DETERMINING THE HELIUM ABUNDANCE OF STELLAR ENVELOPES

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Abstract

Intermediate-degree mode solar oscillation have been used to determine the solar helium abundance to a high degree of precision. However, we cannot expect to observe these modes in other stars. We investigate whether low degree modes that should be available from space-based asteroseismology missions can be used to determine the helium abundance with sufficient precision. We find that the oscillatory signal in the frequencies caused by the depression in Γ_1 in the second helium ionisation zone can be used to determine the envelope helium abundance of low mass stars.

Key words: Stars: oscillations; Stars: abundances

1. Introduction

The second helium ionisation zone causes a distinct, localised depression in the adiabatic index Γ_1 (see Fig. 1). This depression increases with increasing helium abundance and it also affects the sound speed in the stellar interior. Helium abundance in the solar envelope has been successfully determined using the sound speed in the solar interior obtained from inversion techniques (Gough 1984; Däppen et al. 1991; Basu & Antia 1995). These studies all use the intermediate-degree modes to measure the sound speed in He II ionisation zone. These modes of oscillation are not likely to be observed on other stars where we expect to be able to detect only low-degree modes.

A localised feature in the sound speed inside a star, such as that caused by a depression in Γ_1 at the He II ionisation zone or the change in temperature gradients at the base of the convection zone, introduces an oscillatory term in frequencies as a function of radial order n, that is proportional to

$$\sin(2\tau_m\omega + \phi), \qquad \tau_m = \int_{r_m}^{R} \frac{dr}{c}, \tag{1}$$

(e.g., Gough 1990) where τ_m is the acoustic depth of the localised feature, c the speed of sound, r_m is the radial distance where the feature is located; and ω the angular frequency of oscillation modes, ϕ being a phase. This oscillatory signal can be extracted from the frequencies of low degree modes. The amplitude of the oscillatory term caused by the depression in Γ_1 depends on the amount of helium present. In principle, if this amplitude is measured, then it can be compared against models to estimate the helium abundance.

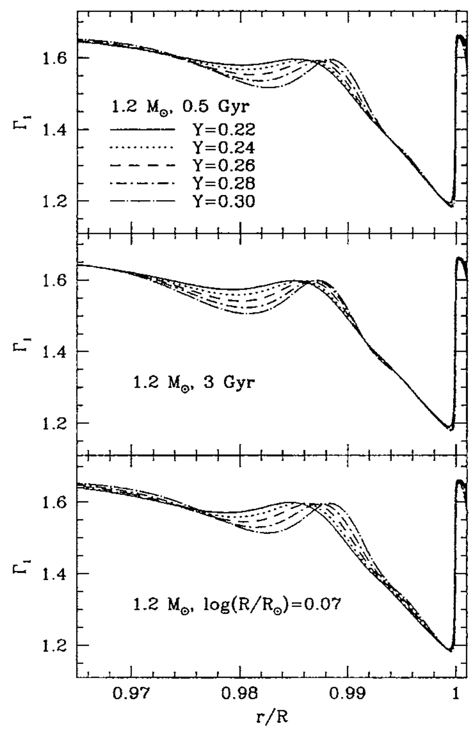

Figure 1. The adiabatic index Γ_1 as a function of fractional radius for 1.2 $M_\odot$ models. The upper two panels are models with different Y evolved to the age indicated in the panels. The bottom panel shows Γ_1 for 1.2 $M_\odot$ models evolved till the surface is at a radius of $\log(R/R_\odot) = 0.07$. The lines of different types represent models with different helium abundances as marked in the figure.

In this work we explore whether we can use low-degree modes to determine the helium abundance in envelopes of low mass stars by measuring the amplitude of the oscilla-

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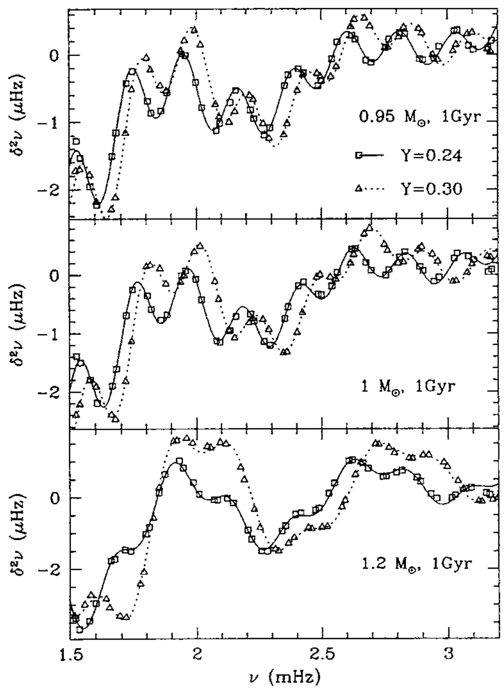

Figure 2. A sample of the fits to the second differences of the scaled frequencies. The points are the 'data', and the line the fits to the points. The examples shown are for models evolved to an age of 1 Gyr.

tory term in the frequencies introduced by the reduction of Γ_1 in the He II ionisation zone.

2. Stellar models and technique

In this preliminary work, we restrict ourselves to stars with masses close to the solar mass. Specifically we have chosen stars with $0.95\,M_\odot$, $1\,M_\odot$ and $1.2\,M_\odot$. We look at main sequence models evolved to different ages for each of these masses. The models were constructed using YREC, the Yale Rotating Evolution Code in its non-rotating configuration (Guenther et al. 1992). These models use the Yale equation of state (Guenther et al. 1992), OPAL opacities (Iglesias & Rogers 1996) and nuclear reaction rates as used by Bahcall & Pinsonneault (1992). At low temperatures we use the Alexander & Ferguson (1994) opacity tables. The models do not include diffusion (so that the surface helium abundance does not change with age) and the models had been constructed with $Z = 0.018$. We have constructed models with five different values of Y (0.22, 0.24, 0.26, 0.28, 0.30). For each of these stellar models we calculate the frequencies of low degree modes and attempt to extract the oscillatory term as explained below.

Because the amplitude of the oscillatory signal is very small, we amplify it by taking the second difference of the frequencies for modes with the same degree. We use modes of degrees $\ell = 0$–3. In addition to amplifying the oscillatory signal the process of taking second differences suppresses the dominant smooth trend of the frequencies to a large extent, thus making it easier to fit the oscillatory term. The second differences are then fitted to the form used by Basu (1997), but without the degree dependences (which is not relevant at low degree), i.e.

$$\delta^2\nu = a_1 + a_2\nu + \frac{a_3}{\nu^2} + \left(a_4 + \frac{a_5}{\nu^2}\right)\sin(4\pi\nu\tau_{\rm He} + \phi_{\rm He}) + \left(b_1 + \frac{b_2}{\nu^2}\right)\sin(4\pi\nu\tau_{\rm CZ} + \phi_{\rm CZ}), \quad (2)$$

where, the coefficients a_1, a_2, a_3 define the smooth part of the second difference, the next term is the oscillatory signal from the He II ionisation zone, while the last term is the oscillatory signal from the base of the convection zone. The parameters a_1, a_2, a_3, a_4, a_5, $\tau_{\rm He}$, $\phi_{\rm He}$, b_1, b_2, $\tau_{\rm CZ}$ and $\phi_{\rm CZ}$ are determined by least-squares fits to the second differences of the frequencies. The He II ionisation zone actually has a small but finite width and the first derivative of the squared sound speed (after some scaling) can be approximated by a triangular function. Monteiro & Thompson (1998) have shown that the amplitude of such a signal would be modulated with an oscillatory factor of the form $\sin^2(\omega\beta)$, where β is the acoustic half-width of the He II ionisation zone. We have not included this factor in our fits, but the quadratic variation in the amplitude should be able to account for this modulation over the small frequency range that is used in fitting.

For this work we assume that we know M/R^3 for the star. M/R^3 can be found reasonably well from the large frequency separation, $\nu_{n+1,\ell} - \nu_{n,\ell}$ (Mazumdar & Antia 2001). Since the stellar model frequencies generally scale as $(M/R^3)^{1/2}$ and models have different M/R^3, the frequencies are scaled by $(MR_\odot^3/M_\odot R^3)^{1/2}$ before taking the differences and fitting.

Since the amplitude is frequency dependent, we determine the average amplitude in the fitting interval, which is chosen to be 1.5 mHz to 3.2 mHz (after scaling by $(MR_\odot^3/M_\odot R^3)^{1/2}$). This range of frequency is found to be optimal for the solar oscillation data. At higher frequency the errors in observed frequency tends to increase, while at lower frequencies there is significant departure from the form assumed in this work.

It can be easily shown that the process of taking the second differences magnifies the amplitudes by a factor $4\sin^2(2\pi\tau\Delta\nu)$ where $\Delta\nu = \langle\nu_{n+1,l} - \nu_{n,\ell}\rangle$ is the large frequency separation. Thus we divide the amplitude calculated for the second differences by this factor to obtain the amplitude of the oscillatory signal in the frequencies. This scaling is found to remove some variation in amplitude between different models (Mazumdar & Antia 2001). All results shown in this work are for this scaled amplitude of the oscillatory term in the frequencies.

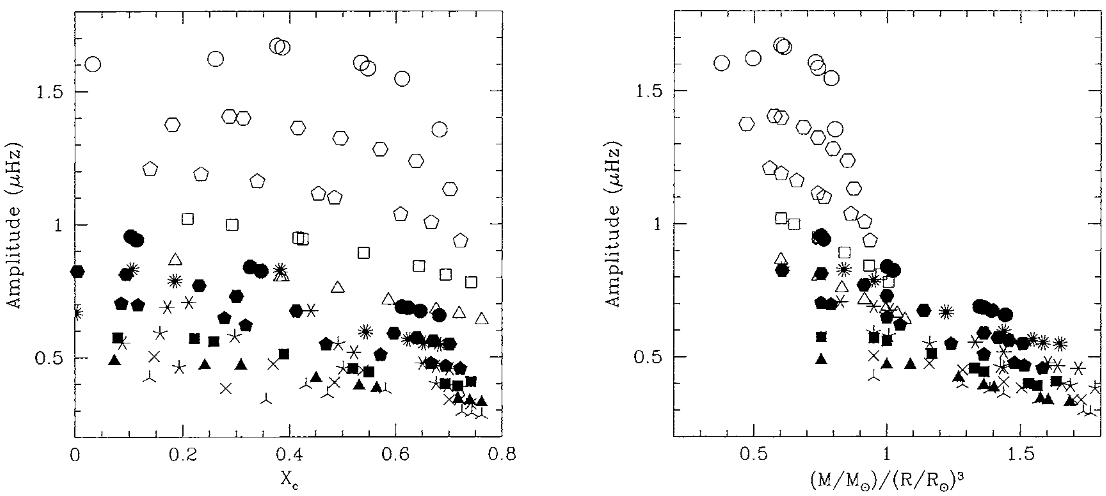

Figure 3. The scaled amplitude of the oscillatory signal due to He II *ionisation zone as a function of central hydrogen abundance, X_c (left panel) and M/R^3 (right panel). Points with 3, 4, 5, 6 and 12 sides respectively show models with* $Y = 0.22, 0.24, 0.26, 0.28, 0.30$. *Filled symbols show models for solar mass star, while crosses are for* $0.95\,M_\odot$, *and open symbols for* $1.2\,M_\odot$ *stars.*

3. Results

Fig. 2 shows the fits to the oscillatory signal in the second differences of the frequencies for a few models. The higher frequency term is the signal from the base of the convection zone, the dominant lower frequency term is that from the helium ionisation zone. Such fits are calculated for each of the stellar models to obtain the mean amplitude, A and the acoustic depth, $\tau_{\rm He}$ for the oscillatory signal.

Fig. 3 shows the mean amplitude as a function of the central hydrogen abundance (as a proxy for age) for different stars of different masses. It can be seen that for a star of any given mass and helium abundance, the amplitude does not change much with age. The amplitudes are systematically higher the higher the helium abundance. The change in amplitude with change in helium abundance increases with increase in mass and age. Fig. 3 also shows the mean amplitude as a function of M/R^3. It can be seen that for a given mass and M/R^3 (i.e., radius) the amplitude increases with Y as expected.

Fig. 4 shows the fitted acoustic depth, $\tau_{\rm He}$, of the He II ionisation zone as a function of M/R^3. These numbers are all scaled by $(R^3 M_\odot / M R_\odot^3)^{1/2}$. It can be seen that after scaling τ is almost constant for all models.

Although, in general, the amplitude of the oscillatory term increases with helium abundance, it is difficult to compare models with different Y as they would be in different evolutionary phases. Our task is made easier if the radius, R for the star is known independently from conventional observations and if we have a reasonable idea of

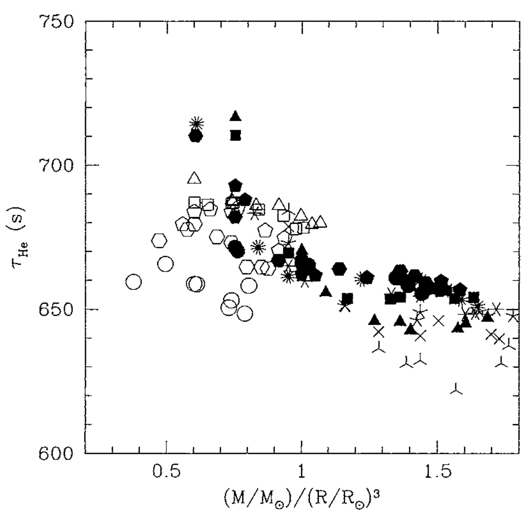

Figure 4. The scaled τ of the oscillatory signal due to He II *ionisation zone as a function of M/R^3. Points with 3, 4, 5, 6 and 12 sides respectively show models with* $Y = 0.22, 0.24, 0.26, 0.28, 0.30$. *Filled symbols show models for solar mass star, while crosses are for* $0.95\,M_\odot$, *and open symbols for* $1.2\,M_\odot$ *stars.*

the mass. Even if mass is not known independently, we can estimate M/R^3 from the frequency separation and combining that with known radius will give an estimate of the mass. This would enable us to make calibration models of a given radius and mass. A plot of the mean amplitude as

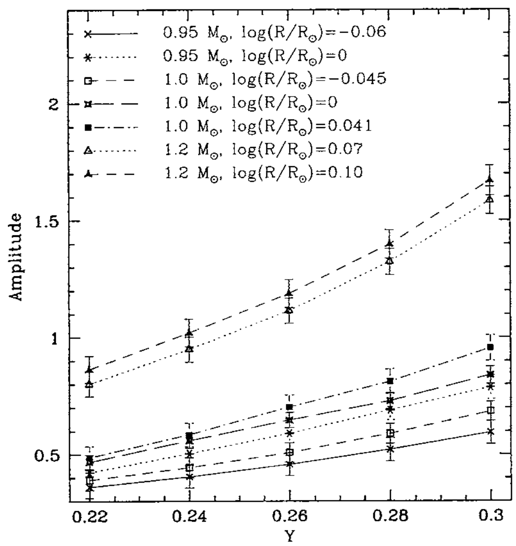

Figure 5. The scaled amplitude of the oscillatory signal due to He II ionisation zone as a function of Y for stars of known mass evolved to a known radius. The error-bars were obtained assuming an error of 1 part in 10^4 in the frequencies.

a function of Y for different masses and radii is shown in Fig. 5. In this case all we would need to do is interpolate to determine Y. Also shown in the figure are the error bars on the points assuming a constant relative error of 1 part in 10^4 in the frequencies. The accuracy to which we can use the amplitude to determine Y increases with increase in mass and age. From the error bars it appears that difference in Y of approximately 0.02 should be measurable for a relative error of 1 part in 10^4 in the frequencies. The acoustic depth τ of the He II ionisation zone is found to be essentially independent of Y for a given M and R in most cases. It should be noted that the error in τ will be about 22 s.

4. Conclusions

We have presented preliminary results of our investigation on how low-degree oscillation frequencies may be used to determine the helium abundance in stars with solar-type oscillations. We find that the oscillatory signal in the frequencies due to the helium ionisation zone can be used to determine the helium abundance of a low-mass main-sequence star, provided the radius is known independently. The precision to which we may be able to determine the helium abundance increases with increase in mass. Using reasonable error estimates it appears that a difference of 0.02 in Y is detectable in most cases.

The next step in this investigation will be the expansion of the mass range and an investigation into systematic errors due to uncertain stellar parameters such as the mixing length and metallicity. We also need to extend this study to stars where the radius is not known independently.

References

Alexander, D.R., Ferguson, J.W., 1994, ApJ, 437, 879
Bahcall, J.N., Pinsonneault, M.H., 1992, Rev. Mod. Phys., 60, 297
Basu, S., 1997, MNRAS, 288, 572
Basu, S., & Antia, H. M. 1995, MNRAS, 276, 1402
Däppen, W., Gough, D. O., Kosovichev, A. G., & Thompson, M. J. 1991, in Challenges to theories of the structure of moderate-mass stars, eds. D. O. Gough & J. Toomre, Lecture Notes in Physics, vol. 388, (Springer, Heidelberg) 111
Gough, D. O. 1984, Mem. Soc. Astron. Ital., 55, 13
Gough, D.O., 1990, Lecture Notes in Physics, vol. 367, (Springer) p283
Guenther, D.B., Demarque, P.R., Kim, Y.-C., Pinsonneault, M.H., 1992, ApJ, 387, 372
Iglesias, C.A., Rogers, F.J., 1996, ApJ, 464, 943
Mazumdar, A., Antia, H.M., 2001, A&A, 377, 192
Monteiro, M. J. P. F. G., & Thompson, M. J. 1998, in Proc. IAU Symp. 185, New eyes to see inside the Sun and stars, ed. F.-L. Deubner, J. Christensen-Dalsgaard & D. W. Kurtz (Kluwer, Dordrecht) 317