HELIUM ABUNDANCE IN THE SOLAR ENVELOPE

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ABSTRACT

The variations in the adiabatic index in the second helium ionization zone of the Sun allow us to infer the helium abundance in the solar envelope using the observed solar oscillation frequencies. These variations leave their signature on the sound-speed in this region, hence, techniques based on solar sound speed inversion can be used to determine the abundance of helium. These techniques are known to be sensitive to the equation of state used in the reference models. Sensitivity of the helium abundance measurements to the equation of state is studied using models constructed with MHD or OPAL equations of state. Recent observations of high degree solar p-modes yield helium abundance $Y = 0.246$ and 0.249, respectively using reference models with MHD and OPAL equations of state. Further, the models constructed using OPAL equation of state are found to be in better agreement with the inferred sound speed in the Sun, particularly below the second helium ionization zone.

1. INTRODUCTION

Direct measurement of the abundance of helium in the Sun are very difficult, hence, helioseismology plays a major role in determining the helium abundance in the solar envelope. The abundance is obtained from the variation of the adiabatic index of the solar material in the second helium ionization zone. Various techniques have been used to infer the helium abundance from the observed frequencies of solar oscillations (Kosovichev et al. 1992, Pérez Hernández & Christensen-Dalsgaard 1994, Antia & Basu 1994a and references therein), but the results have not always been consistent with each other.

It is found that the helioseismic measurement of helium abundance is sensitive to the equation of state of stellar material, which is used in translating the variation of Γ_1 to the difference in Y. Recently the OPAL equation of state (Rogers 1994) has become available. It would thus be interesting to test the sensitivity of helium abundance measurement to equation of state. It would also be interesting to check which of the equations of state is closer to that of the solar material.

Apart from the equation of state uncertainties in the model of solar surface layers also introduce errors in measurement of helium abundance (Antia & Basu 1994a). Recently, there has also been a significant improvement in modeling the structure of surface layers of the Sun. In particular, it has been demonstrated (Basu & Antia 1994) that if the convective flux is calculated using the prescription of Canuto & Mazzitelli (1991) then the resulting solar models are in much better agreement with the observed frequencies.

In the present work we use the techniques described in Antia & Basu (1994a) to determine the abundance of helium in the Solar envelope using an improved set of reference models as well as observed frequencies (Bachmann et al. 1995). Therefore we expect to obtain a more reliable estimate of the helium abundance in the solar envelope.

2. THE TECHNIQUE AND MODELS

Asymptotically, the frequency differences between a solar model and the Sun, or between two solar models can be written as (Christensen-Dalsgaard, Gough & Thompson 1989)

$$S(w)\frac{\delta\omega}{\omega} = H_1(w) + H_2(\omega), \tag{1}$$

where $S(w)$ is a known function of the sound speed of the reference model and $w = \omega/(\ell + 1/2)$. The functions $H_1(w)$ and $H_2(\omega)$ can be found by a least squares fit to the known frequency differences. $H_1(w)$ which contains the information about differences in internal structure of the two models, can be inverted to obtain the sound-speed difference between the two models, or between the reference model and the Sun (Christensen-Dalsgaard et al. 1989). While $H_2(\omega)$ is essentially determined by the differences in the surface layers.

Difference in the Helium abundance leaves its signature on both $H_1(w)$ and $H_2(\omega)$. For two solar envelope models which differ only in their X and Y values, $H_1(w)$ and $H_2(\omega)$ depend only on the difference in X. If $\phi(w)$ is the $H_1(w)$ between two envelope models with a known difference in X, then

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$H_1(w)$ for any other pair of models can be written as

$$H_1(w) = \beta\phi(w) + H_s(w), \tag{2}$$

where $H_s(w)$ is a smooth component of $H_1(w)$ which arises from differences in the equation of state, surface physics etc, while the first term gives the contribution arising from difference in X. Thus if β is determined by a least squares fit, the unknown helium abundance in the Sun can be obtained (AB). We can use $H_2(\omega)$ in a similar manner.

If we complete the inversion of $H_1(w)$ to calculate $\delta c/c$ and hence c, the sound speed in the Sun, we can calculate the function

$$W(r) = \frac{r^2}{Gm}\frac{dc^2}{dr}, \tag{3}$$

This function shows a peak at the HeII ionization zone, with the peak height depending on the helium abundance and can be calibrated to find the unknown helium abundance in the Sun (Gough 1984).

We have used all three functions $H_1(w)$, $H_2(\omega)$ and $W(r)$ to determine the helium abundance in the solar envelope. Since we are only interested in the region around helium ionization zone it is enough to use solar envelope models to calibrate the signature of helium abundance in these functions. Apart from the abundances a solar envelope model would also depend on the mixing length parameter, but we fix the mixing length parameter to obtain a prescribed depth of convection zone thereby reducing one unknown parameter. This is justified since the depth of the convection zone is known accurately (Christensen-Dalsgaard, Gough & Thompson 1991).

For the purpose of calibration we have constructed 5 solar models each using the MHD (Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988; Däppen et al. 1988) and OPAL equations of state with $X = 0.68$, 0.70, 0.72, 0.74 and 0.76. In addition, we have constructed models with $X = 0.73$ with both the equations of state to act as test models. For convenience, the models are labeled by the EOS used and their X value, e.g., MHD68, OPAL72, etc.

All these models have base of convection zone at 0.71 $R_\odot$ which is the depth of convection zone as determined by Christensen-Dalsgaard etal (1991). All these models use OPAL opacities (Rogers & Iglesias 1992) for $T > 20000$ K, and opacities from Kurucz (1991) at lower temperatures. The convective flux in these models has been calculated using formulation of Canuto & Mazzitelli (1991). The MHD models have $Z = 0.020$, while OPAL models have $Z = 0.019$.

Thus for each equation of state we have 4 calibration curves each for $H_1(w)$ and $H_2(\omega)$ (e.g, MHD68-MHD70, MHD70-MHD72, MHD72-MHD74 and MHD74-MHD76, similarly for the OPAL models too).

To determine the helium abundance in the solar envelope we have used the observed frequencies given by Bachmann et al. (1995). We use all modes in the frequency range $1.8 \le \nu \le 5.0$ mHz, with $w(=\omega/(\ell+1/2))$ in the range $-2.44 \le \log w \le -1.4$ mHz. We have excluded the f-modes, since the asymptotic relations used in the present study are not expected to be valid for these modes. This gives us a set of 4546 modes. All these modes have their lower turning points well above the base of the convection zone. For consistency we have used the same set of modes to determine the helium abundance in test models also.

3. RESULTS

The results of helium abundance for the test models and observed frequencies are shown in Table. 1, where all numbers represent the helium abundance by mass in terms of percentages. The errors shown in the table include uncertainty in Y due to errors in the data and the fitting process, but does not include systematic errors due to difference in EOS and other physical inputs. From the results for the test models we can see that when the equation of state in the test model is the same as that in the reference models, there is no difficulty in determining the helium abundance using any of the techniques. In this case $W(r)$ appears to give the best estimate.

From the results for test model using an equation of state that is different from that in reference models, it is clear that the method based on the height of $W(r)$ hump in the helium ionization zone is extremely sensitive to differences in the equation of state. This difference is caused by the fact that the height of the HeII hump in $W(r)$ depends on the equation of state as well, and not just the helium abundance. Figure 1 shows $W(r)$ for models MHD72, MHD74, OPAL72 and OPAL74 as computed from their known sound speed profiles. We can see that the height of the peak is significantly different for the two EOS even if the models have the same helium abundance. Further, from the difference in heights it appears that there would be an error of approximately 1% in determining the helium abundance using reference models with different equation of state.

In addition, $W(r)$ for the OPAL models shows some humps apart from the main HeII peak. We believe these are due to interpolation errors caused by the rather coarse grid in T, ρ, and X in the equation of state tables. Such humps are also present in the function $S(w)$ that is used in asymptotic inversion and are transmitted to $H_1(w)$ and $H_2(\omega)$ through the least squares solution. These humps can interfere with the determination of Y using models with the OPAL equation of state. Thus it would be desirable to define the equation of state over a denser grid of points in T, ρ and X.

Table 1, Determining Y for test models and the Sun

Method	MHD73	OPAL73	Obs. frequencies
Exact	25.00	25.10	
	MHD reference models		
$W(r)$	24.97 ± 0.11	23.83 ± 0.14	23.53 ± 0.47
$H_1(w)$	25.04 ± 0.05	24.89 ± 0.23	24.51 ± 0.06
$H_2(\omega)$	24.92 ± 0.30	24.90 ± 0.30	24.84 ± 0.30
	OPAL reference models		
$W(r)$	26.17 ± 0.15	25.00 ± 0.11	24.80 ± 0.49
$H_1(w)$	25.30 ± 0.22	25.11 ± 0.05	24.79 ± 0.34
$H_2(\omega)$	25.26 ± 0.51	25.18 ± 0.37	25.04 ± 0.49

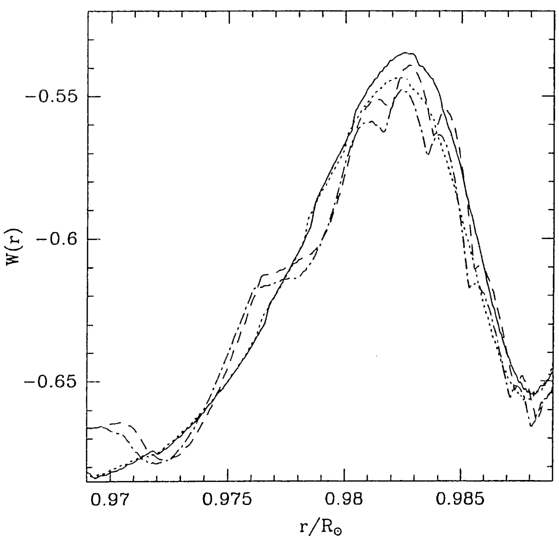

Figure 1. The function $W(r)$ for MHD72 (continuous line), MHD74 (dotted line), OPAL72 (dashed line) and OPAL74 (dot-dashed line) reference models.

Nevertheless, the techniques based on $H_1(w)$ and $H_2(\omega)$ are not particularly sensitive to the difference in equation of state and are thus more reliable for determining the helium abundance. Considering the results for test models with different equations of state it appears that if the difference between the MHD or OPAL equation of state and that of the solar material is comparable to the difference between the OPAL and MHD EOS, then the helium abundance can be determined quite reliably from $H_1(w)$ and $H_2(\omega)$, with an error of around 0.3%.

Considering the observed frequencies we find that the result obtained using $W(r)$ is extremely sensitive to the equation of state in the reference models and hence is unlikely to be reliable. Discarding the results obtained using $W(r)$ we get a weighted mean of $Y = 0.2456 \pm 0.0007$ for the Sun with the MHD reference models and $Y = 0.2489 \pm 0.0028$ with the OPAL equation of state.

Figure 2 shows the function $H_1(w)$ between the reference models and the Sun. From this figure it is clear that the solar helium abundance is between 0.24 and 0.26. Figure 3 shows the function $H_2(\omega)$ between the various solar models and the Sun.

To compare the two equations of state, we first compare the function $H_1(w)$ between the Sun and the calibration models using the two equations of state (Figure 2). Since the variation of $H_1(w)$ within the HeII ionization zone depends on the helium abundance, we concentrate on layers just below this. If we consider the curves for X=0.72 and 0.74, which bracket the solar hydrogen abundance, it is clear that the curves using OPAL equation of state are in general flatter than that using MHD equation of state. Since the gradient of $H_1(w)$ determines the difference in sound speed between the reference model and the Sun, we can expect the sound speed in the OPAL models to be closer to that in the Sun. This is borne out by the results of sound speed inversion, displayed in Figure 4. In particular, the hump in the sound-speed difference between the Sun and the MHD reference models around 0.95 $R_\odot$, which arises from a possible difference between equation of state of the solar material and that used in the model (Dziembowski, Pamyatnykh & Sienkiewicz 1992, Antia & Basu 1994b) is significantly reduced in the sound speed difference between the Sun and the OPAL reference models. It may be noted that we have used different heavy element abundance for the models with MHD and OPAL equations of state. We have verified that this hump is not caused by the difference in Z, by considering a model with $Z = 0.02$ using the OPAL equation of state. It may be noted that apparent departure near $r = 0.85$ in the sound speed of the models is most probably due to lack of modes penetrating to those depths among the observed frequencies.

Thus, we conclude that below the HeII ionization zone, the OPAL EOS is closer to the EOS of solar material. For $r > 0.99 R_\odot$ the sound speed inversion is not reliable because of uncertainties in extrapolating $H_1(w)$ beyond $\log w = -2.44$ mHz as no observed modes are available in this range. Apart from extrapolation the asymptotic inversion technique also breaks down in this region, where scale heights are comparable to or smaller than the vertical scale of oscillations. By considering the sound speed differences in the HeII ionization zone around $r \lesssim 0.98 R_\odot$, it appears that OPAL models are closer to the Sun in terms of the sound speed. However, considering the errors in inversion in this region and the variation in sound speed with the helium abundance, we are not sure if the result is significant.

By comparing Figures 2 and 3 with Figures 13 and 16 in AB, it is clear that the present models are significantly better than those used by AB. The steep trend in $H_1(w)$ at low w is significantly re-

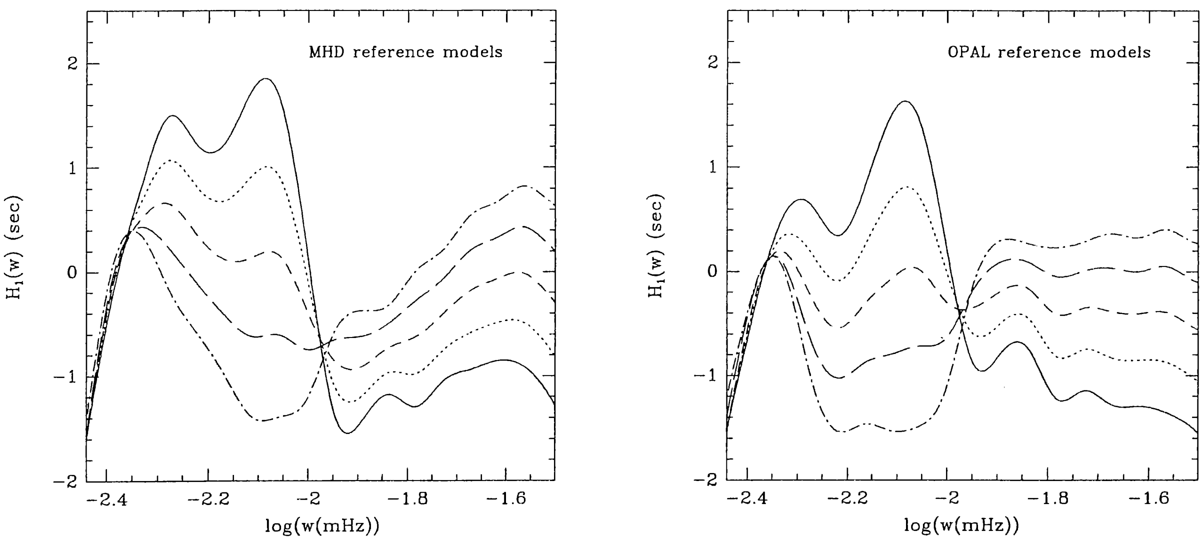

Figure 2. The function $H_1(w)$ between the reference models and the Sun. The various curves are for reference models with $X = 0.68$ (solid line), $X = 0.70$ (dotted line), $X = 0.72$ (short dashed line), $X = 0.74$ (long dashed line), and $X = 0.76$ (dot-dashed line).

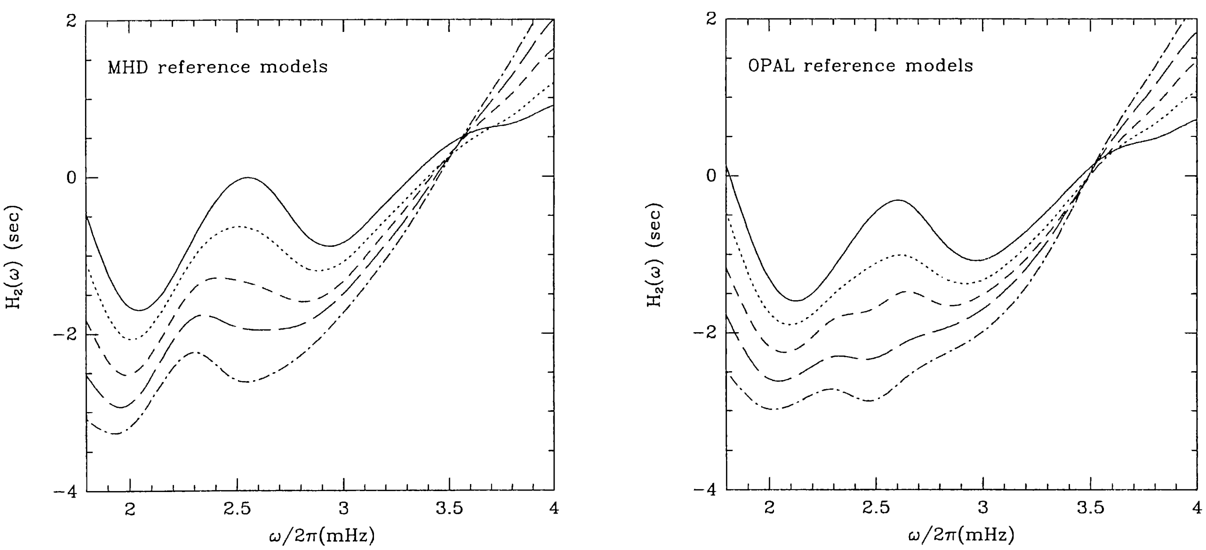

Figure 3. the function $H_2(\omega)$ between the reference models and the Sun. The various curves are for reference models with $X = 0.68$ (solid line), $X = 0.70$ (dotted line), $X = 0.72$ (short dashed line), $X = 0.74$ (long dashed line), and $X = 0.76$ (dot-dashed line).

duced, while the variation in $H_2(\omega)$ which measures the contribution from differences in surface layers is reduced from nearly 20 s over the frequency range shown in the figure to about 3 s in the present models. Similarly, the frequency differences are reduced from about 180 μHz in the models of AB to 13 μHz in present models.

In order to demonstrate the improvement more clearly, in Figure 5 we show the frequency differences between the Sun and model OPAL73. We can see that for most modes the difference is less than 13 μHz. Further, for $\nu \lesssim 3.5$ mHz the computed frequencies are smaller than the observed frequencies, while at higher frequencies they are larger than the

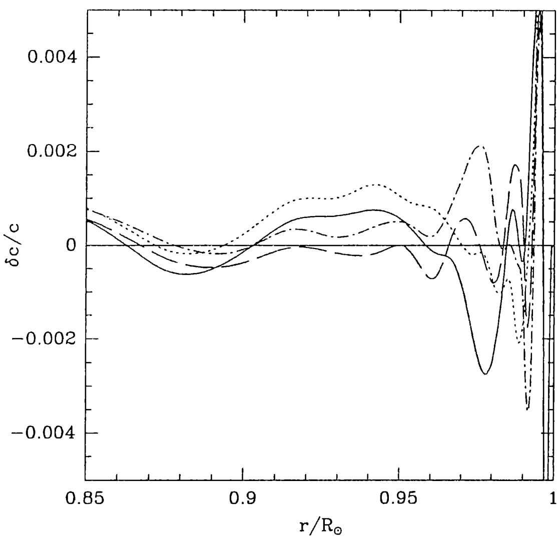

Figure 4. Relative sound speed difference $(c_{\rm model}-c_{\rm Sun})/c_{\rm model}$ between the Sun and the reference models MHD72 (continuous line), MHD74 (dotted line), OPAL72 (dashed line), and OPAL74 (dot-dashed line).

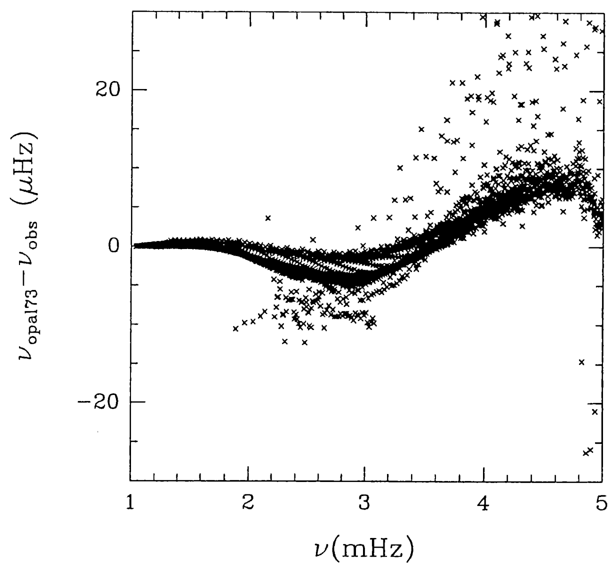

Figure 6. The frequency differences between model OPAL73 and the Observed frequencies of Libbrecht et al.. (1990).

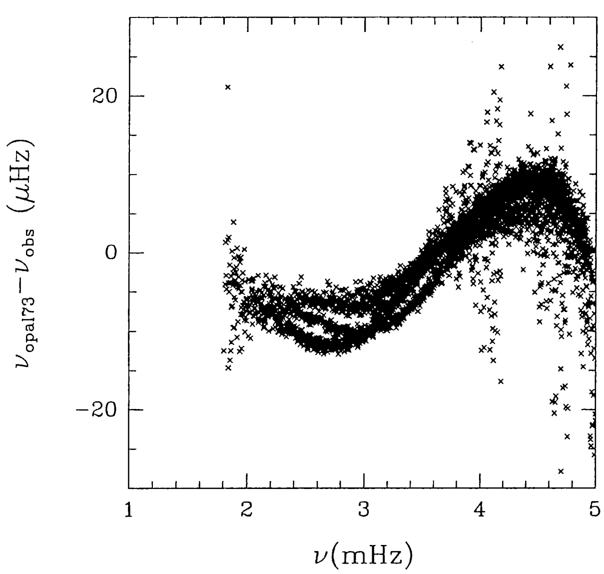

Figure 5. The frequency differences between model OPAL73 and the Observed frequencies of Bachmann et al.. (1995).

observed frequencies. The different bands in the figure correspond to various values of the radial harmonic number n, with the lowest one corresponding to $n = 1$. The f-modes ($n = 0$) which have somewhat larger differences have been excluded from this figure as their frequencies are essentially independent of the solar model. The thickness of the bands in this figure should give an indication of errors in observed frequencies. There appear to be significant errors in observed frequencies at low frequencies as well as at high degrees. The bunch of points spreading out around $\nu \approx 4.2$ mHz is due to very high degree modes with $n = 1$, while the second and third bunch around $\nu \approx 4.8$ mHz and $\nu \gtrsim 5$ mHz are due to $n = 2$ and 3 modes respectively. The sole point that is standing out around 2 mHz is due to $n = 1$, $\ell = 166$ mode.

It is clear that the differences between the observed and computed frequencies is significantly larger than the errors in observations. Of course, the non-adiabatic corrections will alter the frequencies, but at the moment there is no acceptable treatment of non-adiabatic effects to provide even a rough estimate of this effect. Apart from non-adiabatic effects the frequencies could also be affected by uncertainties in the atmospheric model. In the present study we have used the atmospheric model from Vernazza et al. (1981).

Although in this study we have not used observed frequencies from any other source, it would be interesting to study the effect of systematic errors in observed frequencies on the helium abundance, by considering other sets of observed frequencies. Figure 6, shows the frequency difference between the same OPAL73 model and the observed frequencies of Libbrecht, Woodard & Kaufman (1990) which have been extensively used in helioseismic investigations. This figure includes all p-modes with $\ell > 3$ and $\nu < 5$ mHz from the tables of Libbrecht et al. (1990). The scattered points lying above the main bands are due to high degree ($\ell > 400$) modes. These frequencies have been measured using a different technique and the errors are much larger. The few points lying below the main band on the low frequency side are due to modes with $140 < \ell \leq 400$. There is clearly some systematic errors between the measured frequencies of these modes as compared to lower degree modes. This systematic errors become very clear if the frequency differences for individual n values are plotted separately. The thinner band lying above the thick one is due to modes penetrating below the convection zone, thus indicating that the depth of the convection zone used in this model does not agree exactly with that in the Sun. There is probably a discrepancy of approximately $0.003R_{\odot}$ in the depth of convection zone. By comparing Figures 5 and 6, it is clear that there is a significant systematic difference between the two sets of observed frequencies. Further, errors in frequencies of high degree modes are clearly much larger than those in intermediate degree modes.

4. CONCLUSIONS

In this work we have tried to determine the helium abundance in the solar envelope using the three tracers described in Antia & Basu (1994a). We have used improved solar models as reference models to calibrate the tracers with two different equations of state, i.e., the MHD and the OPAL. The solar envelope models that we have used for this work show very small frequency differences with the Sun with most modes having frequency difference of less than 13 μHz for the model OPAL73.

From tests with solar models we find that $H_1(w)$ and $H_2(\omega)$ give a more reliable estimate of Y. The observed frequencies of Bachmann et al. (1995) yields a value of $Y = 0.2456 \pm 0.0007$ and 0.2489 ± 0.0028 with the MHD and OPAL calibration models respectively. At this stage it is not feasible to get very accurate results with the OPAL equation of state since it is available on a very coarse grid.

We find that to be able to make any definitive statement about the solar helium abundance we need to be certain about the equation of state of the solar material. From the function $H_1(w)$ obtained for the Sun with our calibration models and the resulting sound speed, we believe that the OPAL equation of state is closer to that in the solar material in the region below the HeII ionization zone. We can not, at this stage, make any definitive statement about the equation of state within HeII ionization zone, but even in that region OPAL equation of state appears to be closer than the MHD equation of state.

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