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Helioseismic detection of overshoot below the solar convection zone

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It is generally accepted that there is no adequate theory to describe astrophysical convection. In particular, there is no agreement among different theories about the extent of overshoot from stellar convection zones. The solar photosphere seems to show, substantial overshoot. On the basis of this result it is sometimes concluded that there is substantial overshoot below the solar convection zone also. Since this layer is not directly observable, it would be interesting to verify this conjecture using the available helioseismic data (Libbrecht *et al.* 1990).

All reasonable models of convection dynamics indicate that the overshoot layer below the base of solar convection zone is almost adiabatically stratified and is followed by an almost discontinuous transition to radiative stratification below the base of the overshoot layer. Gough (1990) has shown that this contributes a characteristic oscillatory component to the frequencies $\nu_{n,\ell}$ of those p-modes which penetrate beyond the base of overshoot layer. This oscillatory component can be extracted by taking second differences of the frequencies for a given degree ℓ. The amplitude of these oscillations is found to depend on the depth of adiabatically stratified region comprising the convection zone and the overshoot layer, and on the 'severity' of the discontinuity, which in turn depends on the extent of overshoot.

Using an asymptotic theory it can be shown that the frequency of p-modes can be written as (Monteiro *et al.* 1992a,b)

$$\nu_{n,\ell} = \nu_0(n, \ell) + A\nu \cos\left(\tau\nu - \gamma\frac{\ell(\ell+1)}{\nu} + \phi\right) \qquad \dots (1)$$

where $\nu_0(n, \ell)$ is the smooth part of ν and

$$\tau = 2\int_{r_d}^{R} \frac{dr}{c}, \qquad \gamma = \int_{r_d}^{R} \frac{c\,dr}{r^2} \qquad \dots (2)$$

Here r_d is the radial distance at which the discontinuity occurs, R the solar radius and c the sound speed. In the second difference, the oscillatory component will dominate over the smooth part and hence can be easily detected. The amplitude A can be calibrated as a function of the extent of overshoot by constructing solar models with different amounts of overshoot.

Using this technique Monteiro *et al.* (1992a,b) found that the amplitude estimated using the observed frequencies is slightly smaller than that for a solar model without overshoot.

Hence it appears that the actual sun is even smoother than the model. However, since the small amplitude makes the effect of errors in observed frequencies difficult to estimate, and since it is also difficult to remove the smooth part in an unambiguous manner, it is necessary to perform an independent analysis of the data to verify their conclusions.

For the present work we have constructed a number of solar models with varying extent of overshoot and depth of convection zone using different opacities. Their properties are summarized in table 1. All these models have the correct radius and luminosity.

Table 1. Properties of solar models

Model	Depth of discontinuity (km)	Extent of overshoot (km)	τ $(\mathrm{mHz})^{-1}$	γ (mHz)
M1	200200	0	26.409	0.01367
M2	206000	5600	26.699	0.01448
M3	211700	11200	27.014	0.01537
M4	217600	16800	27.339	0.01632
M5	223800	22400	27.656	0.01736
M6	229500	28000	27.954	0.01837
M7	193500	0	25.992	0.01270
M8	200100	6500	26.364	0.01362
M9	210100	16000	26.918	0.01512
M10	220100	25400	27.456	0.01674
M11	193200	0	26.012	0.01266
M12	211000	0	27.083	0.01526
M13	200000	5600	26.385	0.01361
M14	201300	0	26.458	0.01382

For each of these models we compute the frequencies $\nu_{n,\ell}$ for $\ell \leq 20$. The modes with higher degree ℓ are not very useful for this study since these modes do not penetrate deep enough to be affected by the discontinuity. Using the computed frequencies it is in principle possible to compute the amplitude of the oscillatory component. The main difficulty in this exercise is that for a given value of ℓ the number of modes is not sufficient to define the oscillatory curve, while it is difficult to combine modes with different values of ℓ because the smooth part of the frequency has some ℓ dependence. In order to overcome this problem we take higher differences of the frequency, this decreases the contribution due to smooth part but the oscillatory component is enhanced. Although the error in the observations get enhanced in higher differences, it is compensated by a similar increase in the amplitude of oscillations.

To compute the amplitude of the oscillatory part, we combine the data for $5 \leq \ell \leq 20$ in the frequency interval 2 mHz $\leq \nu \leq$ 3.5 mHz and remove the smooth part using a cubic spline data smoother. The modes with $\ell < 5$ have too large an error to be useful. After the smooth component is removed we perform a least squares fit to a function of the form

$$\delta^{2k}\nu = \left(a_0 + \frac{a_1}{\nu} + \frac{a_2}{\nu^2}\right)\cos\left(\tau\nu - \gamma\frac{\ell(\ell+1)}{\nu} + \phi\right) \qquad \ldots (3)$$

to determine the coefficients a_0, a_1, a_2, τ, γ and ϕ. It may be noted that although the asymptotic theory predicts that the amplitude is proportional to ν the computed frequencies show that the amplitude actually decreases with ν. In order to follow this decrease we use the above form for the amplitude. The results are shown in figure 1. For the purpose of comparison, we use the average amplitude in the given frequency interval $\nu_1 \leq \nu \leq \nu_2$. We have also obtained fits with constant amplitude (i.e. $a_1 = a_2 = 0$). While performing the fit each point is weighted according to the errors in corresponding observed frequencies.

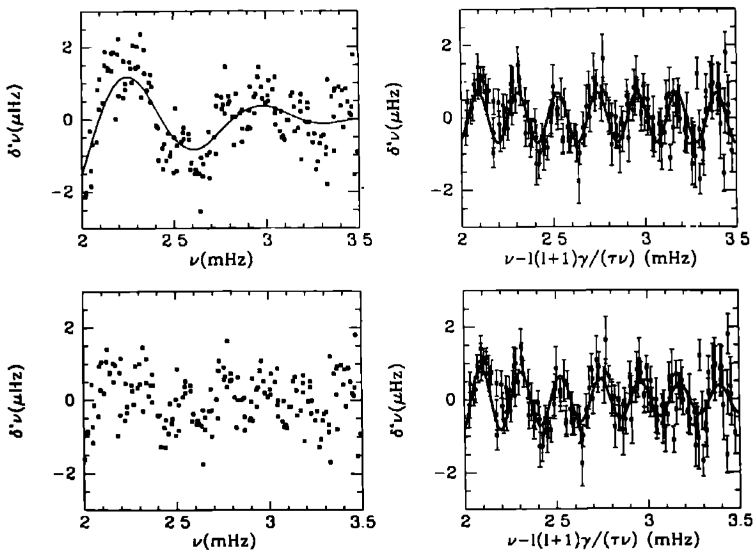

Figure 1. The fourth difference $\delta^4\nu$ for $5 \leq \ell \leq 20$ and frequency ν in the range 2-3.5 mHz for the observed frequencies. Filled squares in the top left figure represent $\delta^4\nu$ for different modes, while the curve defines the non-oscillatory part in the data which is subtracted to get the oscillatory part which is shown in the figure at the bottom left. The figures on the right side show the result of least square fit to the residuals after the smooth part is removed. In these figures the frequencies have been modified to remove the ℓ dependent shift in the oscillatory part. The top right figure shows the result with constant amplitude, while the bottom right figure shows that with variable amplitude.

In order to estimate the amplitude of oscillatory component in the solar frequencies the same treatment is applied to the observed data of Libbrecht *et al.* (1990). The errors in computed amplitude and τ can be estimated by performing a simulation using artificially generated data sets that are generated by adding errors with quoted variance to the computed model frequencies. From the distribution of amplitude in a set of 20 simulations we can estimate the mean value and the variance.

It is clear from figure 2, that the present technique is able to distinguish between models with different extent of overshoot as well as with different depths of the convection zone. The amplitude mainly depends on the extent of overshoot, apart from a slight decrease with the depth of convection zone, while τ essentially measures the depth of the adiabatic layer. The Solar data appear to be close to the model M1, which has no overshoot. The depth of convection zone in this model is the same as that inferred for the Sun by Chistensen-Dalsgaard *et al.* (1991). Nevertheless, for a given extent of overshoot the amplitude does depend on the opacity at the base of the convection zone, because the extent of discontinuity in the temperature gradient depends on the opacity gradient. Neglecting systematic errors due to uncertainties in opacity, we can put a two sigma upper limit of $0.1H_p$ on the extent of overshoot.

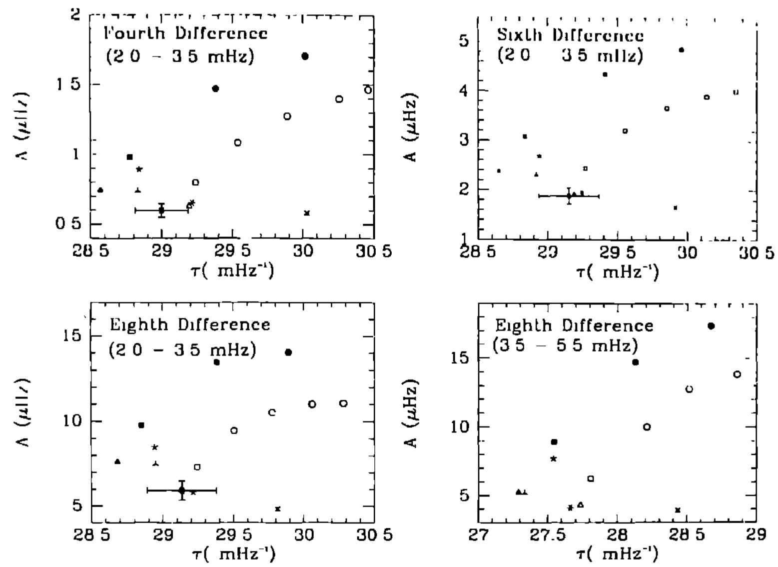

Figure 2. The amplitude A and the acoustic depth τ of the discontinuity for the different models for various differences are shown in the four figures. The different symbols are as follows : open triangle (model M1), open square (M2), open pentagon (M3), open hexagon (M4), open heptagon (M5), open octagon (M6), filled triangle (M7), filled square (M8), filled pentagon (9), filled hexagon (M10), star with three vertices (M11), cross (M12), star with five vertices (M13) and asterisk (M14). The star with error bars represents the observations. Models M1, M7, M11, M12 and M14 have no overshoot.

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A matter of gravity

Members of the ASI, Ladies and Gentlemen,

Before I begin my scheduled talk I wish to thank the organizers for inviting me for this occasion, and also to say how well they have organized this meeting. Of course, having attended the Srinagar meeting in 1989 I knew and expected the same high level of organizational skill and warmth of hospitality here too with Bhat and Razdan in command. On this occasion let me share my dream with all of you. The dream is that in a not too distant future the ASI will meet in Gulmarg with this pair welcoming us there.

To come to my talk, I am reminded of a statement by King Dushyanta to his close confidant the Vidushaka in Kalidasa's Abhijnanshakuntalam. He says : 'Parihasavijalpitam sakhe paramarthen na grihyatam vachah". "Friend! I was only joking—do not take my words seriously'. I will paraphrase this statement and say : "Friends! I am talking about a matter of gravity but you need not take my words seriously".

I was in two minds about whether or not to write down what I have to say by way of a pre-dinner speech. On the one hand writing down helps focus one's own ideas and serves to control the timing mechanism which is specially important for a pre-, rather than an after-dinner speech. In an after-dinner speech the listeners have already performed the main activity of the evening and so are in an indulgent mood to spare some time to the food for thought provided by the speaker. The pre-dinner speaker does not have that advantage and is essentially facing an impatient if not a hostile audience telling him in their own minds to get it over with fast so that they can come to the main business of the session.

Hence the need to be brief and precise. But the danger of writing up the speech is also formidable. For with his editorial predatory instinct Professor Abhyankar would be after me to publish it in the Bulletin. So, what is the problem, you may ask, if you have already written your speech? Isn't the problem with those speakers who are pressured to write what they said afterwards... after they have forgotten what they actually said on the occasion? No, in this particular situation the danger lies in my handing over the manuscript right away to Professor Abhyankar. For, every word that I have written will be transformed into rupees and eventually appear on my plate as the bill for page charges which the Director of my institution will refuse to honour!

But seriously, having weighed the two sides of the argument I decided to opt for writing up... I know how to deal with Professor Abhyankar who is a friend of long standing and with my Director whom I think I know pretty well. So here it is for what its worth.

The title suggested by the organizers rang a bell... a distant bell on my past light cone. When I was a research student at Cambridge the astronomer R. A. Lyttleton was writing a play entitled **'A Matter of Gravity'** based on the discovery of Neptune. Lyttleton's interest in the topic was astronomical as well as literary. For he had then written a paper showing an alternative and much shorter way in which the existence of Neptune could have been deduced by its original discoverers Adams and Leverrier. I do not recall if the play was ever produced but the title remained in my mind to resurface on this occasion.

Let us briefly recall what had happened regarding Neptune. The path of the newly discovered planet Uranus had shown perturbations which seemed to be real and non-trivial. Either they demanded a reexamination of the Newtonian law of gravitation or, as both Adams and independently Leverrier found, they implied the existence of a perturbing body in the vicinity. Could it be a new trans-Uranian planet?

Both Adams and Leverrier opted for the latter alternative and calculated the position of the hypothetical planet. Adams who got the answer first, appealed to Airy the Director of the Royal Greenwich Observatory and Challis, the Director of the Cambridge Observatory to look for the predicted planet. Both of them, however, ignored the request. Meanwhile, Leverrier at Paris fared no better with the directors of the leading French observatories. But he then approached the Berlin Observatory. It so happened that the Director was on a holiday to celebrate his birthday and only the junior staff was around. That is how Galle who actually took the suggestion seriously looked for and found the planet.

There is a moral in all this. If you want something done bypass the director and approach a younger man. For those who want something done at IUCAA I declare that the Director's birthday falls on July 19.

As some of you may know, the Neptune episode was described in the last issue of Khagol. We were pleasantly surprised to receive a letter from Phillip Morrison. With the permission of the Editor of **Khagol** I reproduce extracts from it :

"Bart Bok, a well-known and disarmingly friendly elderly astronomer, now gone, who often referred to himself as 'just an old telescope pusher', was long the Director of Steward Observatory on Kitt Peak, part of the University of Arizona. There in early 1969 three young astronomers, one a theorist, one an electronics man, none familiar with that observatory, deliberately searched for and at once found the optical pulses of the Crab Nebula pulsar. It was justly a sensation.

Soon a meeting took place at which the valuable discovery was reported. Bart Bok was chairman of the session. He began by saying that the three men (Cocke, Disney and Taylor) had received all the credit for this fine discovery, but that he wanted to emphasize the contribution of the Director as well. The audience grew uneasy, even embarrassed; could this be good old Bok, so anxious to claim some formal credit for himself? "Yes", he continued. "My contribution was absolutely necessary. You see, I was out of town at the time; if I had been there, I wouldn't have let such inexperienced observers near the telescope!"

So Director Bok played an essential role; being off station. More power to the young readers of Khagol; take chances!..."

During the rest of this talk I wish to air my own prejudice that I do my best to keep out of the way of the young readers and writers of Khagol...even when I am not off-station. Here it is for what its worth...

First, let us not forget that in retrospect the discovery of Neptune was, so far as I know, the only clearcut, positive and uncontroversial detection of dark matter. Whether it be the black hole in Cygnus X-1, or the flat rotation curves of spirals or the velocity distribution in a cluster of galaxies, there is always some 'if' or 'but' attached to the nature and extent of the unseen mass.

Nevertheless the current fashion for dark matter in cosmology inspired me to concoct a parody of the famous story **The Emperor's New Clothes**. My story goes something like this... with apologies to the dark matter experts present here. The principal characters in the story are the Emperor, of course, and the Great Ranee (G.R.), Official Consort (O.C.) and the Progressive Princess (P.P.). Any resemblance between these latter characters and general relativity, observational cosmology and particle physics is purely intentional.

Once upon a time there was a kingdom where a new prince was born amid great rejoicings. He took over as king in due course but was guided all the time by his mother the Great Ranee. The G.R. set very high aesthetic standards and conforming to them the king became rather remote from his subjects. But eventually he got married and his Official Consort was a very observant lady who tried and succeeded partially in bringing the king back to reality. With her help the king made large scale surveys of his raj and ensured that he was in touch with his subjects. The kingdom prospered and expanded and the king became Emperor in due course.

While things were proceeding in a smooth and steady way there came a beautiful visitor from remote lands. Progressive Princess as she was called she brought new ideas and new fashions to the kingdom. The Emperor fell prey to her charms and began to do as she dictated.

"Heavens! What outmoded clothes you are wearing. Anybody can see them. Back where I come from we wear clothes that no one can see." She said.

"But I can see what you are wearing" remarked G.R. caustically.

"What you see are my visible clothes. I am actually wearing a lot more that you can't see. In my country the status of a person is determined by the ratio of the invisible to the visible clothing he or she has. An Emperor must wear totally invisible clothes."

"Certainly, My dear. I will do that right away." The Emperor was about to throw off all his clothes when the commonsense of O.C. prevailed. She had read the Hans Andersen story. "Nonsense. Everybody will laugh at you." There was a tussle between the modern and the traditional but the P.P. had the better of the debate because she unleashed on them new jargon that the two old fashioned ladies had never heard before.

"You need these clothes because inflation demands it" she said.

"Inflation? That was a dirty word in my time", said G.R. "We learnt to balance our budgets and always managed to save."

"That is where you were wrong. Your budget balancing was an act of fine tuning. Besides, inflation is here to stay... you can't live without it." P.P. replied.

"Thanks to you not only the common man but even the royal household is threatened with an empty treasury," continued G.R. "In the good old days money was energy and we always went by the positive energy condition... now we may have to borrow to sustain your new fashions."

"Maam, you are only now realising the power of the vacuum. If there were no empty treasury there would have been no inflation. How then could you have solved your flatness and horizon problems?"

"The...what problems? We never heard of them before," confessed O.C. thoroughly bewildered.

"Let me explain. Before inflation weight reduction was a problem. Now everbody eats less and people have flat tummies. Also, earlier you could travel far and wide and noticed the economic imbalances which led to political repercussions. With rising travel costs you can't travel far and are therefore confined to your own local homogeneous regions. By so limiting your horizons you are much better off now." P.P. explained patiently.

"I see", said O.C. who really did not. She felt there was something wrong somewhere and that inflation could not possibly be beneficial. But all around her everybody was singing praises of it. Seeing their confusion P.P. pressed her advantage further by giving an elaborate argument why inflation required everyone to wear invisible clothes. Finally O.C. agreed that may be there should be some invisible clothing and that it may even exceed the ordinary visible one: but she stubbornly stuck to some minimum component of the latter.

So, a bargain was finally struck. Henceforth the Emperor would wear only ten percent of his dress in a visible form. The P.P. noted that the G.R. was hostile to her and vowed to replace her by someone Super Grand one of these days. So far as O.C. was concerned she would matter less and less anyway. But her immediate concern was to dazzle the Emperor with the latest fashions in the invisible.

She showed the Emperor hot clothes and cold clothes the likes of which he had never seen before...because they were invisible. Because the Emperor admired them so did the courtiers. How wonderful the imported clothes were. They were supposed to achieve what ordinary clothes could not.

"The Emperor should have a smooth aura about him. It gets spoilt if you wear ordinary clothes...it looks too patchy" said P.P. The Emperor bought that argument and asked for the variety of samples. This is where things began to go wrong. For P.P.'s fashion designers could not agree on what to recommend. Some wanted to sell 'hot' clothes while some made a pitch for the 'cold' ones.

"Sire, you are fond of large scale jogging... you should wear our hot clothes. These cold-wallahs cannot help you there."

"No Sire. Only the cold clothes will give your body the real scale independent shape that is so manly and regal."

"Wear hot clothes and you will feel comfortable top down, Your Majesty!"

"Your Highness, only the cold clothes worn suitably biassed near your visible clothes would give you the right appearance."

And so on and so forth. There were also middle-liners who recommended varying combinations of hot and cold clothes. Then there were others who had clothes made of strings. The Emperor was getting more and more jittery with this pandemonium wondering whether he would be able to arrive at any decision.

Meanwhile two children were watching all this, one from the Emperor's country and the other from the land of the Progressive Princess.

"Do you know what I know?" asked the latter.

"What am I supposed to know?"

"Why are all my people here trying to sell these things to your people?"

"Because they are nice and useful?"

"That is the big joke! Let me tell you a secret. We were never able to make any of these things in our own land. You see our mills are not powerful enough!"

Well, this is an ongoing story and I do not know how it is going to end. The bottom line is that in these days of market economy you can sell practically anything to anybody provided you are clever enough with your ads.

That, Ladies and Gentlemen is indeed a matter of gravity.

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