STUDYING ASPHERICITY IN THE SOLAR SOUND SPEED FROM MDI AND GONG DATA

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ABSTRACT

We study the variation of the frequency splitting coefficients describing the solar asphericity in both GONG and MDI data, and use these data to investigate temporal sound-speed variations as a function of both depth and latitude during the period 1995–2000. The temporal variations in even splitting coefficients are found to be correlated with the corresponding component of magnetic flux at the solar surface. The sound-speed variations associated with the surface magnetic field appear to be superficial. Temporally averaged results show a significant excess in sound speed around $r = 0.92R_\odot$ and latitude of 60°.

Key words: Sun: oscillations; Sun: activity; Sun: interior.

1. INTRODUCTION

Helioseismology – the study of acoustic oscillations in the Sun – allows us to probe solar interior structure and rotation in two dimensions, depth and latitude, by taking advantage of the different spatial distribution of the various modes. Recently, two projects – the Global Oscillation Network Group (GONG) and the Solar Oscillations Investigation using the Michelson Doppler Imager (MDI) instrument aboard the *SOHO* spacecraft – have provided nearly five years of continuous helioseismic data, allowing the rise of the current solar cycle to be followed in unprecedented detail. Two dimensional inversions for rotation profile have shown that the torsional-oscillation pattern seen at the surface penetrates substantially into the convection zone (Howe et al. 2000a,b; Toomre et al. 2000; Antia & Basu 2000). Thus it would be interesting to check if the structural changes also penetrate deeper than has previously been thought. In this work we examine changes in the helioseismically estimated solar asphericity as the solar cycle progresses.

We have used data sets from GONG and MDI. These sets consist of the mean frequency and the splitting coefficients, $a_j(n,\ell)$, defined by:

$$\nu_{n\ell m} = \nu_{n\ell} + \sum_{j=1}^{j_{\max}} a_j(n,\ell)\,\mathcal{P}_j^{(\ell)}(m), \qquad (1)$$

where $\nu_{n\ell}$ is the mean frequency of the (n,ℓ) multiplet, and $\mathcal{P}_j^{(\ell)}(m)$ are orthogonal polynomials in m (Ritzwoller & Lavely 1991; Schou et al. 1994). The odd-order splitting coefficients, $a_1, a_3, \ldots$, are used to determine the rotation rate in the solar interior, while the even coefficients, which are much smaller, are sensitive to second order contributions from rotation, any possible magnetic field and any possible departure of the solar structure from the spherically symmetric state. While the effect of rotation can be estimated from the known rotation rate, it is not possible to distinguish between the contributions from magnetic field and other aspherical perturbations to solar structure (Zweibel and Gough 1995).

It is well known that the even splitting coefficients show temporal variations related to solar activity measures (Kuhn 1988; Libbrecht 1989; Libbrecht & Woodard 1990). More recently, Howe et al. (1999) have found linear relations between the even-order a coefficients and the Legendre decomposition components of the surface magnetic flux. Dziembowski et al. (2000) found a good correlation between the even-a coefficients and the corresponding even components of the Ca II K data from Big Bear Solar Observatory (BBSO).

In this work we use more extensive data sets from both GONG and MDI to study asphericity in the solar interior. We have analysed 43 overlapping 108-day time series of GONG data covering the period 1995 May 7 to 1999 October 12, centered on dates one 36-day 'GONG month' apart. The MDI data consist of 18 non-overlapping 72-day time series covering the period 1996 May 1 to 2000 April 9, al-

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though with some interruptions due to problems with the *SOHO* satellite. For the purpose of inversion we have used only p-mode splittings for modes with frequencies between 1.5 to 3.5 mHz. Further, only coefficients up to a_{14} are used as the higher order coefficients do not appear to be significant, at least for GONG data.

2. COEFFICIENT ANALYSIS

The latitudinal distribution of the surface magnetic flux can be expressed as a sum of Legendre polynomials in $\cos\theta$,

$$B(t, \cos\theta) = \sum_k B_k(t) P_k(\cos\theta). \tag{2}$$

where $B_k(t)$ are time-varying coefficients. These coefficients can be compared with the corresponding frequency splitting coefficients. Global helioseismic measurements are sensitive only to the latitudinally symmetric part of any departures from spherical symmetry — that is, to the even components of the expansion. We compare these Legendre components, B_{2k}, with the mean splitting coefficients weighted by the mode inertia, E_{nl} and another factor arising from the integral over the surface, l/Q_{lk} (cf. Eq. 7). These factors remove the main l and k dependent part. Thus we define the quantity,

$$< b_{2k} > (t) = \frac{\sum_{n,l} a_{2k}(n,l,t) l E_{nl}/W^2 Q_{lk}}{\sum_{n,l} 1/W^2}, \tag{3}$$

where t is time,

$$W = l E_{nl} \sigma_{a_{2k}}(n,l,t)/Q_{lk}$$

and σ_{a_k} is the estimated uncertainty in the coefficient a_k, and the sum is over the approximately 400 (n,l) multiplets that are common to all the mode sets. We then express the variation of the $< b_k >$ as a function of the B_k, for even k, as

$$< b_k > (t) = c_k + m_k B_k(t) \tag{4}$$

and perform linear least-squares fits to obtain the gradient m_k and intercept c_k for each even k. Fig. 1 shows the resulting fit for both GONG and MDI data. The slope m_k is found to be essentially independent of k with the weights we have adopted.

3. ASPHERICITY INVERSIONS

In order to study the variation of asphericity with depth and latitude we apply an inversion technique to the even-order splitting coefficients. We use the variational principle to analyse the departures from a spherically symmetric solar model (e.g., Gough 1993), in order to study aspherical perturbations to the sound speed and density in the solar interior. For simplicity, we only consider axisymmetric perturbations (with the symmetry axis coinciding with the rotation axis) that are symmetric about the equator. In this case, using the variational principle, the difference in frequency between the Sun and a solar model for a mode of a given order, degree and azimuthal order (n, l, and m) can be written as:

$$\begin{aligned}\frac{\delta\nu_{nlm}}{\nu_{nlm}} = \int_0^R \mathrm{d}r \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \qquad\qquad (5)\\ \left(\mathcal{K}^{nl}_{c^2,\rho}(r)\frac{\delta c^2}{c^2}(r,\theta) + \mathcal{K}^{nl}_{\rho,c^2}(r)\frac{\delta\rho}{\rho}(r,\theta)\right) Y_l^m (Y_l^m)^*\end{aligned}$$

where r is radius, θ is colatitude, $\delta\nu_{nlm}/\nu_{nlm}$ is the relative frequency difference, and $\mathcal{K}^{nl}_{c^2,\rho}(r)$ and $\mathcal{K}^{nl}_{\rho,c^2}(r)$ are the kernels for spherically symmetric perturbations (Antia & Basu 1994), and Y_l^m are spherical harmonics denoting the angular dependence of the eigenfunctions for a spherically symmetric star. The perturbations $\delta c^2/c^2$ and $\delta\rho/\rho$ can be expanded in terms of even order Legendre polynomials, e.g.,

$$\frac{\delta c^2}{c^2}(r,\theta) = \sum_k c_k(r) P_{2k}(\cos\theta). \tag{6}$$

The spherically symmetric component ($k = 0$) gives frequency differences that are independent of m and thus only contribute to the mean frequency of the (n,l) multiplet. Higher order terms give frequencies that are functions of m and thus contribute to the splitting coefficients.

The angular integrals in Eq. 5 can be evaluated to give

$$\begin{aligned}\int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, Y_l^m (Y_l^m)^* P_{2k}(\cos\theta)\\ = \frac{1}{l} Q_{lk} \mathcal{P}^{(l)}_{2k}(m) \qquad (7)\end{aligned}$$

where Q_{lk} depends only on l, k and $\mathcal{P}^{(l)}_{2k}(m)$ are the orthogonal polynomials defined by Eq. 1. The extra factor of $1/l$ ensures that the Q_{lk} approach a constant value at large l. Thus with this choice of expansion the inversion problem is decomposed into independent inversions for each even splitting coefficient and $c_k(r), \rho_k(r)$ can be computed by inverting the splitting coefficient a_{2k}. This is similar to the 1.5d inversion to determine the rotation rate (Ritzwoller & Lavely 1991), called so because a two dimensional solution is obtained as a series of one dimensional inversions. Alternatively, we can perform a 2d inversion to directly determine the asphericity as a function of r, θ. In this paper we present the results of 2d inversions only.

In practice, we also need to account for the contribution to the frequency splittings that arises from uncertainties in the treatment of surface layers in the model. It is known that this contribution to frequency splittings should be a slowly varying function of frequency once it is corrected for differences

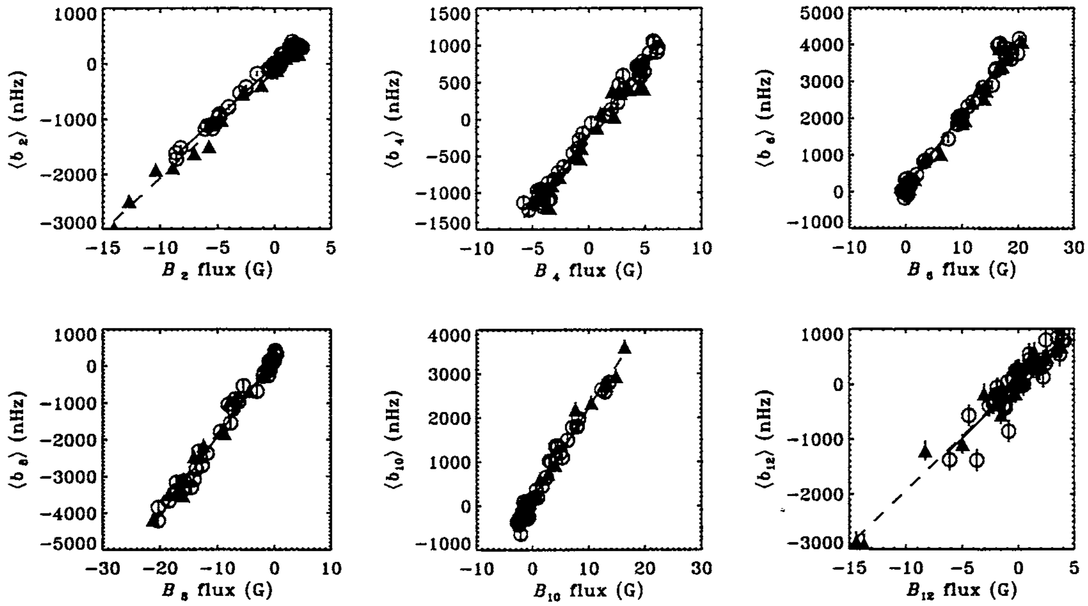

Figure 1. The relations between the GONG (open circles) and MDI (filled triangles) $< b_k >$-coefficients and the corresponding B_k components of the Legendre decomposition of the magnetic flux from the Kitt Peak synoptic maps. The lines show the best-fit results for linear fits to the data, for GONG (solid) and MDI (dashed).

in the mode mass of the modes; no degree dependence is expected to a first approximation. As in the case of inversions to determine the spherically symmetric structure of the Sun, the surface uncertainties are accounted for by assuming that these can be represented by a frequency-dependent function $F(\nu)$. However, in this case we determine a different function $F_k(\nu)$ for each coefficient a_{2k} and write the 2d inversion problem as

$$\frac{l a_{2k}(n,l)}{\nu_{nl}} = \frac{Q_{lk}(4k+1)}{2} \int_0^R dr \int_0^\pi \sin\theta d\theta \left(\mathcal{K}^{nl}_{c^2,\rho} \frac{\delta c^2}{c^2} + \mathcal{K}^{nl}_{\rho,c^2} \frac{\delta \rho}{\rho} \right) P_{2k}(\cos\theta) + Q_{lk} \frac{F_k(\nu_{nl})}{E_{nl}} \qquad (8)$$

The quantities $\delta c^2/c^2$ and $\delta\rho/\rho$ can be expanded in terms of basis functions (e.g., splines) in r and θ, $F_k(\nu)$ can be expanded in terms of basis functions in ν and the three functions can be determined by inverting Eq. 8 using the usual inversion techniques. We use a Regularized Least Squares (RLS) technique for this purpose.

In the first two columns of Figure 2 we show the results of 2d sound speed inversions of GONG and MDI data at selected radii as a function of time and latitude. There is no signature of significant temporal evolution at these radii. Both data sets show a persistent sound-speed excess at around 60 degrees, apparently extending well down into the convection zone, though this is more significant in the MDI than in the GONG data.

The inversions show the deviation from a spherically symmetric model. It turns out that the temporal mean of these deviations is non-zero and has a dependence on depth and latitude, as illustrated in Figure 3. However, it may be noted that the current data extend over only a small fraction of a magnetic cycle and the temporal mean over this limited period may not have much significance. The second order contribution from rotation would also contribute to the temporal mean. There is some difference between the mean calculated from GONG and MDI data sets, which could be due to systematic differences between the two sets, or to the larger uncertainties in the GONG data. In both cases there is a peak around depth of $0.08R_\odot$ and the peak is more pronounced in MDI data.

The sound-speed variation after subtraction of the mean profile is shown in the right-hand columns of Figure 2. No systematic structure is evident. It is clear from these figures that there is no significant temporal variation in the asphericity at the depths we can resolve.

The strong variation in the even a coefficients is evidently not reflected in the inversion results. Instead, it has been absorbed in the surface terms. To illustrate this, we show in Figure 4 the reconstruction of the surface term for a frequency of 2.5 mHz, with overlaid contours of the magnetic flux. As we would expect, since the dependence of the individual surface terms on frequency and on the magnetic field

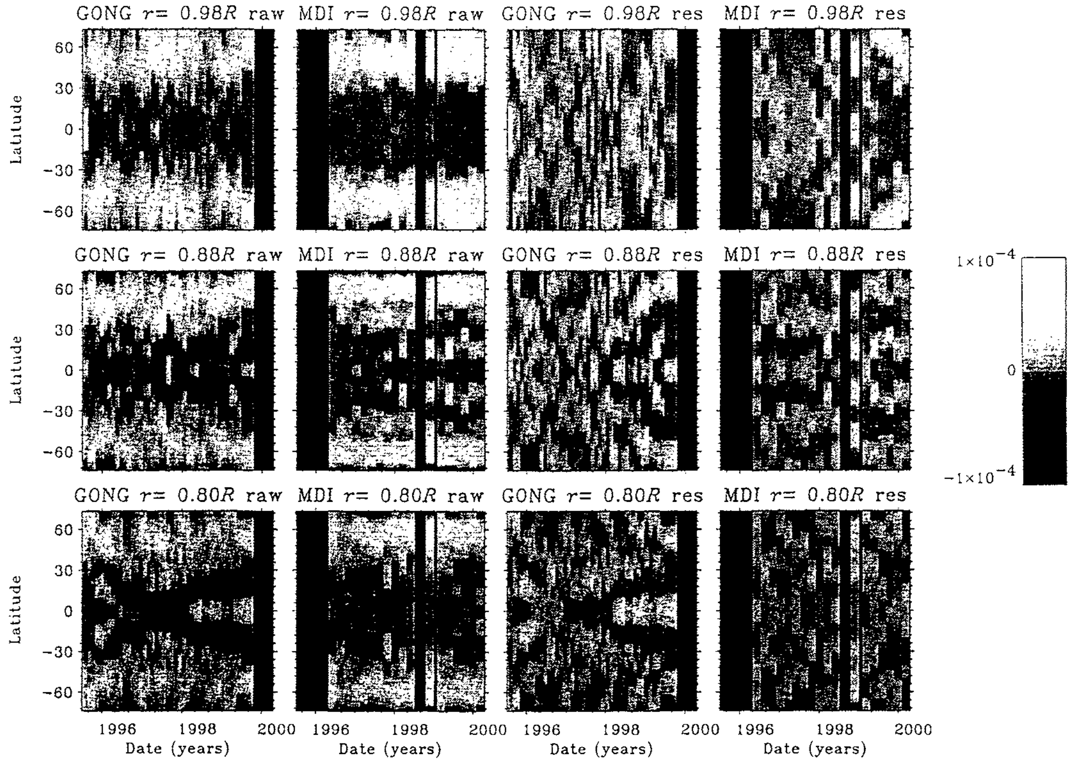

Figure 2. Grey scale maps showing the results of 2d sound-speed inversions of GONG (first and third columns) and MDI (second and fourth columns) data at radii 0.98R (top), 0.88R (centre) and 0.80R (bottom). Blank spaces represent periods where there are no data available. Columns 1 and 2 show the 'raw' inversion results and columns 3 and 4 the results after subtraction of the temporal mean.

strength is largely independent of the order of the coefficient, this reconstruction matches the magnetic flux quite well.

In order to test our inversion method and to see how well the inversions can resolve different features, we have conducted tests with artificial data. These tests suggest that with the current data sets we can resolve features in the range 0.8 to 0.98 $R_\odot$.

4. DISCUSSION

We have studied the temporal variations in even a-coefficients from GONG and MDI data during the period 1995–2000. The mean values of the even a-coefficients (after scaling for mode mass and angular integrals) over all modes are found to be well correlated to the corresponding components of the observed magnetic flux at the solar surface. The slope of best linear fit between the mean splitting coefficient and the corresponding Legendre component of the surface magnetic flux is found to be essentially independent of the order of the coefficient. Thus the latitudinal variation in the surface magnetic flux is correlated with that in asphericity as measured by the even a-coefficients.

In this work we have done two dimensional inversions for sound speed using even a-coefficients from GONG and MDI data sets covering the period from 1995 to 2000, which encompass a substantial portion of the rising phase of the current solar cycle. We find no significant temporal variation in the asphericity of the sound speed over this period. Thus it appears that the temporal variations in even a-coefficients arise from changes taking place in surface layers. A similar conclusion has been obtained for the spherically symmetric component of sound speed and density (Basu & Antia 2000) as the changes in mean frequency also appear to be associated with surface effects.

For this work we have assumed that the even a-coefficients arise from aspherical sound-speed distribution, but this is by no means obvious as these coefficients could also arise from a magnetic field since both sets of kernels are very similar in most parts of the Sun and cannot be easily distinguished (see, for example, Figure 10 in Dziembowski et al. 2000). If the signature in even a-coefficients is due to magnetic field, a magnetic field strength yielding $v_A^2/c^2 \sim 10^{-4}$ would be required, where v_A is

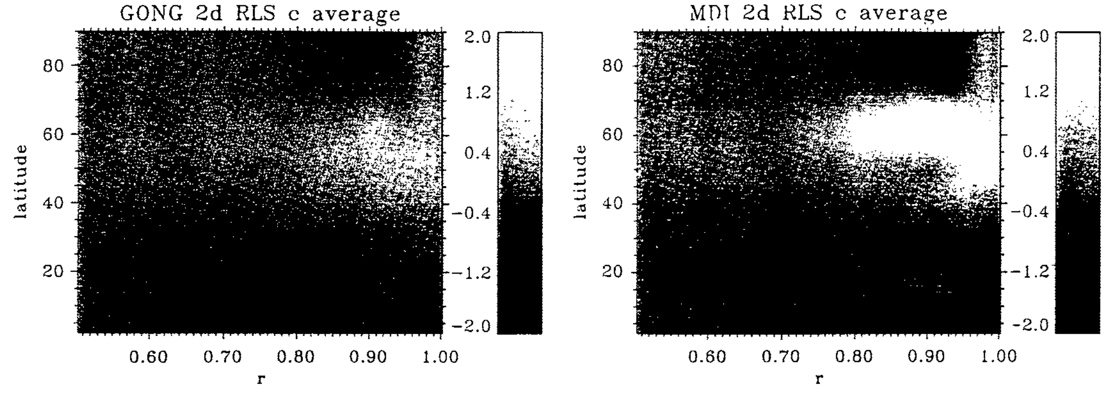

Figure 3. Grey scale maps showing the mean results of sound-speed inversions of GONG (left) and MDI (right) data, mulitiplied by 10^4, as a function of latitude and radius.

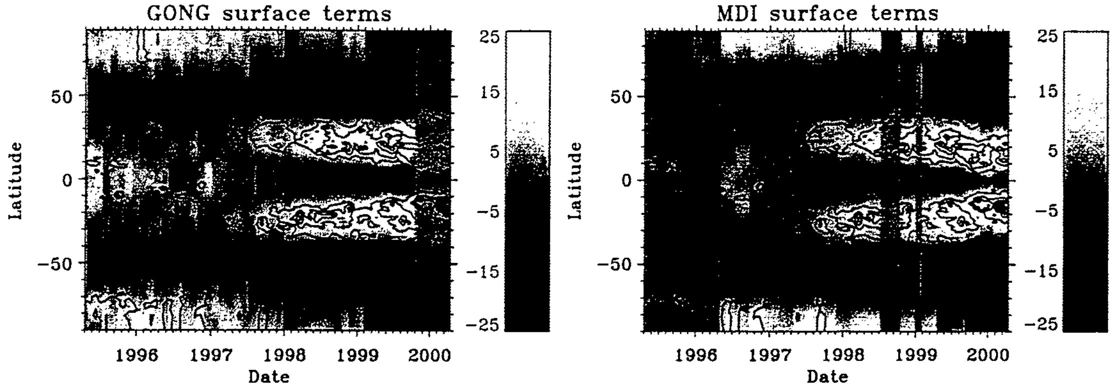

Figure 4. Grey scale maps showing the reconstruction of the latitudinal dependence of the surface term (multiplied by 1000) from GONG (left) and MDI (right). Overlaid contours show the Kitt Peak unsigned magnetic flux with the B_0 term subtracted; contour spacing is 10 G.

the Alfven speed. Inside the convection zone one might not expect an ordered magnetic field over large length scales as turbulence might be expected to randomize the magnetic field. Such a randomized magnetic field can also effectively change the wave propagation speed, giving a signal similar to that from aspherical sound-speed distribution. It may not be possible to distinguish between these two effects from the even a-coefficients. Kuhn (1998) has argued that the observed magnetic field at the solar surface is not sufficient to explain the magnitude of the even a-coefficients. However, one can argue that the magnetic field increases significantly as one goes deeper in the near surface layers and this could explain the observed magnitudes of the splittings.

Although the temporal variation in asphericity in the solar interior is not significant, the mean asphericity is found to be significant and shows a peak around $r = 0.92R_\odot$ which is very pronounced in the inversions of MDI data. This is deeper than the depth to which the surface shear layer in solar rotation rate extends (Schou et al. 1998) and about the same as the penetration depth recently (Howe et al. 2000; Antia and Basu 2000) established for the torsional oscillation pattern. Antia, Chitre and Thompson (2000) attempted to detect a possible signature for magnetic field using the even a-coeffcients, and found that the coefficients a_2 and a_4 have a residual after subtracting the contribution from rotation, peaked around $r = 0.95R_\odot$, which is consistent with our results. Dziembowski et al. (2000) calculated inversions separately for temperature and magnetic field perturbations and detected a significant perturbation in the spherical part at a depth of 25–100 Mm with a maximum at about 45 Mm, which agrees with the depth at which we find the aspherical perturbation in the sound speed. They interpret this perturbation as being due to either a magnetic perturbation of about $(60\ \mathrm{kG})^2$ or a temperature perturbation of about $1.2\ 10^{-4}$ which is of the same order as the result presented here. Gilman (2000) pointed out that a thermal driving mechanism for the observed meridional flow would require a temperature excess at the equator — the opposite of what we observe. Thus this finding poses yet another challenge for theoretical understanding of the dynamics of the convection zone.

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