STRUCTURE INVERSION HARE-AND-HOUNDS

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ABSTRACT

We report some results of a hare-and-hounds exercise for the 1-D structure inversion of p-mode oscillation frequencies. The exercise was carried out to uncover possible uncertainties in the inversion results, and to test the reliability of the error estimates. The results have demonstrated numerically robust structure inversions for both the adiabatic and isothermal sound speeds. The most accurate results were obtained for the squared isothermal sound speed, u ($\equiv p/\rho$) using the optimally localized averages technique and the equation of state to constrain the variations of the adiabatic exponent, γ. Without the additional constraint, the most accurate results were obtained for the adiabatic sound speed by the regularized least squares technique. The estimates of the density and the parameter of convective stability are less accurate than the estimates of the sound speeds. The variations of the adiabatic exponent, the temperature and the helium abundance in the radiative zone were not determined reliably. Additional constraints, such as the equation of state and the equations of thermal balance, are important for improving the diagnostic power of the structure inversions. Understanding the physics of the constraints theoretically and seismologically is essential for making robust inferences about solar properties by inversion.

A detailed report is published elsewhere (Antia *et al.*, 1996).

1. Solar Models

The solar model used as the Sun's proxy was computed under the standard assumptions about the solar evolution (Guzik, Cox & Swenson, 1996). The model age was 4.54 Gyr; it included gravitational settling and diffusion of helium and heavier elements. The nuclear reaction rates were taken from the tables by Caughlan and Fowler (1988). The opacity coefficient and the equation of state were computed using the OPAL tables (Rogers & Iglesias, 1992). The initial abundances of helium, Y, and heavy element, Z, were 0.2740 and 0.0195, respectively. The model chosen for the Sun's proxy had small perturbations of the abundances added after the model evolution was completed. The perturbed model was properly calibrated.

The solar model used as a reference for the exercise was Model S (Christensen-Dalsgaard *et al.*, 1996), which was also used for the inversion of the initial GONG frequencies. This model was also computed under the standard assumption of the solar evolution, but using a different numerical procedure and different microscopic data. The opacity coefficient and the equation of state were taken from the same OPAL tables. However, the interpolation scheme was different from the interpolation used for the proxy. The nuclear reaction parameters were adopted from Bahcall and Pinsonneault (1995). Helium and heavy-element (at the oxygen rates) settling was included, using Michaud and Proffitt's (1993) theory. The initial helium and heavy-element abundances were 0.2713 and 0.01963.

2. Frequencies

The oscillation frequencies were computed in the adiabatic approximation. The data set chosen for the exercise consisted of the frequencies of the p modes of the angular degree from 0 to 150 in the frequency range from 1.5 to 3 mHz (Fig. 1). The frequencies of these modes are least affected by the poorly determined upper convective boundary layer and by other non-adiabatic effects. The same set of modes was used by Gough *et al.* (1996) for inferring the internal structure of the Sun from the initial GONG data. The errors added to the frequencies of the proxy model were random numbers computer-generated for the Gaussian distribution with zero mean and the standard deviation corresponding to 1σ standard error estimated from the initial GONG data (Hill *et al.*, 1996).

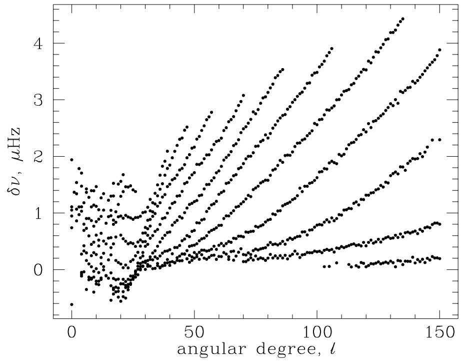

Figure 1: The difference between the frequencies of the proxy with the added noise and the reference model.

3. Inversions

3.1 Sound Speeds

S. Basu and J. Christensen-Dalsgaard used the technique of Subtractive Optimally Localized Averages (SOLA) (cf., Pijpers & Thompson 1992). They carried out inversions for the adiabatic sound speed, c, using the density, ρ as the second helioseismic variable, and also inferred the isothermal sound speed, u, using the equation of state as an additional constraint.

H.M. Antia implemented a Regularized Least Squares (RLS) method with iterative refinement (cf., Antia 1996). All his inversions were done using ρ and γ as the two independent variables and all other dynamical quantities were computed by assuming hydrostatic equilibrium to produce a complete seismic model. This seismic model was then used as the reference model in subsequent iterations to check for convergence.

The results of the inversions are generally in good agreement with the actual difference between the proxy and the reference models. Small deviations of the order of 10^{-3} that exceed the estimated errors are seen in the regions of relatively rapid variation of the first derivatives of $\delta c^2/c^2$ and $\delta u/u$, that is at $0.4R$, $0.5R$, at the base of the convection zone, and near the surface. Obviously, these deviations partly result from averaging the solar properties around the target points in the inversions.

It is interesting that both groups of inverters detected a small jump in the sound speed of the amplitude of $\approx 5 \times 10^{-4}$ at $r \approx 0.86R$. This jump was a numerical artifact, and had no physical meaning. Nevertheless, its successful detection demonstrates the ability to resolve fine variations of the internal structure of the Sun by using the inversion techniques.

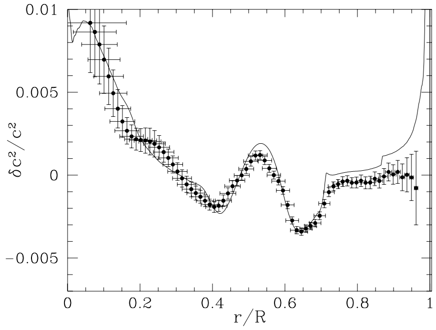

Figure 2: The result of inversion for the sound speed variation by S.Basu and J.Christensen-Dalsgaard (crosses) and the actual variation (solid curve). The horizontal bars show a half-width of the averaging kernels; the vertical bars show the estimated errors.

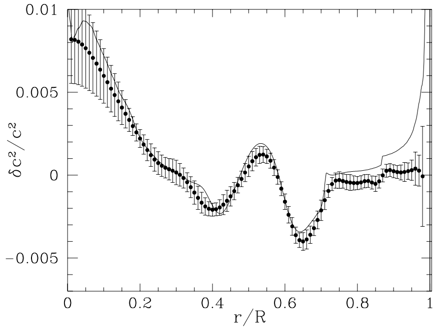

Figure 3: The result of inversion for the sound speed variation by H.M. Antia (points with the errorbars), and the actual difference (solid curve).

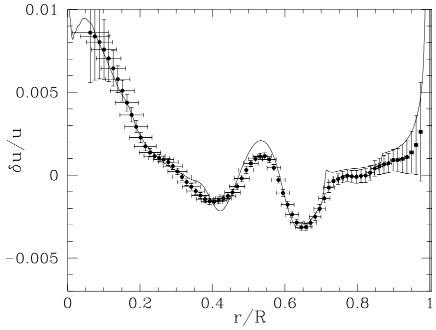

Figure 4: The result of inversion for the variation of u ($\equiv p/\rho$) by S.Basu and J.Christensen-Dalsgaard (crosses) and the actual variation (solid curve).

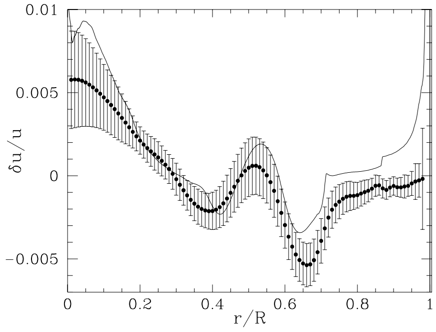

Figure 5: The result of inversion for the variation of u ($\equiv p/\rho$) by H.M. Antia (points with the errorbars), and the actual difference (solid curve).

3.2 Density

The density estimates are considerably less accurate than the estimates of the sound speed. The inverters reported that the accuracy of the density estimates is improved if a broader range of mode frequencies is used in the inversions.

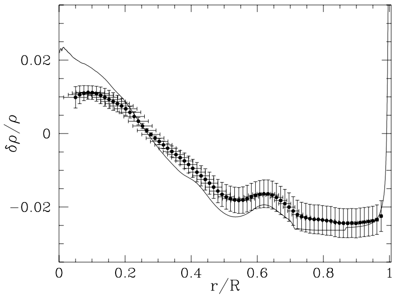

Figure 6: The result of inversion for the density variation by S.Basu and J.Christensen-Dalsgaard (crosses) and the actual variation (solid curve).

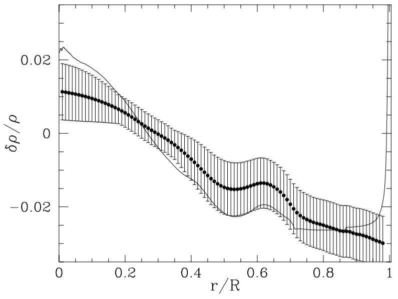

Figure 7: The result of inversion for the density variation by H.M. Antia.

3.3 Adiabatic Exponent

The solar plasma is almost an ideal gas with $\gamma \approx 5/3$ everywhere but the ionization zones. However, the helioseismic inversions are capable, in principle, to detect the variations of γ not only in the subsurface ionization zones, but also in the deep interior where the non-ideal effects are of the order of 10^{-3}. H.M. Antia estimated the variations of γ through out the Sun, using ρ as the second helioseismic variable, whereas J.R. Elliott and D.O. Gough studied variations of γ in the convection zone assuming that the convection zone is adiabatically stratified. Because of the additional constraint, the estimates by J.R. Elliott and D.O. Gough are more precise than the others. However, the inversions for γ are not yet reliable (see Antia, *et al.* 1996).

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