LATITUDINAL VARIATIONS IN THE PROPERTIES OF THE TACHOCLINE

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ABSTRACT

The tachocline or the transition layer where the rotation rate changes from differential rotation in the convection zone to almost latitudinally independent rotation rate in the radiative interior is studied using recent data on splitting of solar p-modes from MDI. Attempt is made to study the latitudinal variation in position and thickness of tachocline using forward modelling techniques. We find that as with the case of the GONG data, there is no compelling evidence for any latitudinal variation in the position and width of the tachocline, although after combining all results it appears that the thickness of tachocline increases slightly with latitude and its mean position also appears to move outwards with latitude.

Key words: Sun: oscillations; Sun: rotation; Sun: interior.

1. INTRODUCTION

Inversion of rotational splittings (Thompson et al. 1996; Schou et al. 1998) has demonstrated that the rotation rate changes from differential rotation within the convection zone to an almost latitudinally independent rotation rate in the radiative interior in the region close to the base of the convection zone. This transition region has been referred to as the Tachocline (Spiegel & Zahn 1992). The exact location and thickness of tachocline is an important constraint on the theories of angular momentum transport in stellar interiors as well as the solar dynamo models (Rüdiger & Kitchatinov 1996; Gilman & Fox 1997; Canuto 1998).

Some inversion results suggest that the position and thickness of the tachocline changes with latitude. However, this could be an artifact of changing resolution of the inversion techniques with latitude. Since the tachocline is not resolved by the inversion techniques with present data, forward modelling techniques have been used to study the tachocline (Kosovichev 1996; Basu 1997; Charbonneau et al. 1997). Corbard et al. (1998) have studied the tachocline using a modified inversion technique which accounts for sharp changes in the rotation rate. However, all these attempts were restricted to average position and thickness of tachocline ignoring the possible latitude dependence. Antia et al. (1998) tried to study the latitudinal variation in the position and thickness of the tachocline using the GONG data for the months 4–14. No compelling evidence for any variation was found as the variations in position and thickness were within the estimated errors. In this work we attempt to repeat these calculations using the MDI data for the first 360 days of its operation. We have used both the simple calibration method and the method of simulated annealing to fit possible variations in tachocline properties. The mode set is restricted to those with frequencies less than 3 mHz.

2. THE TECHNIQUE

Since inversions do not resolve the tachocline adequately, we use forward modelling methods. We first take appropriate combinations, c_θ, of splitting coefficients a_1 to a_{11} (see e.g., Fig. 1) that correspond to rotation rates at latitudes of 0, 15, 45, 60 and 75 degrees. We use two methods for investigating the tachocline: (1) A straightforward calibration with models of known position and width of the tachocline (cf., Basu 1997; Antia et al. 1998) which we refer to as the "calibration method", and (2) Non-linear least squares fit using the method of simulated annealing (Antia et al. 1998). The details of these methods can be found in Antia et al. (1998), we give a brief description below.

For the calibration method we construct a series of models with known jumps, position and thickness of tachocline. The rotation rate in the tachocline is modelled as

$$\Omega = \begin{cases} \Omega_{\rm tach} + \Omega_c + B(r-0.7) & \text{if } r \le 0.95 \\ \Omega_{\rm tach} + \Omega_c - C(r-0.95) + 0.25B & \text{if } r > 0.95 \end{cases} \quad (1)$$

where,

$$\Omega_{\rm tach} = \frac{\delta\Omega}{1+\exp[(r_d - r)/w]}, \quad (2)$$

where $\delta\Omega$ is the jump in the rotation rate across the tachocline, Ω_c is the rotation rate in the radiative interior, w is the half-width of the transition layer, r is the radial distance in units of solar radius, and r_d the mid-point of the transition region. It may

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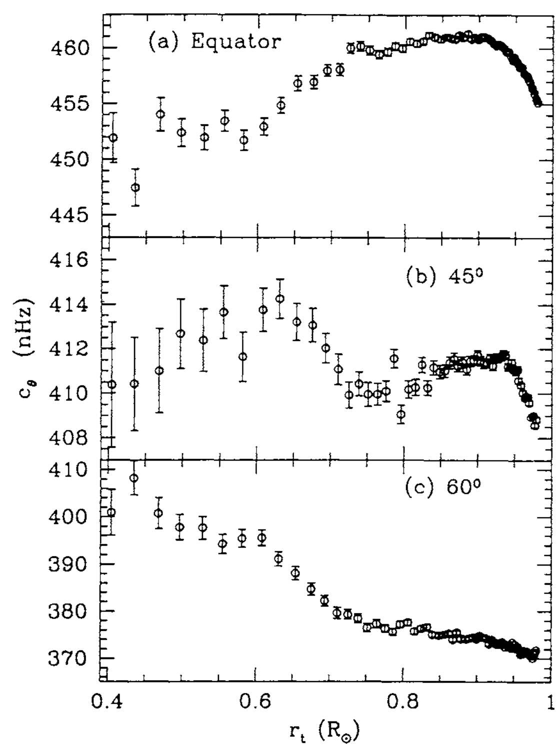

Figure 1. The splitting coefficients for MDI 360-day data combined to obtain the data for different latitudes. The points represent the combinations binned in groups of 15 modes.

be noted that the form used in this study is different from that used by Kosovichev (1996) and Charbonneau et al. (1997) and in particular the width as defined in those studies is about 4.9 times the half-width w used in our work. In this work, the rotation rate increases from a factor $1/(1+e)$ of the entire difference to the factor $1-1/(1+e)$ of $\delta\Omega$ in the range $r = r_d - w$ to $r = r_d + w$. The coefficients Ω_c, B and C which account for the smooth variation in the rotation rate are obtained from inversion for the rotation rate. B is the average gradient of the rotation rate in the lower part of the convection zone, while C is the mean gradient in the near surface shear layer.

The calibration models have r_d of 0.68, 0.69, 0.70, 0.71 and 0.72 $R_\odot$ and half-width w of 0, 0.005, 0.01, 0.015, 0.02 and 0.025 $R_\odot$. A first guess for $\delta\Omega$ is taken from inversion results. All fits are done using modes with lower turning point r_t between 0.6 and 0.9 $R_\odot$. Modes with lower r_t are avoided because of large observational errors in these modes, while modes with higher r_t are not used so that shear layer known to exist just below the solar surface does not affect the results significantly. To determine the tachocline properties, we make identical combinations of the model and observed splitting coefficients which correspond to rotation rate at different latitudes and look for the model which gives a zero difference with the observations after allowing for some smooth variations. The details of the implementation can be found in Antia et al (1998).

We use another forward modeling method, where the jump, position and width can be found simultaneously. The nonlinear least squares fit is done using the method of simulated annealing. The rotation rate at any given latitude is again parameterized by Eqs. 1 and 2 and all the six parameters, $r_d, w, \delta\Omega, \Omega_c, B, C$ are simultaneously determined by a non-linear least squares fit to the combinations of splitting coefficients representing the rotation rate at the required latitude. Once again we use the same set of modes, i.e., those with frequency less than 3 mHz and with lower turning point r_t in the range 0.6–0.9$R_\odot$.

Instead of fitting the rotation rate at each latitude separately and then finding the variation in the tachocline property, we can directly fit a 2D form of the rotation rate with tachocline to obtain the variation in its properties. The form fitted is the same as in the 1D case with the following substitutions in Eqs. 1 and 2:

$$\begin{aligned} B &= B_1 + B_3P_3(\theta) + B_5P_5(\theta), && (3)\\ \delta\Omega &= \delta\Omega_1 + \delta\Omega_3P_3(\theta) + \delta\Omega_5P_5(\theta), && (4)\\ r_d &= r_{d1} + r_{d3}P_3(\theta), && (5)\\ w &= w_1 + w_3P_3(\theta), && (6) \end{aligned}$$

where

$$\begin{aligned} P_3(\theta) &= 5\cos^2\theta - 1, && (7)\\ P_5(\theta) &= 21\cos^4\theta - 14\cos^2\theta + 1, && (8) \end{aligned}$$

are polynomials used to define the latitude dependence. Here θ is the colatitude. For this fit only the first 3 splitting coefficients (i.e., a_1, a_3 and a_5) are used as the form assumed for the latitudinal variation of rotation rate will not fit the higher coefficients.

Use of three different techniques to estimate the tachocline parameters provide checks on the results. Further these methods have been tested on artificial data obtained from a known tachocline model (Antia et al. 1998).

3. RESULTS

The results obtained by the calibration method and 1-D annealing using the MDI data for 360 days, are shown in Table 1. It is clear from these results that although there is some variation in the position and half-width of the tachocline, the variation is within the estimated errors and as such the significance of variation is not clear. The extent of jump across the tachocline of course, has a clear variation with latitude as is also known from the inversion results. This table also shows the χ^2 per degree of freedom as obtained by the simulated annealing fits and the fits for the equator and latitude of 75° are shown in Fig. 2. It appears that χ^2 increases with latitude, which may indicate that the model used to represent the rotation rate is not adequate to explain the data. Alternatively it is possible that the errors in some splitting coefficients have been underestimated or there is some correlation between errors in different splitting coefficients. From the distribution of residuals and from the fact that inversions of these splitting coefficients also yield $\chi^2 \approx 1.5$ per degree of freedom, it appears that the errors have been underestimated,

Table 1. Results for Tachocline properties

Latitude	Results from Calibration			Results from 1-D Simulated Annealing			χ^2
	Jump	Position	Half-width	Jump	Position	Half-width	
0	20.54 ± 1.07	0.6830 ± 0.0115	0.0099 ± 0.0130	26.56 ± 5.71	0.6746 ± 0.0144	0.0031 ± 0.0026	0.954
15	18.69 ± 0.54	0.6981 ± 0.0084	0.0114 ± 0.0092	13.39 ± 2.48	0.7157 ± 0.0070	0.0062 ± 0.0058	1.101
45	-24.97 ± 3.21	0.7093 ± 0.0355	0.0168 ± 0.0141	-31.76 ± 5.53	0.7227 ± 0.0097	0.0110 ± 0.0070	1.253
60	-61.18 ± 9.08	0.7013 ± 0.0120	0.0217 ± 0.0141	-63.95 ± 1.30	0.7190 ± 0.0038	0.0280 ± 0.0100	1.729
75	-89.52 ± 9.96	0.7132 ± 0.0254	0.0202 ± 0.0149	-94.82 ± 8.22	0.7199 ± 0.0071	0.0066 ± 0.0055	2.076

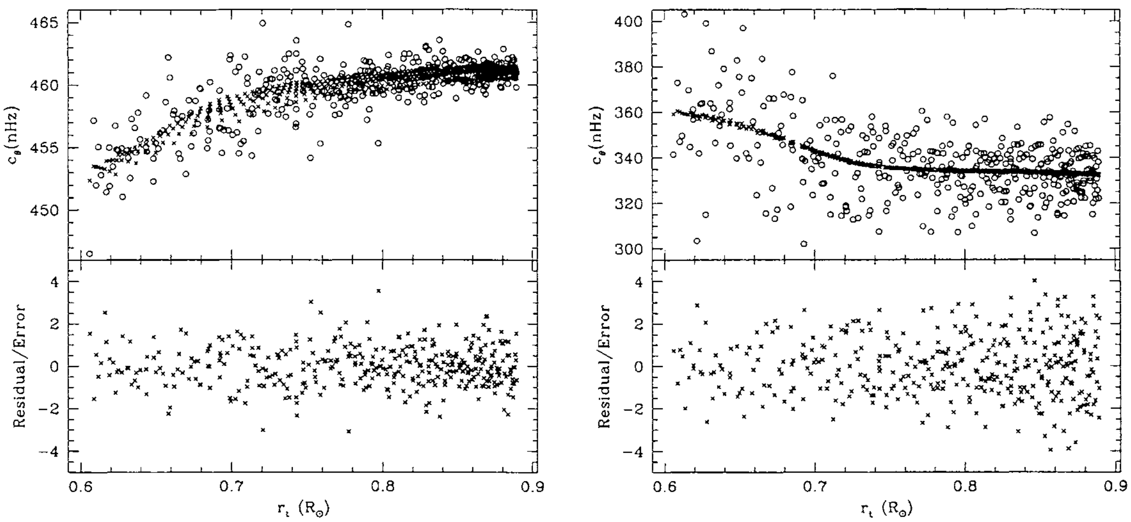

Figure 2. The 1D simulated annealing fit for the solar equator (left panels) and for latitude of 75° (right panels). In upper panels the circles are the observed splitting combinations and the crosses are those obtained by the fit. Lower panels show the normalised residuals for the fits.

though it is not clear if that alone can explain the variation in χ^2 with latitude. At high latitudes it is possible that the simple model for the smooth component assumed in Eq. 1, is not adequate because of possible presence of a jet like feature. This will also increase the χ^2 and could possibly affect the fits.

To obtain an independent estimate of variation in properties of tachocline we fit the 2D variation directly using method of simulated annealing and the results are as follows:

$$r_d = [(0.7046 \pm 0.0066) + (0.0062 \pm 0.0035) P_3(\theta)] R_\odot, \quad (9)$$

$$w = [(0.0055 \pm 0.0053) + (0.0053 \pm 0.0055) P_3(\theta)] R_\odot, \quad (10)$$

$$\delta\Omega = (-2.01 \pm 1.70) - (21.48 \pm 1.24) P_3(\theta) - (2.81 \pm 0.83) P_5(\theta) \text{ nHz}, \quad (11)$$

$$\Omega_c = 437.7 \pm 1.4 \text{ nHz}, \quad (12)$$

$$B = (52.6 \pm 5.0) + (-3.1 \pm 5.4) P_3(\theta) + (-3.5 \pm 3.6) P_5(\theta) \text{ nHz}/R_\odot \quad (13)$$

$$C = 226.4 \pm 46.9 \text{ nHz}/R_\odot \quad (14)$$

It is clear from the results that the variation in position and half-width is comparable to the error estimates. In order to get another measure of the significance we repeat the fit with parameters r_{d3} and w_3 set to zero, which does not allow for any variation in tachocline position or thickness with latitude. This fit gives $r_d = 0.7143 R_\odot$ and $w = 0.0023 R_\odot$ and has χ^2 per degree of freedom of 1.657. With two additional parameters, representing latitudinal variation the χ^2 per degree of freedom reduces to 1.649. Thus it is clear that there is no significant improvement in the fit when two additional parameters are included. Hence, there is no compelling evidence for latitudinal variation of position or width of tachocline.

These results are summarized in Fig. 3, which shows the position, thickness and extent of jump in rotation rate across the tachocline as estimated by all three techniques as a function of latitude. This figure may be compared with Fig. 12 in Antia et al. (1998), which gives the same results using the GONG data. It is clear that the two results are consistent with each other within the estimated errors. It is possible to take a weighted average of all results obtained using the GONG and MDI data at every latitude and

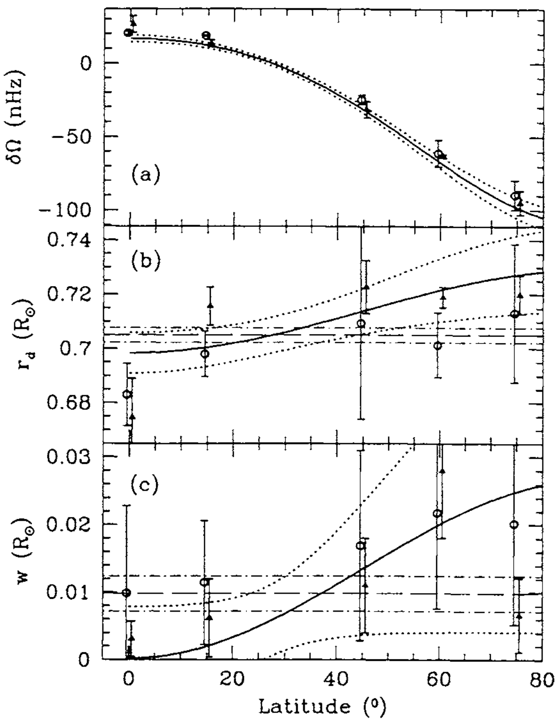

Figure 3. A summary of the tachocline results. Panels (a), (b) and (c) show the results of the jump, position and thickness respectively. In each panel the continuous line is the results of the 2D annealing with the 1σ error bounds shown as the dotted lines. The circles show the results of the calibration method and the triangles are the results obtained by 1D annealing. The symbols are displaced by $\pm 0.5°$ about the true latitude for the sake of clarity. The dashed line in panels (b) and (c) mark the mean values found by Basu (1997), while the 1σ error bounds are shown as dot-dashed lines.

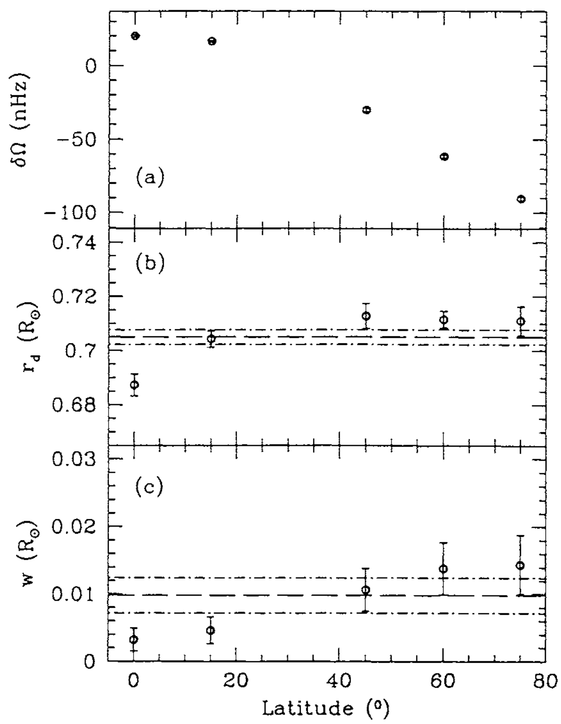

Figure 4. The weighted average of the MDI and GONG tachocline results. Panels (a),(b) and (c) show the results of the jump, position and thickness respectively. The dashed line in panels (b) and (c) mark the mean values found by Basu (1997), while the 1σ error bounds are shown as dot-dashed lines.

these results are shown in Fig. 4. While the weighted average show some variation in position and thickness it is not clear if the variation is significant. The mean position of the tachocline r_d appears to change sharply between the equator and 15° latitude, while beyond that the variation is not significant. It is possible that this apparent variation is due to some systematic errors in determining the position at the equator. This point is also within 2σ of the mean position estimated by Basu (1997). Similarly all the thickness values are more or less consistent with the mean thickness determined by Basu (1997).

The smoothing used in inversions for rotation rate tends to smooth out the steep variation in the tachocline. Apart from this it may introduce ripples away from the tachocline (the Gibbs phenomenon). Thus we have done an inversion after explicitly removing the contribution from tachocline as given by the 2-D annealing fits from the observed splitting before doing inversion. The inversion was done using a 2D-RLS method (see Antia et al. 1998 for details of the method used) using all splitting coefficients up to a_{35} corresponding to modes with frequency less than 3 MHz. Fig. 5 shows the contours of constant rotation rate in the solar interior, while Fig. 6 shows the variation in rotation rate at a few selected latitudes as a function of radial distance. These results also show a possible high latitude jet around $r = 0.95R_\odot$ and a latitude of 75°, which has been seen in some inversion results using MDI data (Howe et al. 1998). It is not clear if this feature is real as no such feature is seen in the GONG results (Antia et al. 1998). Similarly the significance of feature near the pole centered at $r = 0.5R_\odot$ is not clear. Although similar feature is seen in the GONG results (Antia et al. 1998) it has opposite sign as compared to that in the MDI results!

4. CONCLUSIONS

We do not find any compelling evidence for the latitudinal variation in the thickness and position of the tachocline. The variations in the position and thickness of the tachocline is within the errors bars. The tachocline results obtained from the MDI 360-day data agrees with those obtained using GONG months 4–14 data (cf., Antia et al. 1998). The magnitude of the jump shows good agreement, while the position and thickness agree within estimated errors. The mean position and thickness of the tachocline agrees with that obtained by Basu (1997) and Char-

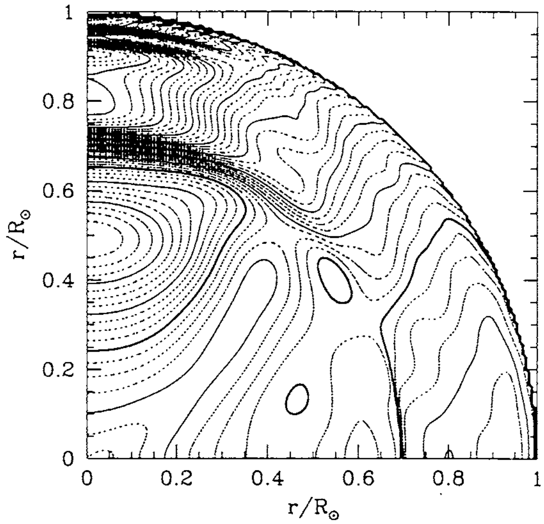

Figure 5. Contour diagram of the solar rotation rate as obtained by a 2D-RLS inversion of the MDI data after removal of the tachocline. The dotted contours have been drawn at intervals of 5 nHz., and the continuous ones at intervals of 20 nHz. The thick continuous line is the contour at a level of 440 nHz. The x-axis represents the solar equator while the y-axis represents the rotation axis.

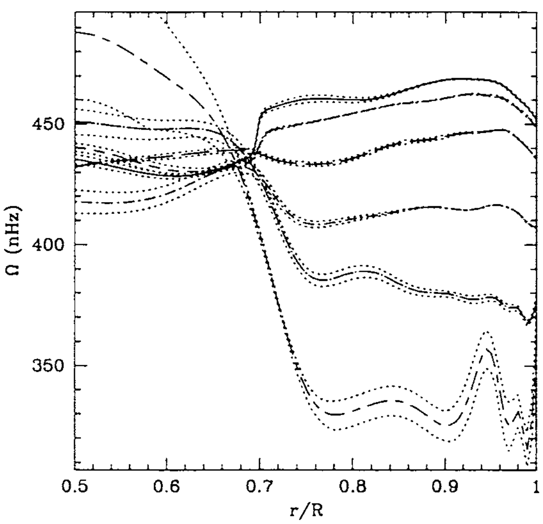

Figure 6. Rotation rate as a function of the radial distance at latitudes of 0° *(continuous line),* 15° *(short dashed line),* 30° *(long dashed line),* 45° *(dot short dashed line),* 60° *(dot long dashed line),* 75° *(short-dashed long-dashed line). In each case the dotted lines mark the* 1σ *error limits.*

bonneau et al. (1997). The weighted average of the MDI and GONG results is shown in Fig. 4, and it appears to show some variation in thickness and position of the tachocline, with the tachocline becoming slightly thicker and shifts outwards at higher latitude. However, the significance of this variation is not very clear.

The mean position of the tachocline appears to be below the base of the solar convection zone, which is located at a distance of $(0.7135 \pm 0.0005)R_\odot$ (Basu 1998). However, a part of the tachocline probably extends into the convection zone, at least, at higher latitudes. Of course, it is possible that the position of the base of the convection zone also has some latitude dependence in which case this situation may be altered.

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