ASTEROSEISMIC DETERMINATION OF HELIUM ABUNDANCE IN SOLAR-TYPE STARS

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ABSTRACT

In this work we investigate whether low degree modes that should be available from space and ground based asteroseismology missions can be used to determine the helium abundance, Y, in stellar envelopes with sufficient precision. We find that the oscillatory signal in the frequencies caused by the depression in Γ_1 in the second helium ionisation zone can be used to determine the envelope helium abundance of low mass main sequence stars.

Key words: Stars: Abundances; Stars: Oscillations; Stars: Interior.

1. INTRODUCTION

The second helium ionisation zone in a star causes a distinct, reasonably localised depression in the adiabatic index Γ_1 (see Fig. 1). This depression increases with increase in the helium abundance. A localised feature in the structure of a star, such as the depression in Γ_1 at the second helium ionisation zone or the change in temperature gradients at the base of the convection zone introduces an oscillatory term in frequencies as a function of radial order n, that is proportional to

$$\cos(2\tau_m\omega + \phi), \qquad \tau_m = \int_{r_m}^{R} \frac{dr}{c}$$

(e.g., Gough, 1990) where τ_m is the acoustic depth of the localised feature, c the speed of sound, r_m is the radial distance where the feature is located; and ω the angular frequency of oscillation modes, ϕ being a phase. The amplitude of the oscillation caused by the depression in Γ_1 depends on the amount of helium present. In principle, if this amplitude is measured, then it can be compared against models to estimate the helium abundance. Basu et al. (2004) have demonstrated that this oscillatory signal can be extracted from the frequencies of low-degree modes to determine the helium abundance in envelopes of low mass stars. In this work we extend their work by using improved $1.4M_\odot$ models with diffusion to study the oscillatory signal in low degree modes.

2. MODELS AND TECHNIQUE

We restrict ourselves to stars of mass close to the solar mass at present. Specifically we have chosen stars with $0.8M_\odot$, $1M_\odot$, $1.2M_\odot$ and $1.4M_\odot$. We consider models evolved to different ages for each of these masses. The models were constructed using YREC, the Yale Rotating Evolution Code (Guenther et al., 1992). These models use the OPAL equation of state (Rogers & Nayfonov, 2002), OPAL opacities (Iglesias & Rogers, 1996) and nuclear reaction rates as used by Bahcall & Pinsonneault (1992). The models include diffusion of helium and heavy elements using the prescription of Thoul et al. (1994). We have constructed models with five different values of initial helium abundance, $Y_0 = 0.24, 0.26, 0.28$, 0.30 and 0.32. All models use initial heavy element abundance $Z_0 = 0.022$. For higher mass stars ($M \gtrsim 1.4M_\odot$) the convection zone is relatively shallow and has very little mass. As a result, helium gets depleted very fast from the outer layers due to diffusion. Our method would not work in such cases where the helium content in the envelope is very low. However, at a later stage, the depth of the convection zone may increase with age and helium may get dredged up once again.

Since the amplitude of the oscillatory signal is rather small, we amplify it by taking the second difference of the frequencies for modes with the same degree. We use modes of degrees $\ell = 0$–3. To test the sensitivity of results to the fitted form, we fit the second differences to the following three forms (Basu, 1997; Monteiro & Thompson, 1998; Basu et al., 2004):

$$\begin{aligned} \delta^2\nu &= \left(a_1 + a_2\nu + \frac{a_3}{\nu^2}\right) \\ &\quad + \left(b_1 + \frac{b_2}{\nu^2}\right)\sin(4\pi\nu\tau_{\rm He} + \phi_{\rm He}) \end{aligned}$$

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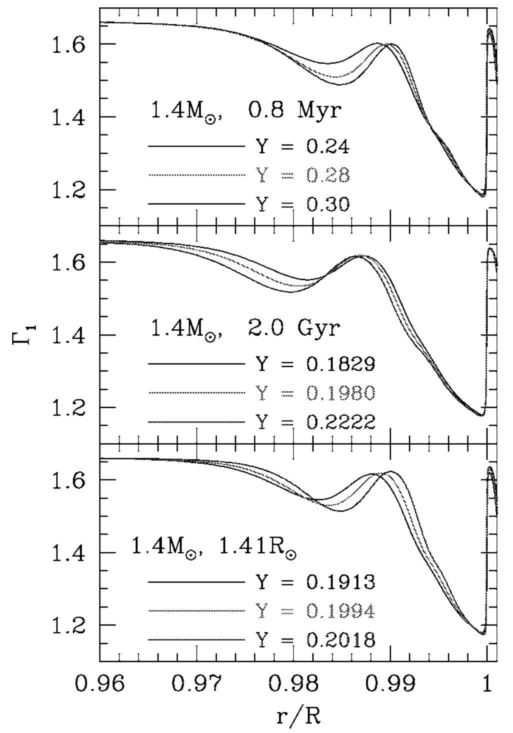

Figure 1. The adiabatic index Γ_1 as a function of fractional radius for $1.4M_\odot$ models with different Y evolved to two different ages and a given radius.

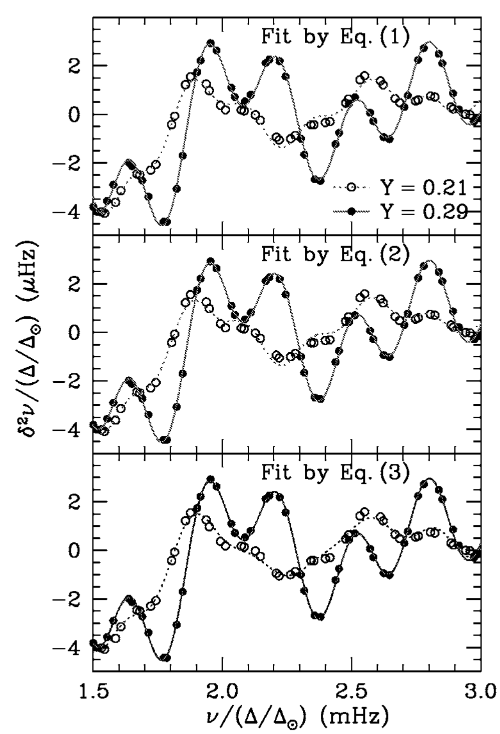

Figure 2. A sample of the fits to the second differences of the scaled frequencies. The points are the 'data', and the line the fits to the points. The examples shown are for $1.2M_\odot$ models evolved to a radius of $1.2R_\odot$.

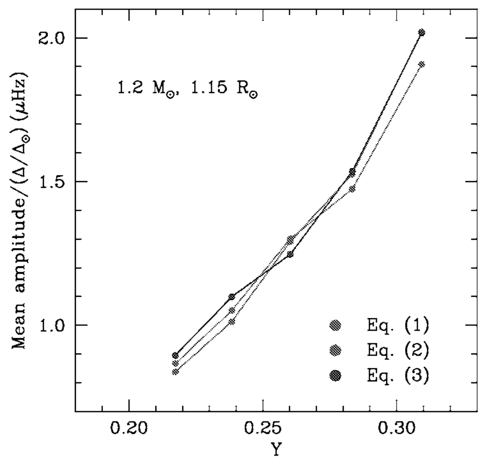

Figure 3. The scaled mean amplitude of the oscillatory signal due to HeII ionisation zone in $1.2M_\odot$ models at a radius of $1.15R_\odot$ as a function of Y obtained with different forms of the fitting function.

$$+\left(c_1+\frac{c_2}{\nu^2}\right)\sin(4\pi\nu\tau_{CZ}+\phi_{CZ}), \qquad (1)$$

$$\begin{aligned}\delta^2\nu &= (a_1+a_2\nu+a_3\nu^2)\\ &+\left(\frac{b_1\sin^2(2\pi\nu\beta)}{\nu\beta}\right)\sin(4\pi\nu\tau_{\rm He}+\phi_{\rm He})\\ &+\left(c_1+\frac{c_2}{\nu}+\frac{c_3}{\nu^2}\right)\sin(4\pi\nu\tau_{CZ}+\phi_{CZ}), \qquad (2)\end{aligned}$$

$$\begin{aligned}\delta^2\nu &= (a_1+a_2\nu+a_3\nu^2)\\ &\left(b_1+\frac{b_2}{\nu}+\frac{b_3}{\nu^2}\right)\sin(4\pi\nu\tau_{\rm He}+\phi_{\rm He})\\ &\left(c_1+\frac{c_2}{\nu}+\frac{c_3}{\nu^2}\right)\sin(4\pi\nu\tau_{CZ}+\phi_{CZ}), \qquad (3)\end{aligned}$$

where, $\sin(4\pi\nu\tau_{\rm He}+\phi_{\rm He})$ is the oscillatory signal from the HeII ionisation zone, $\sin(4\pi\nu\tau_{CZ}+\phi_{CZ})$ is the oscillatory signal from the base of the convection zone, and β is the acoustic half-thickness of the HeII ionisation zone. The parameters a_i, b_i, c_i ($i = 1, 2, 3$), $\tau_{\rm He}$, $\phi_{\rm He}$, τ_{CZ}, ϕ_{CZ} and β are determined by least-squares fits to the second differences of the frequencies.

An example of fits to the three forms of the fitting function for a typical model is shown in Fig. 2. Since stellar model frequencies generally scale as $\sqrt{M/R^3}$ and models have different M/R^3, the frequencies are scaled by this homology factor before taking the differences and fitting. In actual practice, we use the average large frequency separation, $\Delta\nu = \langle\nu_{n+1,\ell}-\nu_{n,\ell}\rangle$ to apply this scaling (Basu et al., 2004). Since the amplitude is frequency dependent, we determine the average amplitude in the fitting interval, which is chosen to be 1.5 to 3.0 mHz (after scaling). At higher frequencies the errors in observed frequency tends to increase, while at lower frequencies the asymptotic theory underlying this work is not valid. It can be easily shown that the process of taking the second differences magnifies the amplitude of a particular oscillatory signal by a factor of $4\sin^2(2\pi\tau\Delta\nu)$ where τ is the acoustic depth of the feature which produces the signal. Therefore, we divide the amplitude calculated for the second differences by this factor to obtain the amplitude of the oscillatory signal in the frequencies. This scaling is found to remove some variation in amplitude between different models (Mazumdar & Antia, 2001). All the results shown here refer to the scaled amplitude of the oscillatory signal in the actual frequencies.

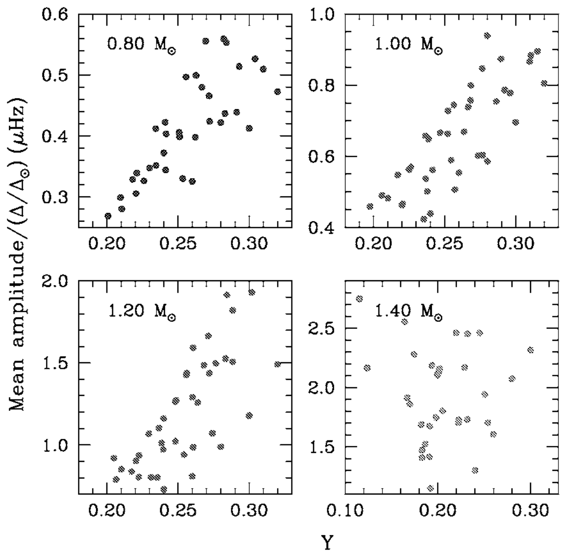

Figure 4. The scaled mean amplitude of the oscillatory signal due to HeII ionisation zone as a function of Y. The individual points in the plots represent models at different evolutionary stages for a given mass in each panel.

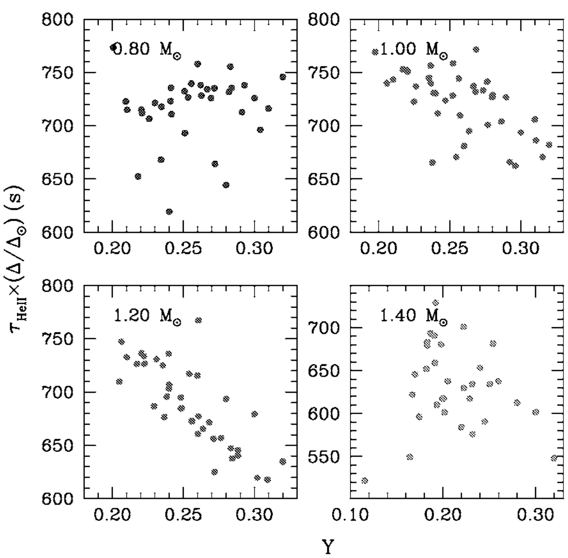

Figure 5. The scaled acoustic depth of the HeII ionisation zone as a function of Y. The individual points in the plots represent models at different evolutionary stages for a given mass in each panel.

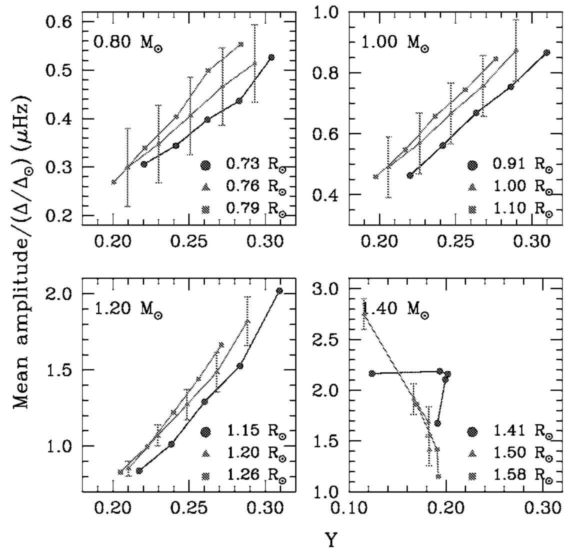

Figure 6. The scaled amplitude of oscillatory signal due to HeII ionisation zone as a function of Y for stars of known mass evolved to a known radius. The error-bars were obtained assuming an error of 1 part in 10^4 in the frequencies.

3. RESULTS

Fig. 2 shows that, in effect, all the three forms produce nearly equivalent fits and hence the amplitude of the oscillatory signal is not sensitive to the particular form of the fitting function. This is further illustrated in Fig. 3, where the amplitudes derived from each of Eqs. (1) to (3) for a set of typical models are found to be very similar. Therefore, we present here only the results obtained from one of the three forms, namely, Eq. (2).

Fig. 4 shows the mean amplitude as a function of Y for different masses. It can be seen that the amplitude increases with Y as expected, except for $1.4M_\odot$ models for which there is no clear trend. Fig. 5 shows the fitted acoustic depth, $\tau_{\rm He}$, of the HeII ionisation as a function of Y. It can be seen that after the homology scaling $\tau_{\rm He}$ lies in the same range for all models. Although in general the mean amplitude increases with helium abundance, it is difficult to compare models with different Y as they would be in different evolutionary phases. To apply this technique most efficiently, we need to construct calibration models of a given stellar mass and radius, which are to be determined independently.

Fig. 6 illustrates the almost linear variation of the mean amplitude with Y for different mass and radii. If both the mass and radius are known, such diagrams can be easily interpolated to obtain the helium abundance. Error bars on the points assuming a constant relative error of 1 part in 10^4 in the frequencies are also shown in the figure. Within this margin of error, difference in Y of approximately 0.02 should be measurable. For $M \gtrsim 1.4M_\odot$,

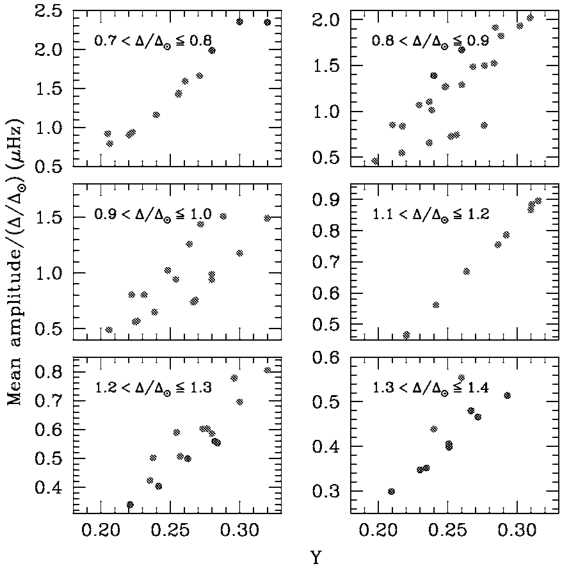

Figure 7. The mean amplitude of the oscillatory signal due to HeII ionisation zone as a function of Y, for models grouped by their average large frequency separation. Blue, red and green symbols show models with 0.8M$_\odot$, 1.0M$_\odot$ and 1.2M$_\odot$, respectively.

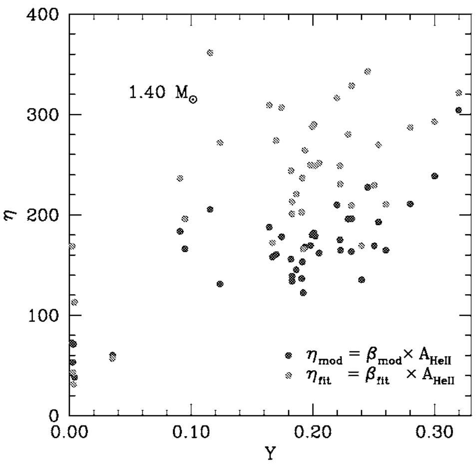

Figure 8. The product of the amplitude of oscillatory signal due to HeII ionisation zone and the half acoustic width of this zone as a function of Y for 1.4M$_\odot$ stars. The two colours represent the product calculated with the same amplitude, but either the theoretical β or its fitted value.

the amplitude is not correlated with Y and it may not be possible to use this technique. If neither the mass nor the radius is known independently, we can still calibrate the mean amplitude against the helium abundance by selecting models with the same mean density (or average large frequency separation, $\Delta\nu$). This is shown in Fig. 7, where models with different mass and radii but similar $\Delta\nu$ exhibit similar variation of amplitude with Y.

Miglio et al. (2003) have suggested that the helium abundance is related to the "area" under the depressed curve of Γ_1, which is proportional to the product of the amplitude of the oscillatory signal, A_{HeII} and the acoustic half-width of this region, β. We find that for $1.4M_\odot$ stars, this product ($\eta = \beta \times A_{\mathrm{HeII}}$) indeed shows a better trend with Y than the amplitude itself (Fig. 8). It turns out, however, that this product has a higher scatter and is systematically overestimated if we attempt to use the β value derived by fitting Eq. (2). Therefore, we have not attempted to calibrate this relationship rigorously.

4. CONCLUSIONS

We have presented results of our investigation on how low-degree oscillation frequencies may be used to determine the helium abundance in stars with solar-type oscillations. We find that the oscillatory signal in the frequencies due to the helium ionisation zone can be used to determine the helium abundance of a low-mass main-sequence star, provided the radius is known independently. The precision to which we may be able to determine the helium abundance increases with increase in mass. Using reasonable error estimates it appears that a difference of 0.02 in Y is detectable in most cases. Even if the mass or radius is not known, it is possible to determine Y from the mean amplitude of this signal, using the mean large frequency separation. For $1.4M_\odot$ models it is difficult to use this technique to determine the helium abundance. Using the product of amplitude and the half-width β may help in this case.

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