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Helioseismic constraints on the proton-proton reaction cross-section

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The primary inversion of the observed frequencies of solar oscillations yields the sound speed and density profiles inside the Sun. In order to infer the temperature and chemical composition profiles, however, additional assumptions regarding the input physics such as opacities, equation of state and nuclear energy generation rate are required (Antia & Chitre 1998; Takata & Shibahashi 1998 and references therein). With the observed luminosity constraint on seismic models it turns out that the cross-section of the proton-proton (pp) nuclear reaction rate, $S_{11} = (4.15 \pm 0.25) \times 10^{-25}$ MeV barns. The main source of error in this estimate is the uncertainties in Z profiles. In this work we, therefore, try to find the region in the (Z, S_{11})-plane that is consistent with helioseismic data. We also explore the possibility of determining the Z profile in addition to the X profile.

We use the observed frequencies from GONG (Global Oscillation Network Group) data for months 4–10 to calculate the sound speed and density profiles. With the help of these inverted profiles, along with a homogeneous Z profile, we obtain the temperature and hydrogen abundance profiles by employing the equations of thermal equilibrium (Antia & Chitre 1998). Using the inverted profiles for ρ, T, X, it is possible to compute the total energy generated by nuclear reactions, which can be compared with the observed solar luminosity, $L_\odot = 3.846 \times 10^{33}$ ergs/sec.

It was emphasized by Antia & Chitre (1998) that there is an (2σ) uncertainty of about 3% in computing the luminosity of seismic models from possible errors in primary inversion, solar radius, equation of state, nuclear reaction rates for other reactions. The uncertainty arising from errors in Z profiles is much larger and hence, in this work we use seismic models with a homogeneous Z profile, with different values of Z covering a wide range. For each central value of Z, we estimate the range of cross-section of the pp nuclear reaction which gives computed luminosity within 3% of the observed value. The results are shown in Fig. 1. It can be seen that current best estimates for Z and S_{11} (Adelberger et al. 1998) are only marginally consistent with helioseismic constraints and probably need to be increased slightly. This figure also shows the limits on value of Z as obtained by Fukugita & Hata (1998), as well as the range of S_{11} as inferred from various theoretical calculations so far (Bahcall & Pinsonneault 1995; Turck-Chiéze & Lopes 1993). One, therefore, expects that the value of Z and S_{11} should fall within the shaded region shown in Fig. 1.

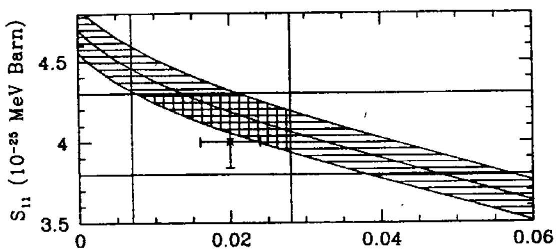

Figure 1. The region in Z–S_{11} plane that is consistent with helioseismic data is marked by the lines with horizontal shading. The central line defines the value where the seismic model matches the observed solar luminosity. The point with 2σ error bars shows the current best estimates for Z and S_{11}. The horizontal and vertical lines mark the limits on central Z value as obtained by Fukugita & Hata (1998) and the limits on S_{11} as obtained by various calculations so far. The region with vertical shading marks the area that is consistent with all data.

Using the density profile along with the equation of hydrostatic equilibrium, it is possible to determine the pressure profile also from primary inversions. It may even be argued that if we use the additional constraint, p = p (T, ρ, X, Z) we can determine the Z besides other profiles. This would require a determination of two of the three unknowns T, X, Z using the two constraints obtained from primary inversions, namely, p(T, ρ, X, Z) and c(T, ρ, X, Z). Since ρ is known independently, we ignore the variation in ρ to write

$$\begin{aligned} \frac{\delta c}{c} &= \left(\frac{\partial \ln c}{\partial \ln T}\right)_{\rho, X, Z} \frac{\delta T}{T} + \left(\frac{\partial \ln c}{\partial X}\right)_{\rho, T, Z} \delta X + \left(\frac{\partial \ln c}{\partial Z}\right)_{\rho, T, X} \delta Z \\ \frac{\delta p}{p} &= \left(\frac{\partial \ln p}{\partial \ln T}\right)_{\rho, X, Z} \frac{\delta T}{T} + \left(\frac{\partial \ln p}{\partial X}\right)_{\rho, T, Z} \delta X + \left(\frac{\partial \ln p}{\partial Z}\right)_{\rho, T, X} \delta Z \end{aligned} \quad (1)$$

It is found that the corresponding derivatives of p and c^2 are almost equal and hence $\delta p/p \approx \delta c^2/c^2$. Thus, for the solar case these two constraints are not independent and it is demonstrably implausible to get any additional information by using the pressure profile. Any attempt to do so will only yield arbitrary results magnifying the errors arising from those in the equation of state and primary inversions.

Even if we do not impose the additional constraint arising from pressure, we can calculate the pressure using the OPAL equation of state (Rogers et al. 1996) with the inferred T, ρ, X and assumed Z profiles. The p-profile can be compared with that inferred from primary inversions using the equation of hydrostatic equilibrium and two profiles are found to agree with each other well within the 1σ error limits. It is, therefore, evident that the pressure profile does not provide an independent constraint.

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References

Adelberger E. C., Austin S. M., Bahcall J. N. et al. 1998, Rev. Mod. Phys. 70, 1265

Antia H. M., Chitre S. M., 1998, A&A 339, 239

Bahcall J. N., Pinsonneault M. H., 1995, Rev. Mod. Phys. 67, 781

Fukugita M., Hata N., 1998, ApJ, 499, 513

Rogers F. J., Swenson F. J., Iglesias C. A., 1996, ApJ 456, 902

Takata M., Shibahashi H., 1998, ApJ 504, 1035

Turck-Chiéze S., Lopes I. 1993, ApJ, 408, 347