A STUDY OF TEMPORAL VARIATIONS OF THE TACHOCLINE

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ABSTRACT

Temporal variations of the structure and rotation rate in the solar tachocline region are studied using helioseismic data from the Global Oscillation Network Group (GONG) and the Michelson Doppler Imager (MDI) obtained during the period 1995–2000. We do not find any significant temporal variation in the depth of the convection zone, the position or thickness of the tachocline or the extent of overshoot below the convection zone. We find evidence to suggest that the mean position of tachocline becomes shallower with increase in latitude, strengthening earlier results. The convection zone depth is found to be essentially independent of latitude.

Key words: Sun: oscillations; Sun: rotation; Sun: interior.

1. INTRODUCTION

Inversions of rotational splittings (Thompson et al. 1996; Schou et al. 1998) have demonstrated that the rotation rate changes from differential rotation within the convection zone to an almost latitudinally independent rotation rate in the radiative interior in the region close to the base of the convection zone. This transition region has been referred to as the tachocline (Spiegel & Zahn 1992). The exact location and thickness of tachocline is an important constraint on the theories of angular momentum transport in stellar interiors as well as the solar dynamo models (Rüdiger & Kitchatinov 1996; Gilman & Fox 1997; Canuto 1998). The tachocline is also the region where the solar dynamo is believed to be located and one may expect some temporal variations in this region associated with the solar cycle.

Frequencies of solar oscillations are known to vary with time and this variation is correlated with solar activity (Elsworth et al. 1990; Libbrecht & Woodard 1990).The frequency differences scaled with mode inertia between frequencies at different phases of the solar cycle appear to be a smooth function of frequency indicating that the cause of the frequency changes lies close to the solar surface (Basu & Antia 2000). The zonal flow pattern which defines the temporal variation in the rotation rate seems shallow too, and does not appear to penetrate below a radial distance of $0.9R_\odot$ (Howe et al. 2000a; Antia & Basu 2000). However, Howe et al. (2000b) have reported 1.3 yr oscillations in the rotation rate in equatorial region at $r = 0.72R_\odot$. It would thus be interesting to check this result independently.

In this work we investigate whether these changes in solar frequencies imply any change in depth of the solar convection zone (CZ), the extent of overshoot, or the rotation rate near the CZ base as a function of the solar activity. Apart from temporal variations we also investigate possible latitudinal variations in the properties of tachocline or convection zone depth.

We have used data sets from GONG and MDI. These sets consist of the mean frequency and the splitting coefficients, $a_j(n,\ell)$, defined by:

$$\nu_{n\ell m} = \nu_{n\ell} + \sum_{j=1}^{j_{\max}} a_j(n,\ell)\,\mathcal{P}_j^{(\ell)}(m), \qquad (1)$$

where $\nu_{n\ell}$ is the mean frequency of the (n,ℓ) multiplet, and $\mathcal{P}_j^{(\ell)}(m)$ are orthogonal polynomials in m (Ritzwoller & Lavely 1991; Schou et al. 1994).

We use GONG data for months 1–47, which cover the period from 1995 May 7 to 1999 December 23. For most of the results we have used the 15 non-overlapping data sets each covering a period of 108 days. The MDI data sets (Schou 1999) consist of 19 non-overlapping data sets each covering a period of 72 days, starting from 1996 May 1 and ending on 2000 June 20.

2. DEPTH OF THE CONVECTION ZONE AND EXTENT OF OVERSHOOT

The change in the temperature gradient, from the adiabatic value inside the convection zone to the radiative value below the base of the CZ, introduces a bump in the sound speed difference between two models (or between a model and the Sun) that have different CZ depths. This signal can be used to determine the depth of the convection zone. We use the method described by Basu & Antia

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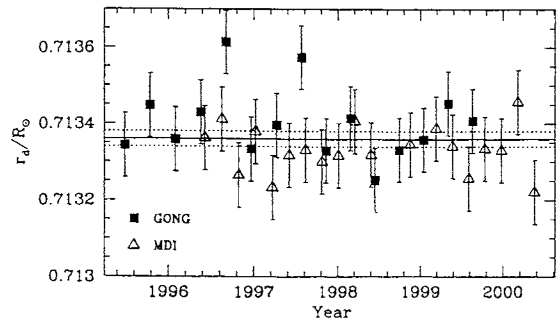

Figure 1. The position of the solar CZ base plotted as a function of time. The continuous line (with dotted lines denoting the 1σ error limits) is the mean value.

Table 1. Mean position of the base of the convection zone (r_d) and the amplitude (A) and frequency (τ) of the oscillatory signal due to overshoot at different latitudes

Lat. (°)	r_d ($R_\odot$)	A (μHz)	τ (s)
Mean	0.71336 ± 0.00002	0.72 ± 0.01	2304 ± 3
0	0.71333 ± 0.00010	0.89 ± 0.10	2330 ± 24
15	0.71338 ± 0.00008	0.76 ± 0.05	2314 ± 12
30	0.71334 ± 0.00008	0.91 ± 0.06	2273 ± 14
45	0.71324 ± 0.00008	0.76 ± 0.06	2329 ± 15
60	0.71359 ± 0.00013	0.60 ± 0.17	2310 ± 97
75	0.71312 ± 0.00080		

(1997) to determine the radial position of the CZ base. Fig. 1 shows how the position of base of the convection zone (r_d) varies as a function of time. We do not find any systematic pattern to suggest that there are any solar cycle related changes in the CZ depth.

The sharp transition of the temperature gradient at the base of the convection zone also introduces an oscillatory signal in frequencies of all modes which penetrate below the base of the convection zone (Gough 1990; Monteiro et al. 1994; Basu et al. 1994). This signal can be amplified by taking the fourth differences of the frequencies as a function of the radial order n. The amplitude of this signal can be calibrated against amplitudes for models with known overshoot extents. The frequency τ of the signal is a measure of the acoustic depth at which the transition occurs. We use the method described by Basu (1997) to determine the amplitude and frequency of the oscillatory signal. We fail to find any systematic trend to indicate solar-cycle related changes in the extent of the overshoot layer.

The latitudinal dependence of the depth of the convection zone or the extent of overshoot, can be studied by using the even-order splitting coefficients (Gough 1993). For this purpose, instead of the mean frequency, $\nu_{n\ell}$, we use

$$\nu = \nu_{n\ell} + \sum_k \frac{\ell a_{2k}(n,\ell)}{Q_{\ell k}} P_{2k}(\cos\theta), \qquad (2)$$

where, $P_k(\cos\theta)$ are the Legendre polynomials, θ is the colatitude and $Q_{\ell k}$ define the angular integrals

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \, Y_\ell^m (Y_\ell^m)^* P_{2k}(\cos\theta) = \frac{1}{\ell} Q_{\ell k} \mathcal{P}_{2k}^{(\ell)}(m), \qquad (3)$$

In the absence of any time-variation, we take the time-average of the CZ depth and overshoot results at each latitude to study latitudinal variation in these quantities. The results are shown in Table 1. There is no significant systematic variation in any of these quantities with latitude.

3. ROTATION RATE IN THE TACHOCLINE

To determine the properties of the tachocline we use the three techniques described by Antia et al. (1998), which are (1) a calibration method in which the properties at each latitude are determined by direct comparison with models; (2) a one dimensional (henceforth, 1d) annealing technique in which the parameters defining the tachocline at each latitude are determined by a nonlinear least squares minimization using simulated annealing method and (3) a two-dimensional (henceforth, 2d) annealing technique, where the entire latitude dependence of tachocline is fitted simultaneously, again using simulated annealing. In all techniques the tachocline is represented by a model of the form (cf., Antia et al. 1998),

$$\Omega_{\rm tac} = \frac{\delta\Omega}{1 + \exp[(r_t - r)/w]}, \qquad (4)$$

where $\delta\Omega$ is the jump in the rotation rate across the tachocline, w is the half-width of the transition layer, and r_t the radial distance of the mid-point of the transition region. The properties we are interested in are the position and the thickness of the tachocline and the change in rotation rate across the tachocline. We study these properties as a function of latitude and time using all the techniques listed above and the results are shown in Figs. 2 and 3. There is no significant temporal variation in any of these properties.

The jump obtained using the calibration technique appears to show a steady increase with time at low latitudes, which is comparable to error limits. This trend is not seen in results obtained using other techniques and its significance is not clear. The values of the jump in the rotation rate across the tachocline obtained using the 2d annealing technique appear to show an oscillatory behaviour at low latitudes. This can be traced back to oscillations in the latitudinally independent component of the jump. Following Antia et al. (1998), $\delta\Omega$ in the 2d annealing technique can be expanded as

$$\delta\Omega(\theta) = \delta\Omega_1 + \delta\Omega_3 P_3(\theta) + \delta\Omega_5 P_5(\theta), \qquad (5)$$

where

$$P_3(\theta) = 5\cos^2\theta - 1, \qquad (6)$$

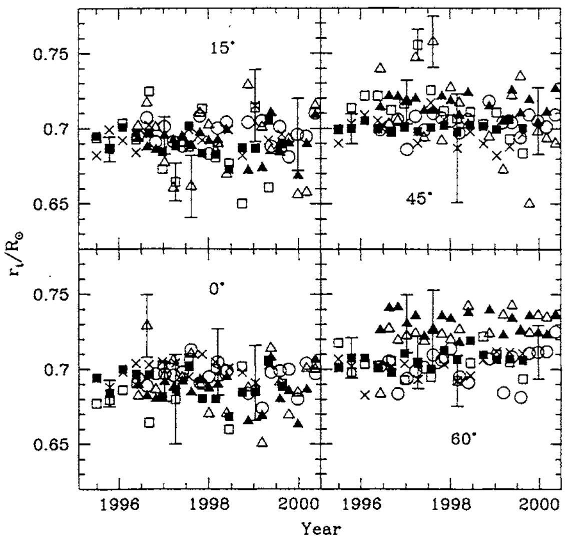

Figure 2. The mean radial position of the tachocline at a few selected latitudes that are marked in each panel. The crosses and circles show the results from calibration method for GONG and MDI data, the open squares and triangles show the 1d annealing results from GONG and MDI data, while the filled squares and triangles show the results from 2d annealing for GONG and MDI data. For clarity only one representative error-bar is shown for each technique.

$$P_5(\theta) \quad = \quad 21\cos^4\theta - 14\cos^2\theta + 1. \qquad (7)$$

It turns out that $\delta\Omega_1$ is anti-correlated with Ω_c, the rotation rate in the radiative zone below the tachocline. This anti-correlation is due to the fact that it is difficult to distinguish between the effects of these two parameters in the tachocline model that is used for fitting. As such the significance of these oscillations is not clear.

In order to test if the oscillatory variation in $\delta\Omega_1$ or Ω_c are periodic, we take the Fourier transform of these results (after subtracting out the temporal mean) and the resulting power spectra show a broad peak around a frequency of 0.7 yr^{-1} or a period of 1.4 years, which is comparable to the period found by Howe et al. (2000b) but the peak is not statistically significant. Since the latitudinally independent component of rotation rate should contribute only to the splitting coefficient a_1, we also try to fit a_1 separately to check if the oscillations are present. Furthermore, following Howe et al. (2000b) we fit all GONG data sets centered at GONG months 2–46, including those that overlap in time. These results are shown in Fig. 4 which also shows the Fourier transform of $\delta\Omega_1$. The peak height in spectra obtained from independent (non-overlapping) data sets is comparable to error estimates, while the peak obtained using all data sets from GONG is about 1.5σ, but its significance is not clear since the errors are estimated by assuming that all data sets are independent.

To study latitudinal variations in the properties of the tachocline we take the temporal average of the tachocline properties obtained from all data sets. Further we take weighted average over all six measurements at each latitude and the results are shown in Table 2. There is clearly a systematic variation in the position of tachocline with latitude while the variation in thickness is not clear.

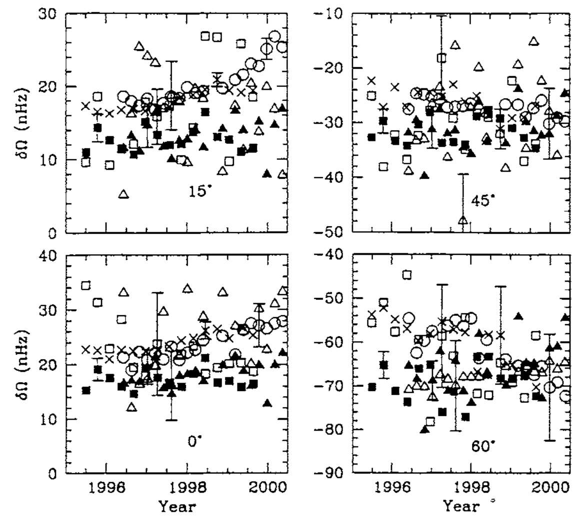

Figure 3. The jump in rotation rate across the tachocline at a few selected latitudes. The different styles of the points have the same meaning as in Fig. 2

Table 2. Mean properties of the tachocline at different latitudes

Lat. (°)	$\delta\Omega$ (nHz)	r_t ($R_\odot$)	w ($0.01R_\odot$)
0	21.22 ± 0.27	0.6896 ± 0.0023	0.61 ± 0.12
15	17.54 ± 0.17	0.6903 ± 0.0021	0.60 ± 0.10
45	-30.90 ± 0.39	0.7077 ± 0.0021	1.11 ± 0.12
60	-67.23 ± 0.58	0.7097 ± 0.0027	1.34 ± 0.15

4. CONCLUSIONS

We find no clear evidence of solar cycle related change in the position of the convection zone base or the extent of overshoot below the convection zone. The changes if any in the depth of the convection zone must be less than the error limits in our estimate, i.e., $0.00008R_\odot \approx 56$ km. There is no significant latitudinal variation in the depth of the convection zone or the extent of overshoot below the convection zone. Any latitudinal variations in depth of convection zone should be less than $0.0004R_\odot$.

There is no significant temporal variation in the position or the width of the tachocline, nor is their any variation in the change in the rotation rate across the tachocline. The results obtained using 2d annealing show some oscillations in the jump across the tachocline at low latitudes,

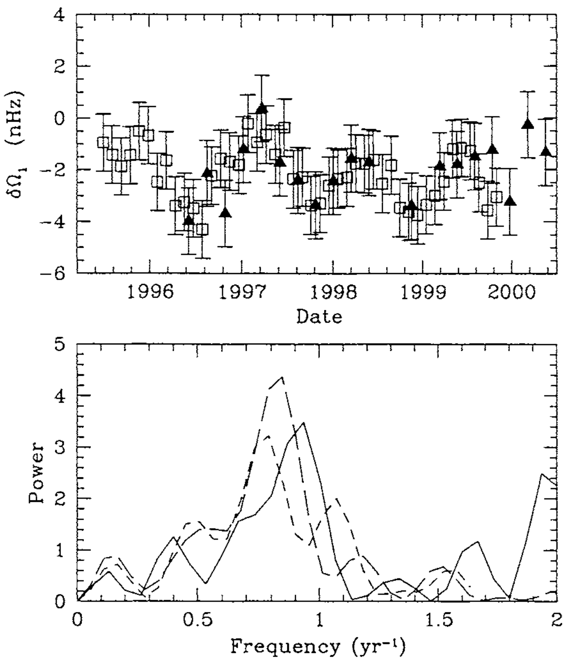

Figure 4. The upper panel shows the jump in latitudinally independent component of rotation rate across the tachocline, $\delta\Omega_1$ obtained by fitting only the splitting coefficient a_1. The squares and triangles, respectively represent the values obtained from GONG and MDI data. The lower panel shows the Fourier transform of $\delta\Omega_1$ from MDI (continuous line) and GONG (short-dashed line) data using non-overlapping data sets, while the long-dashed line shows the spectra using all data sets from GONG.

which can be traced to the latitudinally independent component of the jump, $\delta\Omega_1$. To study $\delta\Omega_1$ we fit only the splitting coefficient a_1 and the results show some periodicity with period around 1.2 and 1.1 years in GONG and MDI data, respectively. The corresponding peak in Fourier transform is not significant. Interestingly, this period in GONG data coincides with that of the well-known Chandler wobble of the Earth, which is irregular variation in the rotation axis by about 10 m. It is not clear if this small wobble can affect GONG data.

The position of tachocline shows a significant variation with latitude, and the tachocline is found to be prolate, with a difference of 0.020 ± 0.003 in position between the equator and a latitude of 60°, which is consistent with results obtained by Charbonneau et al. (1999). It is not clear if the latitudinal variation is a continuous function of latitude as is assumed in 2D annealing technique. Results in Table 2 suggest that the tachocline may actually be composed of two parts, one at lower latitude where $\delta\Omega > 0$ and the other at higher latitudes where $\delta\Omega < 0$. These two parts may have different locations and thickness, but there may be no significant latitudinal variation within each part.

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