar 1982; Rammal and Toulouse 1983). Consider a sample of macroscopic linear of size L. Its DOS $\rho_L(\omega)$ is

$$\rho_L(\omega) = \Sigma_i\delta[\omega-\omega_i(L)]. \tag{16}$$

Let the size of the sample be changed by a factor b: $L \to bL$. Then $\omega_i(L) \to \omega_i(L)/b^z$. Now since DOS is an extensive variable

$$\rho_{bL}(\omega) = b^{\bar{d}}\rho_L(\omega),$$

$$= \Sigma_i\rho\delta(\omega-\omega_i/b^z) = b^z\rho_L(b^z\omega), \tag{17}$$

or

$$\rho_L(\omega) = b^{z-\bar{d}}\rho_L(b^z\omega). \tag{18}$$

Equation (17) implies that $\rho(\omega) \propto \omega^{\bar{d}/z-1}$ and

$$x = \frac{t-\beta_p-(\bar{d}-2)\nu_p}{t-\beta_p+2\nu_p} = \frac{t-(d-2)\nu_p}{t-\beta_p+2\nu_p}, \tag{19}$$

which is the same as Shender's result. This seems to us to be a more natural way to arrive at Shender's result.

Alexander and Orbach (1982) addressed this problem by relating it to diffusion problem and anomaly of diffusion on a fractal. Dhar (1977) and Alexander and Orbach (1982) define a spectral dimension $\bar{\bar{d}}$, through the relation $x = 1-\bar{\bar{d}}/2$ and find that $\bar{\bar{d}} = 2\bar{d}/(2+\theta)$, where θ is the index of anomalous diffusion. Comparison with above relations yields $z = 2+\theta$ and $\theta = (t-\beta_p)/\nu_p$. Such relations further underscore the relationships between physical processes like conduction, diffusion, spin-waves and phonons etc. The basic equations of motion involved in these processes are linear and contain the finite difference analog of the Laplacian operator. So all the processes are governed by the spectral dimension of the IPC, which incorporates the dynamic exponent z, as well as the anomalous diffusion index θ.

To conclude, we have pointed out how the two formulations of dynamic scaling for spin waves in dilute ferromagnets can be reconciled through an explicit realization that the IPC on which the spin waves propagate is a fractal at small length scales and the DOS on a fractal depends upon the dynamic exponent z and the fractal dimension $\bar{d}$.

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References

Alexander S and Orbach R 1982 *J. Phys. (Paris) Lett.* **43** L625

Brenig W, Wölfle P and Dohler G 1971 *Z. Phys.* **246** 1

Buyers W J L, Bertrand D, Locke K and Stager C V 1982 in *Excitations in disordered systems* (ed.) M F Thorpe (New York: Plenum Press) p. 411

Cowley R A and Buyers W J L 1972 *Rev. Mod. Phys.* **44** 406

Dhar D 1977 *J. Math. Phys.* **17** 577

Halperin B I and Hohenberg P C 1969 *Phys. Rev.* **177** 952

Harris A B and Kirkpatrick S 1977 *Phys. Rev.* **B17** 542

Kirkpatrick S 1973 *Rev. Mod. Phys.* **45** 574

Kirkpatrick S 1979 in *Ill-condensed matter* (eds) R Balian, R Maynard and G Toulouse (Amsterdam: North Holland) p. 323

Kumar D 1982 *Phys. Rev.* **B25** 2069

Lewis S J and Stinchcombe R B 1984 *Phys. Rev. Lett.* **52** 1021

Mandlebrot B B 1982 *The fractal geometry of nature* (San Fransisco: Freeman) Chap. 13

Rammal R and Toulouse G 1983 *J. Phys. (Paris) Lett.* **44** L13

Shender E F 1976 *Zh. Eksp. Teor. Fiz* 2251 (1976 *Sov. Phys. JETP*) **43** 1174

Stauffer D 1976 in *Amorphous magnetism II* (eds) R A Levy and R Hasegawa (New York: Plenum Press) p. 17