SEISMIC DETERMINATION OF SOLAR HEAVY ELEMENT ABUNDANCES

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ABSTRACT

The recent downward revision of solar photospheric abundances of Oxygen and other abundant heavy elements has caused a serious discrepancy between the standard solar model and the seismically determined solar structure. In this work we attempt to determine the heavy element abundance in the solar convection zone, using the dimensionless sound-speed derivative. This should provide an independent check of the spectroscopically determined abundances.

Key words: Sun: oscillations; Sun: abundances; Sun: interior.

1. INTRODUCTION

Recent analyses of spectroscopic data using modern atmospheric models have suggested that the solar abundance of oxygen and other abundant elements needs to be revised downwards (Asplund et al. 2004). Asplund et al. (2005; henceforth ASP) claim that the oxygen abundance should be reduced by a factor of about 1.48 from the earlier estimates of Grevesse & Sauval (1998; henceforth GS). The reduction in Z reduces opacity, in turn reducing the depth of the convection zone (CZ) in solar models computed using the new abundances (Bahcall & Pinsonneault 2004).

The new abundances along with the current OPAL opacity tables (Iglesias & Rogers 1996) are not consistent with seismic constraints (Basu & Antia 2004; Antia & Basu 2005; Bahcall et al. 2004), and that the opacity needs to be increased by 11–20% to restore agreement. Independent calculation by the OP project (Badnell et al. 2005) finds that the opacity values near the base of CZ are within 1–2% of OPAL values.

Ayres et al. (2006) find O abundance close to the old GS value. Thus it is possible that there are significant uncertainties in spectroscopic determination of abundances. In this work we try to get an independent estimate of heavy element abundances in the Sun using seismic data in a manner similar to that used to determine Helium abundance (e.g., Däppen et al. 1991; Antia & Basu 1994).

2. THE TECHNIQUE

The adiabatic index and hence the sound speed is lowered in the ionization zone. This can be seen in the dimensionless gradient of sound speed, which can be used, for example, to measure the helium abundance (Gough 1984). We define the dimensionless sound-speed gradient

$$W(r) = \frac{1}{g}\frac{dc^2}{dr}, \tag{1}$$

where, g is the acceleration due to gravity. In the adiabatically stratified region of the CZ, $W(r)$ is related to the adiabatic indices and $W(r) = -2/3$, if the material is fully ionized. The deviation of $W(r)$ from this value in the ionization zones can in principle, be used to measure the heavy element abundances. Since different ionization stages of various heavy elements overlap with each other, it is difficult to isolate the effect of individual heavy element, but it should be possible to estimate the total heavy element abundance from $W(r)$ in the lower part of the CZ.

To infer $W(r)$ in the Sun we use the Regularized Least Squares (RLS) inversion to infer the sound-speed in the Sun (e.g., Antia 1996). The RLS technique is used here, since it is convenient to differentiate the resulting sound speed profile to compute $W(r)$. The inverted $W(r)$ can be calibrated for heavy element abundances using solar models with different abundances. In particular, we use the average value of $W(r)$,

$$\langle W(r)\rangle = \frac{\int_{r_1}^{r_2} W(r)\,dr}{r_2 - r_1}, \tag{2}$$

in different radius intervals to determine the heavy element abundance.

The $W(r)$ profile will also depend on the EOS and other parameters in the solar model. We study the sensitivity of $W(r)$ to various parameters, to estimate the systematic errors in this technique. In order to use this technique we need to construct solar models with different heavy element mixtures. Unfortunately, the standard EOS like OPAL (Rogers & Nayfonov 2002) use a fixed mixture of heavy elements and hence cannot be used for this purpose. Hence we use solar models constructed using the CEFF EOS (Eggleton et al. 1973; Guenther et al. 1992; Christensen-Dalsgaard & Däppen 1992).

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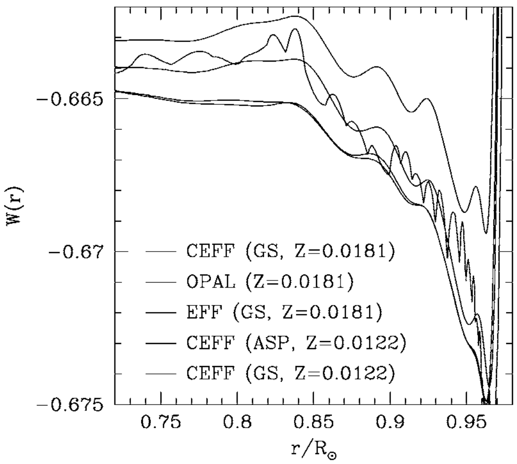

Figure 1. The function $W(r)$ for solar envelope models constructed with different equations of state, heavy-element mixtures, and abundances as marked in the figure. All these models have their CZ base at $r_b = 0.7133R_\odot$, and have a hydrogen abundance of $X = 0.739$.

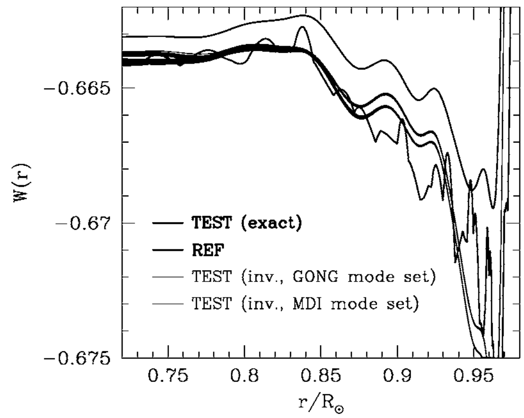

Figure 2. The actual $W(r)$ for a test model constructed with the OPAL equation of state compared to that inferred from inversions using an EFF model as the reference model. The inversion results using 50 realizations of random errors from GONG and MDI mode-sets are shown. The blue/green line shows the result for each of the 50 realizations for the GONG and MDI sets. Since the lines are close together, the result appears as one thick line.

3. RESULTS

To study the sensitivity of $W(r)$ to EOS and heavy element abundances, we construct solar models with different EOS and heavy element abundances and the results are shown in Fig. 1. It is clear that CEFF EOS yields results which are close to OPAL and hence can be used to determine Z. The difference between models with different Z is clear and it should be possible to use this to determine Z. To study the effect of different sources of systematic errors, we look at the difference in $\langle W(r) \rangle$ between solar models constructed with different parameters (and input physics) and a standard model constructed with CEFF equation of state (Antia & Basu 2006).

To test the inversion technique we construct a test model using OPAL EOS and attempt to infer $W(r)$ using the same set of frequencies as are available in observed data sets. The result using an EFF model as the reference model is shown in Fig. 2. Errors were added to the model frequencies before inversions. It is clear that inversions are fairly successful in inferring $W(r)$. To get a quantitative estimate of Z we plot in Fig. 3 $\langle W(r) \rangle$ in three different radial intervals.

To infer the heavy element abundance in the Sun we use the observed frequencies from GONG and MDI to infer $W(r)$. We use 97 sets of frequencies from GONG and 42 sets from MDI and for each set we use three different reference models using EFF, OPAL and CEFF EOS. Using the inverted $W(r)$ shown in Fig. 4, and the calibration curves shown in Fig. 3 we infer the heavy element abundance in the Sun. The results for both the test models and the Sun are shown in Table 1. All abundances are in terms of percentages.

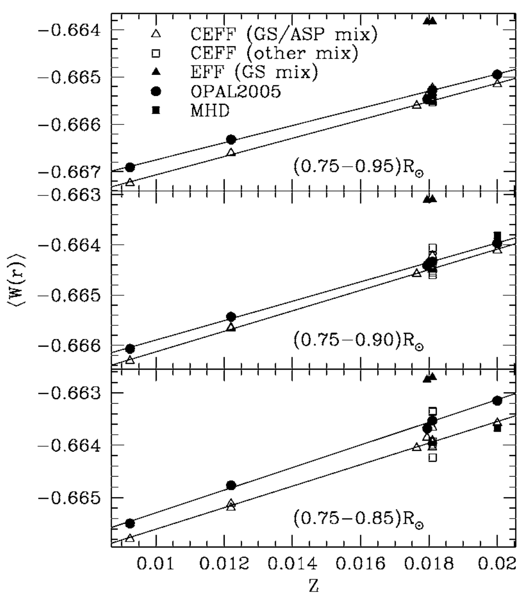

Figure 3. The average value of $W(r)$ in three different radius ranges plotted as a function of Z for solar models with different equations of state and mixtures of heavy elements as marked in the figure. The CEFF models with 'other mixtures' include models constructed with the abundance of some individual elements increased by a factor of 2 as compared to GS value. The reference and test models used in this study are also included in this figure. The blue line is the best-flt line through CEFF models of different Z, the green line is that through OPAL models, and these lines are used to determine Z of the test models and the Sun using inverted $\langle W(r) \rangle$.

Table 1. The heavy element abundance, Z (in percentages) as inferred using average values of $W(r)$ in different radius ranges for two test models and for the Sun. The errorbars quoted in the table are the respective standard deviation of all estimates and do not include other systematic errors. The mean value for each test model and observed frequencies is an average over all estimates listed here and the error bar is the standard deviation of these values.

Test	Calib.	GONG data sets			MDI data sets		
model	model	$(.75,.85)R_\odot$	$(.75,.90)R_\odot$	$(.75,.95)R_\odot$	$(.75,.85)R_\odot$	$(.75,.90)R_\odot$	$(.75,.95)R_\odot$
OPAL	CEFF	$1.91 \pm .01$	$1.93 \pm .01$	$1.95 \pm .01$	$1.91 \pm .01$	$1.88 \pm .01$	$1.85 \pm .01$
$Z = 1.795$	OPAL	$1.72 \pm .01$	$1.77 \pm .01$	$1.84 \pm .01$	$1.72 \pm .01$	$1.72 \pm .01$	$1.73 \pm .01$
				Mean value $Z = 1.83 \pm 0.09$			
OPAL	CEFF	$1.36 \pm .02$	$1.36 \pm .01$	...	$1.36 \pm .02$	$1.28 \pm .02$	$1.40 \pm .01$
$Z = 1.22$	OPAL	$1.20 \pm .02$	$1.23 \pm .01$	...	$1.20 \pm .02$	$1.16 \pm .02$	$1.25 \pm .01$
				Mean value $Z = 1.28 \pm 0.09$			
Obs.	CEFF	$1.79 \pm .04$	$1.83 \pm .09$	$1.81 \pm .03$	$1.75 \pm .05$	$1.81 \pm .10$	$1.80 \pm .04$
	OPAL	$1.61 \pm .04$	$1.67 \pm .08$	$1.69 \pm .03$	$1.57 \pm .05$	$1.66 \pm .09$	$1.68 \pm .04$
				Mean value $Z = 1.72 \pm 0.09$			

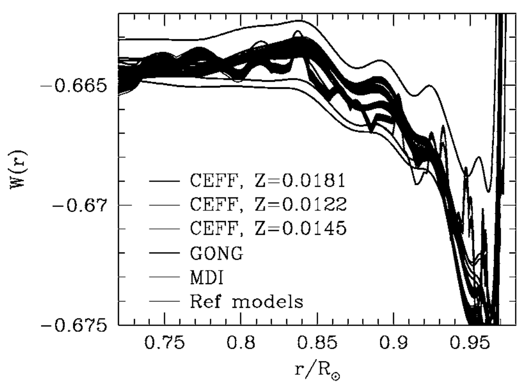

Figure 4. The function $W(r)$ for the Sun obtained using three different reference models for the different sets of solar oscillations frequencies from GONG and MDI are compared with that in solar models with different heavy element abundances. The model with $Z = 0.0145$ uses ASP mixture with Ne abundance increased by a factor of 2 (Antia & Basu 2005).

To study the possibility of determining the abundances of individual heavy elements we construct solar models by varying the abundance of one element at a time. Results are shown in Fig. 5. From this figure one can identify some peaks in $W(r)$ due to C, O and Ne around $r = 0.89R_\odot, 0.82R_\odot$ and $0.92R_\odot$ respectively. In principle, these can be used to determine the abundances of these elements. Since the contribution of different elements is not the same at all depths, the average of $W(r)$ in different depth ranges will be affected differently by each element. This can be converted to difference in inferred Z value using the calibration curves and the results are shown in Table 2. Since the abundance using the entire region $(0.75, 0.95)R_\odot$ is rather insensitive to these changes, we take the differences w.r.t. to these values for all models. For models with the ASP mixture, the differences are small as may be expected from the fact that the abundances of the most important elements are decreased

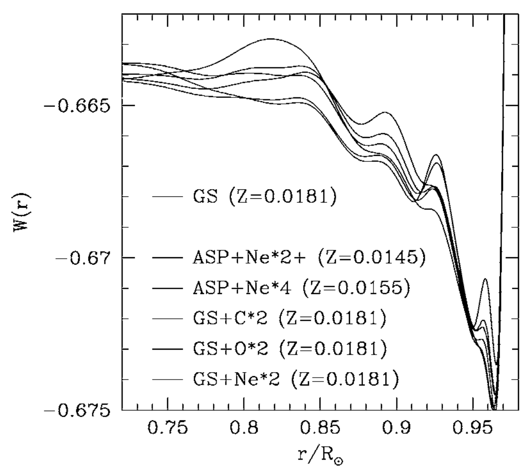

*Figure 5. The function $W(r)$ for solar envelope models constructed with the CEFF equation of state. The models have heavy-element mixtures and Z as marked in the figure. These mixtures include cases where the relative abundance of C, N or O are increased by a factor of 2 over the GS or ASP value. The mixture ASP+Ne*2+ is with Ne abundance increased by a factor of 2 and CNO abundances increased by 1σ as considered by Antia & Basu (2005). All these models have the CZ base at $r_b = 0.7133R_\odot$, and have a hydrogen abundance $X = 0.739$.*

by similar factors. For observed frequencies, the differences are comparable to the errorbars and it is difficult to infer anything about abundances of individual elements from these.

Table 2. The heavy element abundance, Z (in percentage) as inferred using average values of $W(r)$ in different radius ranges for different models and for the Sun.

Model difference	$(.75, .85)R_\odot$ Z_1	$(.75, .90)R_\odot$ Z_2	$(.75, .95)R_\odot$ Z_3	$Z_1 - Z_3$	$Z_2 - Z_3$
GS Mix. with C incr. by factor 2	1.664	1.775	1.805	-0.141	-0.031
GS Mix. with N incr. by factor 2	1.813	1.832	1.821	-0.009	0.011
GS Mix. with O incr. by factor 2	2.083	1.937	1.836	0.247	0.101
GS Mix. with Ne incr. by factor 2	1.785	1.748	1.794	-0.009	-0.045
GS Mix. with Mg incr. by factor 2	1.765	1.788	1.807	-0.042	-0.019
GS Mix. with Si incr. by factor 2	1.776	1.801	1.821	-0.045	-0.021
GS Mix. with S incr. by factor 2	1.824	1.838	1.839	-0.014	-0.001
GS Mix. with Fe incr. by factor 2	1.769	1.794	1.814	-0.046	-0.021
ASP mix. with $Z = 1.26$	1.200	1.228	1.238	-0.038	-0.011
ASP mix. with Ne incr. by factor 4	1.463	1.417	1.518	-0.055	-0.100
ASP mix. with Ne incr. by factor 2+	1.419	1.416	1.454	-0.035	-0.038
GONG + MDI data sets	$1.778 \pm .009$	$1.824 \pm .018$	$1.804 \pm .006$	$-.026 \pm .011$	$.020 \pm .019$

4. SUMMARY AND DISCUSSION

The results using test models show that it is possible to infer $W(r)$ through inversion and determine the heavy element abundance using appropriate calibration models. Using observed frequencies with 3 different reference models for inversion and 2 sets of calibration models we get $Z = 0.0172 \pm 0.002$ where the errorbars include systematic errors from different sources (Antia & Basu 2006). This value is consistent with GS but much larger than the revised estimate of Asplund et al. (2005).

In principle, it is possible to determine the abundance of individual heavy elements using this technique, but that requires more reliable EOS and better understanding of systematic errors. The solar models using ASP composition with Ne abundance enhanced (Antia & Basu 2005) do not appear to match the $W(r)$ value for the Sun (Fig. 4) in the region $0.8R_\odot < r < 0.9R_\odot$.

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